An Experimental Test of an Optimal Growth Model

By VIVIAN LEI AND CHARLES N. NOUSSAIR*

This paper describes the design and behavior of an experimental economy with the structure of the Ramsey-Cass-Koopmans model of optimal growth. The experiment includes three different implementations of the model: a decentralized implementation with multiple agents and a market for capital, a treatment where individual subjects are placed in the role of social planners, and a treatment where the social planner consists of five agents making a joint decision. The findings highlight the role of market institutions in facilitating convergence to the optimal steady state. (*JEL* C91, C92, O40)

Understanding the process of economic growth is a fundamental task of modern economics. Macroeconomists have certainly recognized the importance of questions of economic growth and have devised an impressive array of theoretical models, analyzing the relationships between current consumption, saving, and investment decisions of agents and future economic activity (see Costas Azariadis [1993] and Robert J. Barro and Xavier Sala-i-Martin [1995] for extensive surveys of growth models). One of the best-known models, and one that has provided a framework widely used in subsequent work, is the optimal growth model of F. P. Ramsey (1928), further developed by David Cass (1965) and Tjalling C. Koopmans (1965). In the model, the economy is assumed to behave like a representative agent, who can be viewed as a benevolent social planner or an "average" agent in the economy. The planner makes optimal investment and consumption decisions over an infinite time horizon, given a utility of consumption and a fixed production technology. If concave production and utility functions are assumed, there is a unique optimal steady state, toward which consumption and capital stock levels converge monotonically over time.

In this paper, we introduce an experimental design that we use to study some basic ideas of growth theory. We construct a simple economy with the structure of the optimal growth model of Ramsey-Cass-Koopmans. The version of the model we use is described in Section I. The empirical prediction tested in the experiment is whether an economy populated by human decision makers converges to the optimal steady-state level. We test the prediction under two different levels of initial endowment: *High Endowment,* in which the starting level of capital stock is greater, and *Low Endowment,* in which the starting level is lower than at the optimal steady state. Under High Endowment, the model predicts that both consumption and capital stock converge to the optimal steady state from above, whereas under Low Endowment, convergence is predicted to occur from below.

The experiment is not designed to assess whether the optimal growth model is a good description of how particular *field* economies grow, nor is it designed to simulate any national economies or the world economy. Rather, the structure of the experimental economy is specified to conform closely to the model, and to allow straightforward comparisons between the

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numerical predictions of the model and the observed data.[1] One of our treatments, the *Social Planner treatment,* is designed to correspond as closely as possible to the literal formulation of the theoretical model. In this treatment, individual agents are given the role of the social planner and a monetary incentive to maximize the discounted sum of the utility of consumption for the economy.

However, in addition to describing the behavior of a social planner, the model can also be interpreted as describing the outcomes in a decentralized economy. The optimal steady state can be supported as a competitive equilibrium by decentralized, competitive markets in which the price of capital equals its marginal product (Barro and Sala-i-Martin, 1995). In the *Market treatment* of our experiment, we explore whether the economy converges to its optimal steady state when it has a decentralized structure. The Market treatment includes two features which depart from the literal formulation of the theoretical model as a social planner's problem: (a) The economy is populated with multiple heterogeneous agents, and (b) a market for capital is present with a structure that we believed would enhance the efficient allocation of resources between investment and consumption purposes. Trade in the market follows continuous double auction rules, and these rules are known to be conducive to attaining the competitive equilibrium prices and quantities exchanged in a wide class of market environments (Vernon L. Smith, 1962).[2] Furthermore, the heterogeneity of agents ensures that gains from trade exist in the market and perhaps makes it more likely that the market would be active. Recognizing that the Market treatment has features that create relatively favorable conditions for convergence to the optimal steady state, the first hypothesis we advance for evaluation is the following:

Hypothesis 1: The decentralized economy of the Market treatment converges to its optimal steady state.

As we describe in detail in Section III, the economies of the Market treatment do have a strong tendency to converge to the optimal steady-state levels of consumption and capital stock. The price of capital also converges to the optimal steady-state level. This is true regardless of whether the initial level of capital is above or below the optimal steady-state level. Our second hypothesis is that the Social Planner treatment generates the same outcomes as the Market treatment, as predicted by the theoretical model.

Hypothesis 2: The Social Planner treatment generates outcomes that are not different from those of the Market treatment.

Our data do not support Hypothesis 2. We find that the social planner's consumption level and capital stock holdings are farther from the optimal steady state than the economies of the Market treatment. There is more variance in the consumption level over time and lower overall welfare in the Social Planner treatment. This difference indicates that the institutional structure can have an effect on outcomes even when not predicted by the model. This result is consistent with earlier research (Noussair and Kenneth Matheny, 2000), which has shown that dynamic optimization problems with the structure of the optimal growth problem studied here are very difficult for individual subjects to solve when they are placed alone in the role of the social planner or representative agent.[3] As de-

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[1] Although this paper is the first experimental study of a growth model, there is an active literature on laboratory testing of macroeconomic models. See John Duffy (1998) for a recent survey of experimental studies of monetary economics.

[2] The capital that trades in our markets has a different and more complex structure than the goods traded in previous studies, in which the competitive equilibria are observed. In previous studies, the good traded in the market typically has an exogenous value of consumption specified by the experimenter and consumption occurs at the end of the current market period. As we describe in detail in Section II, the capital traded in our markets has two possible uses, consumption and investment, so that a consumer's willingness to pay is a function of the value of both uses. Calculating that value is complicated by the fact that the value of capital used in investment depends on activity in future periods. Thus, it is a priori by no means obvious from the results of previous studies that our markets for capital will operate at or near their competitive equilibrium.

[3] Other researchers have also found that dynamic optimization problems are difficult for individual subjects to solve. See, for example, Richard H. Thaler (1981), John D.

scribed in later sections of the paper, the decentralized economy achieves outcomes closer to the optimum despite the fact that it allows several potential sources of inefficiency that the Social Planner does not. In the Market treatment, (1) production output can be inside the production possibilities frontier, (2) units produced can be consumed by agents other than those who value them most highly, (3) individual agents do not have an incentive to maximize welfare at the aggregate level, and (4) each agent has private information about her own valuations and costs.

The data from an additional treatment, the *Planning Agency treatment,* allows us to refine our conclusions. The only difference between the Planning Agency and the Social Planner treatments is that a group of five planners, rather than just one, makes the consumption and investment decisions of the economy. The Planning Agency achieves outcomes that are closer to the optimal steady state than the single Social Planner. Therefore, some of the inefficiencies of the Social Planner treatment occur because there is only a single agent.

Our findings underscore the role that markets and decentralized decision-making can have in helping an economy to attain its potential level of output. The market allows the five agents of our decentralized economy to behave as if the incentives of each of them were to maximize group welfare, they each had full information about the economy, they could consult each other before making their decisions, and they were constrained to produce along their frontier and to allocate the output they produce efficiently. The market prices appear to be an effective instrument to enable the economy to make the proper trade-off between consumption and investment.

I. The Model

In the theoretical model corresponding to our experiment, the economy behaves like a representative agent, who maximizes the present discounted value of the utility of consumption $u(c_t)$ over an infinite horizon:[4]

$$(1) \qquad \max \sum_{t=0}^{\infty} (1+\rho)^{-t} u(c_t)$$

where t indexes time period, ρ is the discount rate, c_t is consumption at time t, and $u(c_t)$ is the utility of consumption at time t. Equation (1) is maximized subject to the constraints given in equations (2) and (3):

$$(2) \quad c_t + k_{t+1} \leq f(k_t) + (1-\delta)k_t \qquad \forall t \geq 0.$$

$$(3) \qquad k_{t+1} \geq (1-\delta)k_t \qquad \forall t \geq 0.$$

Equation (2) is a resource constraint. k_t is the capital stock at time t. Depreciation of the capital stock occurs at the rate $\delta \in (0, 1]$. The capital stock at time t can be transformed, using the production function $f(k_t)$, into output, which can be consumed in period t or used to augment the next period's capital stock k_{t+1}, as in equation (2). Utility and production functions u and f are assumed to be strictly increasing, concave, and differentiable. Equation (3) rules out negative gross investment in capital stock. We also assume that the initial level of capital stock, k_0, is strictly greater than 0.

The first-order conditions of the maximization problem in (1) require:

$$(4) \quad u'(c_t) = (1+\rho)^{-1}[1-\delta + f'(k_{t+1})]u'(c_{t+1}) \qquad \forall t \geq 0$$

and the resource constraint (2) to be binding. Under the transversality condition (5) that the discounted value of period t's capital stock approaches 0 as time approaches infinity,

$$(5) \qquad \lim_{t \to \infty} (1+\rho)^{-t} u'(c_t) k_{t+1} = 0$$

there are unique steady-state values of consumption

Hey and Valentino Dardanoni (1987), Gary Gigliotti and Barry Sopher (1997), Ernst Fehr and Peter K. Zych (1998), and the survey by Colin Camerer (1995).

[4] Equations (1)–(3) are written as an optimization problem for an individual agent. However, under the assumption that the individual is a representative agent, the optimization of aggregate welfare by a social planner has the same structure.

and capital stock, $c_t = \bar{c}$ and $k_{t+1} = \bar{k}$, $\forall t \geq 0$, which satisfy:

$$\bar{c} = f(\bar{k}) - \delta\bar{k} \tag{6}$$

and

$$f'(\bar{k}) = \rho + \delta. \tag{7}$$

If the initial levels of capital stock and consumption are equal to $(\bar{c}, \bar{k})$, they will remain the same in subsequent periods. If they are not equal to $(\bar{c}, \bar{k})$, the dynamics from period t to $t + 1$ exhibit the properties that (a) for any given initial capital stock level, optimal sequences of consumption and capital stock are unique, (b) convergence to the steady state of both consumption and capital are strictly monotonic,[5] and (c) changes in the capital stock (net investment) are larger the further k_t is from the steady state.

II. The Market Treatment

A. *Parameters*

In the experiment, the economy's aggregate production capability was a discrete function approximated by the continuous production function,

$$f(K_t) = 6.96 * (K_t)^{1/2} \tag{8}$$

and the economy's aggregate marginal utility for consumption good was approximated by

$$u'(C_t) = 310 - 10C_t \tag{9}$$

corresponding to a utility function of $u(C_t) = 310C_t - 5(C_t)^2$. The approximations were chosen so that $(\bar{C}, \bar{K}) = (12, 10)$ was a solution to (6) and (7) for both the actual parameters of the experiment and the continuous functions approximating them. We set $\rho = 1/9$ and $\delta = 1$.[6]

There were two sets of parameters used. Under Low Endowment, the initial level of capital stock in the economy was $K_0 = 5$, and under High Endowment, the initial level of capital stock was $K_0 = 20$. There were no other differences between High and Low Endowment.

B. *Individual Production and Consumption*

In each period, which corresponds to a time period t in the theoretical model, each of the five subjects was endowed with a production function indicating his ability to transform capital into output. We will denote individual i's production capability as $f^i(k_t^i)$, where k_t^i is agent i's capital stock holding in period t. $f(K_t)$, where $K_t = \Sigma_i k_t^i$, is the economy's production capability, as defined in Section I. Agents could only make consumption and investment decisions in integer amounts. The production function of each agent, shown in Figure 1, remained constant for each subject from period to period. In other words, there were no exogenous shocks to production.

Each of the first five panels in Figure 1 indicates the quantity that each individual agent could produce with a given amount of input. In these panels, *Units of Input* denotes k_t^i and *Output* denotes $f^i(k_t^i)$. For example, if agent 1 used one unit of capital (k_t^i) in production in a period, he could produce seven units of output ($c_t^i + k_{t+1}^i$), as indicated in the first panel. If he used a total of two units in production in a period, he could produce eight units of output, implying a marginal product of one unit of output for the second unit of input.

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[5] Capital stock and consumption converge from the same direction toward the optimal steady state. That is, if $k_0 < \bar{k}$ then $\forall t$, $k_t < \bar{k}$ and $c_t < \bar{c}$. If $k_0 > \bar{k}$ then $\forall t$, $k_t > \bar{k}$ and $c_t > \bar{c}$.

[6] We use the notation c_t and k_t to denote the consumption and capital stock holdings of a representative agent, as in the theoretical model described in Section I. C_t and K_t denote the aggregate quantities in the Market treatment, which is the sum of the individual holdings of the five heterogeneous members of the economy. The consumption and capital stock of an individual i at time t will be denoted as c_t^i and k_t^i. Subjects make decisions only observing the total of output plus undepreciated capital stock. This means that a depreciation rate other than 1 could be used without changing the design of the experiment. The main impact of setting $\delta = 1$ is that it admits the possibility of the capital stock of an individual to fall to 0 at any time, if he consumes all of his output. Therefore, the fact that $\delta = 1$ may make it more difficult to reach the optimal positive steady state, because it permits the economy to exhaust its entire stock of capital, at which point it cannot be reaccumulated. Of course, despite the fact that $\delta = 1$, the economy remains dynamic in structure in that positive gross investment is required in every period to assure future consumption.

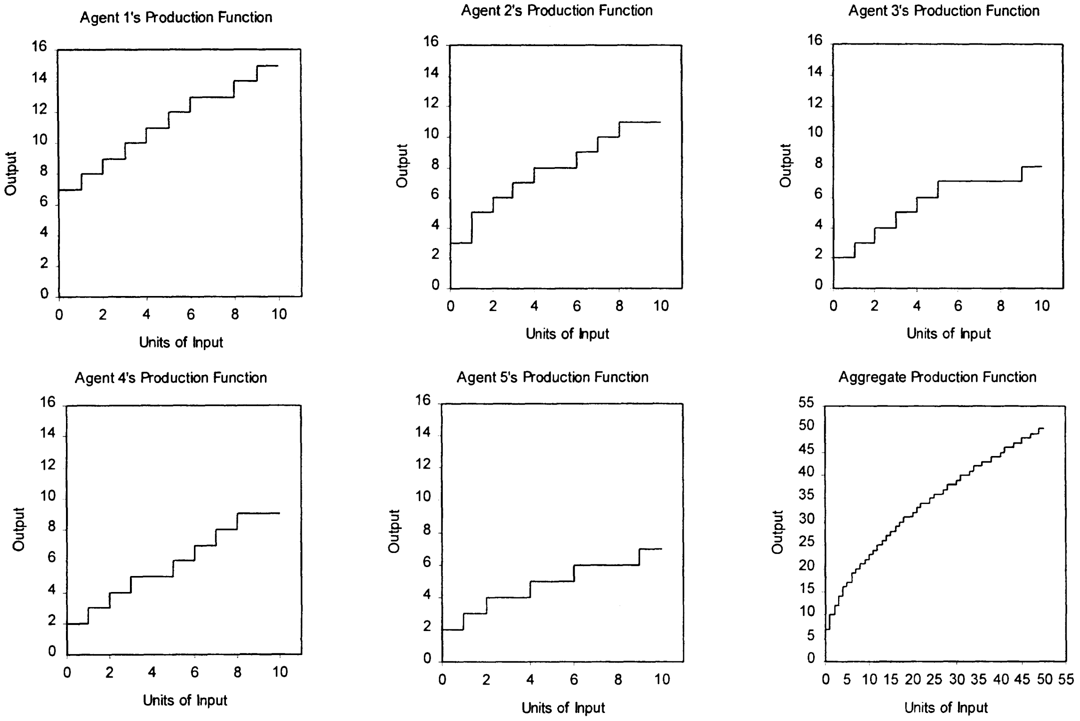

FIGURE 1. INDIVIDUAL AND AGGREGATE PRODUCTION FUNCTION

The last panel in the figure is the economywide production capability, with *Units of Input* and *Output* representing K_t and $f(K_t)$ respectively. At the economywide level, the production function was an approximation to (8). The production capability was allocated among the five agents so that the first unit held by agent 1 produced seven units of output, the first unit held by agent 2 produced three units, etc. Each agent knew only his own production function and did not know the production functions of other agents.

However, for the economy to produce the output given by $f(K_t)$, the particular agents who have the highest marginal product for capital must use their capital in production. It is therefore possible for the economy to produce well inside its production possibility frontier, and thus the constraint in (2) need not bind. Under Low Endowment, each agent was endowed with one unit of capital stock at the beginning of the time horizon for an economywide total of 5. Under High Endowment, each agent was endowed with four units of capital stock at the beginning of the time horizon so that the total initial endowment of the economy was 20 units.

The utility of consumption good c_t was expressed in terms of an experimental currency which could be converted to U.S. dollars at the end of the experiment. The conversion rate differed between agents to compensate for the higher earnings in terms of the experimental currency due to the differing production functions held by individual agents. The marginal utility of consumption of each individual i was an approximation to $U_i(c_t^i) = 300 + 10i - 50c_t^i$, implying an economywide marginal utility of consumption of approximately $U(C_t) = 310 - 10C_t$.[7] Each agent knew his own utility function, but not the utility functions of other agents. In the optimal steady state, agents 4 and

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[7] The marginal valuations, measured in terms of the experimental currency, for c_t^i were 260, 210, 160, 110, 60 and 10 for agent 1. For agent 2, the marginal values were 270, 220, 170, 120, 70, and 20; for agent 3: 280, 230, 180, 130, 80, and 30; for agent 4: 290, 240, 190, 140, 90, and 40; and for agent 5: 300, 250, 200, 150, 100, and 50.

5 each consume three units per period, and agents 1–3 each consume two units per period, for an economywide total of 12 units of consumption per period. For this pattern of consumption to occur, trade must take place in the capital market.

C. *The Market for Capital*

During each period, a computerized continuous double auction market for capital operated. The market was open for a period of time, during which potential buyers and sellers could make public offers to purchase and sell units. An offer consists of a price and a maximum quantity offered for purchase or sale. For example, a buyer may offer to purchase up to 5 units at a per unit price of 100 or a seller may offer to sell up to 3 units at a price of 300. At any time, buyers or sellers may accept offers made by agents on the other side of the market, and an acceptance of an offer means that a binding contact has occurred. Agents are not required to accept the entire quantity offered; they may accept only a portion of the total quantity offered. In this experiment, the market was computerized and used the Multiple Unit Double Auction (MUDA) computer program (see Charles R. Plott and Peter Gray [1990] for details on the operation of MUDA).

An equilibrium market price for capital can be calculated for the optimal steady state of the economy. Because capital K_{t+1} could be substituted for consumption good C_t at a rate of 1 to 1 as in equation (2), the market price for capital must be the same as the marginal utility of consumption. Since the value of an extra unit of C_t in any period is 180, the value of a unit of investment must also equal 180. Therefore the equilibrium price for capital equals 180.

D. *Timing*

There are three notions of time in the experimental design. A *period* corresponds to a time t in the theoretical model. We use the term *horizon* to refer to the entire life of an economy, that is, the entire sequence of interrelated decisions of equation (1). Finally, we use the term *session* to refer to a single day's activity in the laboratory. As described in the next two subsections, a session may include more than one horizon. We use the term *cohort* to refer to each group of five subjects, who participated together as a group in a given session. There were seven cohorts of subjects, many of whom participated as a group in more than one session.

Each session consisted of a sequence of periods. The initial period of the first horizon in which each cohort of subjects participated was for practice. It was the only period during the session which did not count toward final earnings. Each subject was endowed with 10,000 units of currency and 1 (under Low Endowment) or 4 (under High Endowment) units of capital. This currency was convertible to U.S. dollars at the end of the experiment. Purchases (sales) in the market for capital decreased (increased) this cash balance. The 10,000 units were endowed in the form of a loan from the experimenter, which had to be paid back at the end of the horizon.[8] The cash balance and capital were reinitialized to the initial level after the practice period.

Within each period of the experiment, the sequence of events was as shown in Figure 2. At the beginning of each period, production took place mapping input, k_t^i, to output (which would be allocated between k_{t+1}^i and c_t^i at the end of the period) for each participant. Operationally, the experimenter circulated among the subjects and pressed a sequence of keys on their computer terminals. This action transformed the capital held by the agents from the amount that remained at the end of the previous period to the amount available for subjects at the beginning of the current period, according to the relationship in Figure 1. Each subject had a sheet entitled Production Schedule, outlining her production capability.

For the first two (or three) minutes of a market period,[9] subjects were free to buy and sell capital in the market. The market for capital was

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[8] Loaning money to the subjects in this manner creates the possibility that subjects may lose money over the course of the experiment. However, in this study, the profits from consumption provided a sufficient degree of profit each period so that no subject had negative total earnings at the end of any session.

[9] To allow subjects some time to become familiar with the procedures of the experiment, the market phase in the first two periods of the first horizon in which a group participated lasted three minutes, and in all other periods the market phase was two minutes long.

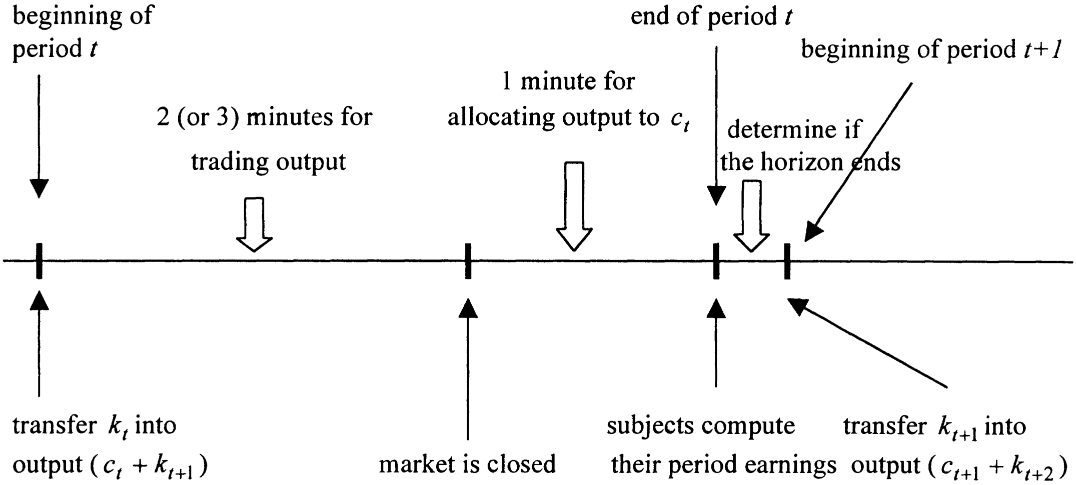

FIGURE 2. TIMING WITHIN PERIOD t

closed with one minute remaining in the period. During the last minute of each period, subjects had an opportunity to allocate any portion of their output to consumption, that is, to choose c_t^i. Through consumption, subjects were awarded a payment, which was added into their period earnings but not into the cash available for future purchases. The period ended after consumption took place. The output that was not consumed became the end of period capital stock, k_{t+1}^i, and was transformed into output for use in the next period. Profits within a period for an agent were given by his utility of consumption for the quantity of units of c_t^i he consumed, plus the change in the agent's cash balance between the beginning and the end of the period. The cash balance at the end of each period was carried over to the next period. Each subject kept the same utility function for the entire horizon.

If the session was the first in which the particular cohort of subjects participated, the sequence of activity in a session was the following: (a) When subjects first arrived at the experiment, they were given approximately 50 minutes to review an interactive tutorial about using the MUDA software. (b) The instructions of the experiment were handed out to each subject. The experimenter read through the instructions for the subjects. Subjects were allowed to ask questions anytime they wanted. (c) The experimenter transformed the initial capital stock of each subject based on his individual production function. (d) The market was opened for period 0 and subjects were able to trade with each other in the market during the first three minutes. (e) In the last minute of period 0, subjects made consumption decisions. Subjects' earnings in period 0 did not count toward their final earnings, though subjects were asked to calculate their hypothetical earnings to ensure that they understood the accounting procedure. (f) After period 0 ended, inventories of cash and capital were both reinitialized to their starting values. (g) Period 1 and subsequent periods proceeded in the same way as described in (c)–(e), except that their earnings in the period did count toward their final U.S. dollar earnings. After period one, the cash and capital stock holdings were not reinitialized for the remainder of the horizon.

If the session represented a continuation of a previous session, the tutorial was not conducted. However, the instructions for the experiment were read. The practice period was skipped and all periods counted toward subjects' earnings. The initial values of capital stock and cash holdings were set at the values of the end of the previous session in which subjects participated. As an illustration, the timing of activity for cohort MktH1 is shown in Figure 3.

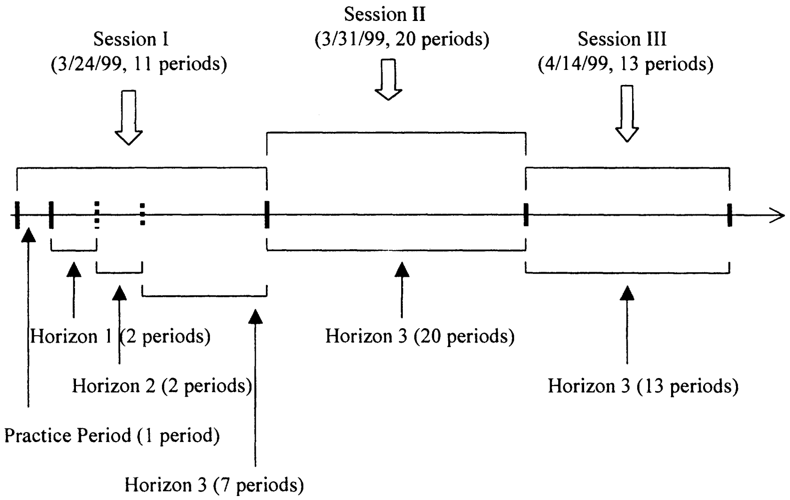

FIGURE 3. ACTIVITY OF COHORT MktH1

E. *Implementing the Infinite Horizon*

To capture the incentive structure of the infinite time horizon in the optimal growth model, we adopted a random ending rule to determine the end of the horizons. To implement the random ending rule, the experimenter rolled a 20-sided die after each period, beginning in period 1, to determine if the horizon would continue. If the die showed numbers 1 or 2, the horizon ended immediately. Otherwise, the experiment continued to the next period within the same horizon. The 10-percent probability of ending implies a $\rho = 1/9$. The infinite-horizon maximization problem described in (1)–(3) is identical when there is a constant probability equal to $\rho/(1 + \rho)$ of the horizon terminating in each period and no discounting of the utility of consumption from period to period.[10]

Each session was scheduled for three hours. If a horizon ended less than one hour before the scheduled end of a session, the session was immediately terminated. If a horizon ended more than one hour before the scheduled end of the session, a new horizon began with the same group, and with the same initial capital stock as the initial level of capital stock in the previous horizon.[11] This meant that any given individual participated only in Low Endowment or only in High Endowment economies.

If the horizon did not terminate before the scheduled session ending time, the horizon continued where it left off during another session. Subjects were offered the opportunity to return for the next session. If a subject chose to return she would resume her previous role, reclaiming her previous utility and production functions. If a subject chose not to return, a substitute would be recruited to take her place. The original subject would also be awarded the amount of earnings made by her substitute. This procedure preserved the incentive for all subjects to make optimal decisions in each period in accordance with the theoretical model.[12] Thus the experi-

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[10] The equivalence of a random ending rule such as the one we use and an infinite horizon requires risk neutrality of agents. If agents are risk averse they would overconsume under a random ending rule, because they would underweight the future uncertain payoff relative to risk-neutral agents. Equivalence of the two decision situations also requires subjects to believe that the random draws that determine termination are independent and identically distributed from period to period. The use of a random ending rule to create an infinite-horizon decision situation with discounting has been used in previous experimental studies. See for example Camerer and Keith Weigelt (1993) or Noussair and Matheny (2000).

[11] Reinitializing in this manner does not affect the optimal solution to the optimization problem in (1), because the probability of a restart is completely independent of any subject's decisions.

[12] One drawback of using the same subjects for multiple sessions is that they may communicate between the ses-

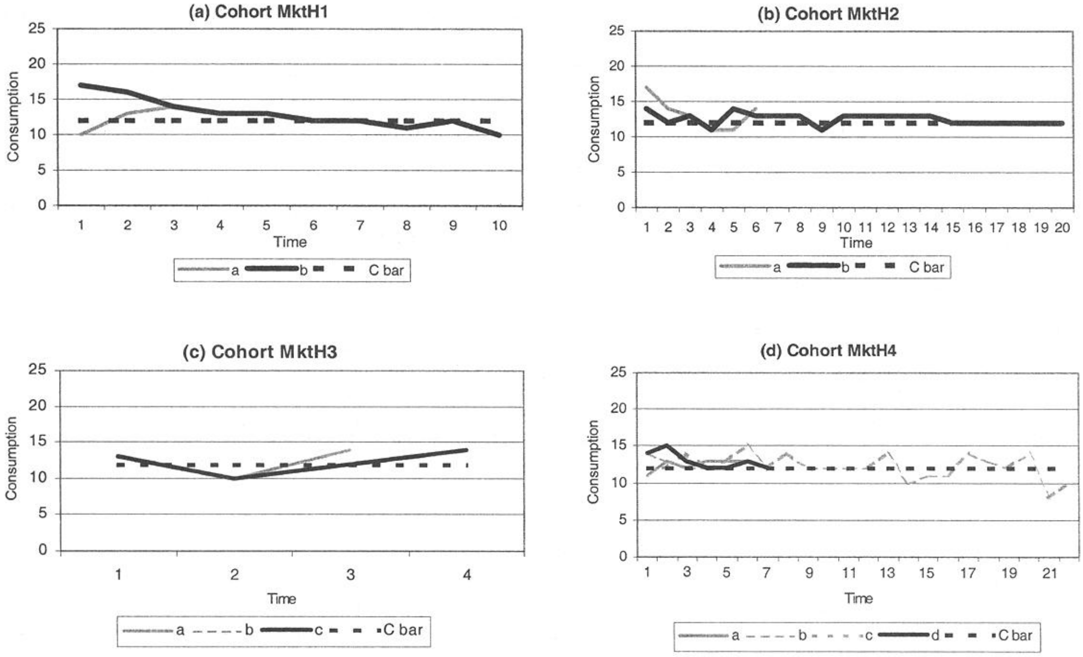

Figure 4. Time Series of Consumption: Market Treatment, All Horizons, High Endowment

Notes: (a), (b), (c), and (d) denote the first, second, third, and fourth horizons that a cohort participated in. C bar denotes the consumption level in the steady-state equilibrium.

menter paid out the substitute's earnings twice, once to the substitute himself and once to the original subject for whom he substituted. Substitutes were recruited from the same subject pool as other members of the group. The substitutes were required to arrive early for the sessions and go through the tutorial in the use of the software.

F. *The Available Data*

Thirteen sessions comprised the Market treatment. All sessions were conducted at Purdue University. None of the subjects had ever participated in a similar experiment before, though some of them had previous experience with the same computer program in other types of experiments. Each of the 13 sessions lasted between two and three hours. There were seven cohorts of subjects. Cohorts MktL2 and MktH3 consisted of graduate students in Management at Purdue University. The other five cohorts consisted of undergraduate students recruited from introductory-level courses in economics at Purdue University.[13]

III. Results from the Market Treatment

Figures 4(a)–(d) illustrate the time path of consumption in all horizons in which each of the four cohorts in the High Endowment treatment participated. Figures 5(a)–(c) show

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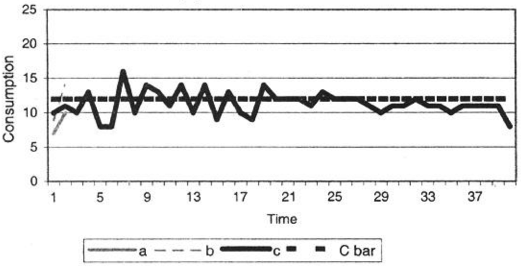
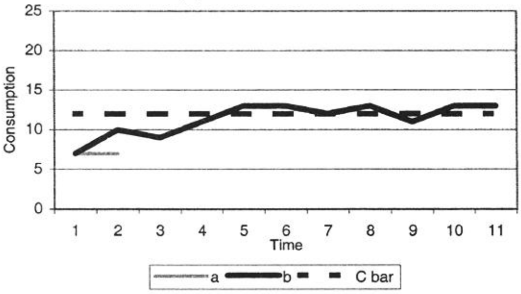
sions, despite our requests that they not do so. However, we felt that the risk was necessary to preserve the incentives of the infinite horizon. Because we did not notice sudden changes in decisions immediately after subjects returned for a second or a third session, we are confident that our results are not due to any communication between subjects between sessions.

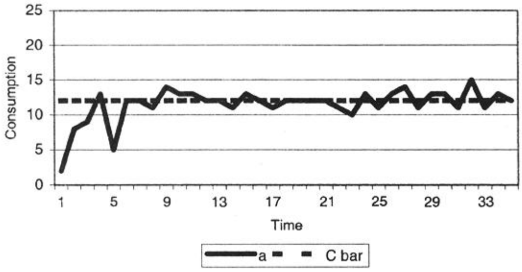
[13] We were quite surprised by the high percentage of subjects who preferred to return for another session, even though they knew that they would be paid the earnings achieved by their substitutes if they did not return.



(a) Cohort MktL1



(b) Cohort MktL2



(c) Cohort MktL3

FIGURE 5. TIME SERIES OF CONSUMPTION: MARKET TREATMENT, ALL HORIZONS, LOW ENDOWMENT

Notes: (a), (b), and (c) denote the first, second, and third horizons that a cohort participated in. C bar denotes the consumption level in the steady-state equilibrium.

analogous data for the Low Endowment treatment.[14] The impression given by the figures is that after the group gains experience with the decision situation, consumption is very close to the optimal steady-state level.

The following linear regression model can be used to estimate the level of consumption toward which any convergence over time is taking place:[15]

$$(10) \qquad C_t^{jm} = \beta_{11}\,\frac{D_1}{t} + \cdots + \beta_{1J}\,\frac{D_J}{t} + \beta_2\,\frac{t-1}{t} + \varepsilon_t^{jm}.$$

In the above equation, C_t^{jm} denotes the economy's consumption level in period t of the mth horizon in which cohort j participated. D_j is a dummy variable for cohort j and t denotes time period within a horizon. For example, D_j equals 1 if the data are generated by cohort j. $t = 1$ in the first period of any horizon, not only in the first one in a session nor only the first horizon in which a given group participates. The model allows for the estimation of the value of the dependent variable at the beginning of each horizon and the value to which the series is converging. In the first period of a horizon populated by cohort j the variable $D_j/t = 1$ and all of the other variables equal 0. Therefore, β_{11} is the estimated value of the time series at the beginning of a horizon populated by cohort 1. The variables D_j/t and coefficients β_{1j} are analogous. The specification assumes that there is a common point of origin for each horizon in which each group participates. For later periods within a horizon of cohort j the D_j/t term decreases toward 0, while the variable $(t - 1)/t$ increases toward 1. If t were projected to the infinite future, $(t - 1)/t$ would converge to 1. Therefore β_2 can be interpreted as the asymptote to which the time series is converging. The specification assumes that there is a common value to which the time series is converging for all horizons and for all groups. In the estimation the complete data from all periods in all horizons is used. We will say that we cannot reject the hypothesis that a variable converges to its optimal steady-state value if the estimated β_2 is not significantly different from that value.

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[14] In the experiment, production and consumption were restricted to integer amounts. A shooting algorithm can be used to compute optimal sequences of capital and consumption without the integer restriction. The algorithm is similar to one used by Robert G. King and Sergio T. Rebelo (1989, 1993). Rounded to the nearest integer, the sequence of consumption levels under High Endowment is 17, 15, 13, 13, 12 for periods 1–5, and 12 thereafter. Under Low Endowment the sequence is 9, 10, 11, 12 in periods 1–4 and 12 thereafter.

[15] This model of convergence was first used by Noussair et al. (1995).

TABLE 1—ESTIMATES OF MODEL OF CONVERGENCE, MARKET TREATMENT, HIGH ENDOWMENT

	β_{1MktH1}	β_{1MktH2}	β_{1MktH3}	β_{1MktH4}	β_2	Model prediction	Rho	Average (periods 6 and later)
Consumption	14.22	14.99	12.85	13.23	12.10	12	0.0851	12.24
(C_t)	(0.84)	(0.83)	(0.74)	(0.60)	(0.22)			(1.32)
$\lvert C_t - \bar{C} \rvert$	3.49	2.99	1.53	1.22	0.74	0	0.1923	0.93
	(0.61)	(0.61)	(0.55)	(0.43)	(0.17)			(0.96)
Capital stock	20.44	15.78	15.92	17.46	11.42	10	0.5956	9.86
(K_{t+1})	(2.04)	(1.36)	(1.37)	(0.94)	(0.51)			(2.07)
Price of K	192.59	166.70	269.84	178.33	182.74	180	0.6345	178.83
(P_t)	(26.00)	(17.05)	(17.45)	(11.91)	(6.70)			(10.55)
Realized $u(C_t)$ as	0.951	1.119	0.922	1.077	0.979	1	0.1695	0.991
percent of optimum	(0.058)	(0.057)	(0.052)	(0.041)	(0.016)			(0.090)

Notes: Standard errors are in parentheses. $N = 87$.

TABLE 2—ESTIMATES OF MODEL OF CONVERGENCE, MARKET TREATMENT, LOW ENDOWMENT

	β_{1MktL1}	β_{1MktL2}	β_{1MktL3}	β_2	Model prediction	Rho	Average (periods 6 and later)
Consumption	8.99	6.74	3.36	12.15	12	−0.1385	11.83
(C_t)	(0.76)	(0.92)	(1.25)	(0.19)			(1.49)
$\lvert C_t - \bar{C} \rvert$	3.69	4.78	9.03	0.85	0	0.2578	1.13
	(0.69)	(0.82)	(1.08)	(0.19)			(0.98)
Capital stock	9.57	7.63	11.72	10.74	10	0.7561	10.35
(K_{t+1})	(1.51)	(1.81)	(2.44)	(0.72)			(4.05)
Price of K	321.92	226.51	512.52	173.76	180	0.3415	178.80
(P_t)	(27.31)	(32.55)	(42.68)	(7.80)			(30.31)
Realized $u(C_t)$ as	0.694	0.591	0.220	0.989	1	−0.1269	0.960
percent of optimum	(0.051)	(0.062)	(0.084)	(0.013)			(0.092)

Notes: Standard errors are in parentheses. $N = 92$.

A. *Consumption*

The model predicts that consumption converges over time toward a value of 12 and a variance of zero. The average per period consumption level, averaged over all periods 6 and greater (to exclude observations that occur early in the horizon, when they would be far away from the optimal steady state even along the optimal trajectory) for all horizons in the treatment, is given in the rightmost column of Tables 1 and 2. The estimates from the regression model for consumption at time t are included in the tables[16] for High and Low Endowment respectively, in the rows labeled Consumption. The standard errors of the estimates are in parentheses.

The estimated values of β_2 for economywide consumption are 12.10 and 12.15 for High and Low Endowment, respectively. The optimal steady-state level of consumption of 12 lies well within a 95-percent confidence interval of the estimated β_2 for both treatments. Therefore, we

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[16] All estimates in Tables 1–6 are feasible GLS estimates from panel data models with the data from each cohort comprising one panel. The panel data format is appropriate since each cohort generates an independent time series from identical initial conditions. The error is assumed to arise from the randomness that exists in experimental markets due to noise in the actions of agents. Our recording technology allows us to assume that there are no errors in measurement of the variables themselves. The estimates assume first-order autocorrelation with a parameter ρ that is common to all cohorts. The assumption of first-order autocorrelation is natural in market experiments of this type, since activity in one period influences decisions in the following period.

cannot reject the hypothesis that the consumption level is converging to the optimal steady-state level. The convergence occurs whether or not the initial value of capital stock is above or below the optimal steady-state level. Under High Endowment, the estimated level of consumption at the beginning of each horizon, the β_{1j} term, is greater than the optimal steady-state level of 12. In every horizon of Low Endowment, the estimated initial value is less than 12. Thus, for all seven groups, we observe convergence to the optimal steady-state level of consumption from the predicted direction. The averages in both treatments are also very close to 12. The regression confirms the visual impression from Figures 4 and 5.

The model is also estimated for the dependent variable $|C_t - \bar{C}|$, the absolute deviation of consumption from the optimal steady-state level. The model predicts that $|C_t - \bar{C}|$ converges to zero. The estimates of the absolute deviations are included in Tables 1 and 2. For both High and Low Endowment, β_2 is smaller than any of the β_{1j} estimates, indicating decreasing variation over time even as period consumption converges to 12. However, both β_2 estimates are significantly different from 0, (0.74 and 0.85 in High and Low Endowment, respectively), so there remains some tendency, even asymptotically, for consumption to fluctuate though it is on average no different from the predicted level.

B. *Capital Stock and Prices*

Tables 1 and 2 also contain the estimates for capital stock. In both treatments, the averages 9.86 and 10.35 are close to 10. Under High Endowment, the estimated value to which capital stock levels are converging, 11.42, is significantly different from the optimal steady-state level of 10. The estimate of 10.74 for Low Endowment is not different from 10. For each of the four High Endowment cohorts, the estimated values for the beginning of the time series are all greater than β_2, reflecting a depletion of capital stock levels over time as predicted in the theoretical model. Under Low Endowment the estimated capital stock at the beginning of two of the sessions is below the optimal steady-state level, as predicted by the model. In the remaining session, it is above the optimal steady-state level, reflecting a high level of investment in the early periods.

The tables also contain the results of a similar estimation for the average price of capital by period. The estimation shows that the price of capital converges to the optimal steady state. The estimated values of β_2 are 182.74 and 173.76 for High and Low Endowment, respectively. Neither is significantly different from the equilibrium price of capital in the optimal steady state, 180. Under Low Endowment, in all three groups, the prices converge to the optimal steady-state level from above as predicted. However, under High Endowment, we do not find that the price of capital converges to the optimal steady-state level from below.

Closer inspection of the capital market sheds light on the ability of the economy to converge over time to the optimal steady state. It appears that the prices established in the market for capital provide signals that induce the economy to allocate resources between consumption and investment in a way that pushes it toward the optimal steady state. In the last horizon that each group of subjects participated in, the correlations between P_t, the average transaction price in period t, and subsequent net investment, $K_{t+1} - K_t$, were 0.52 for the Low Endowment data and 0.08 for the High Endowment data. The positive correlations indicate that the higher the price of capital, the more positive was net investment immediately following the closing of the market. The correlation between P_t and C_t, consumption in period t, is -0.54 for Low Endowment and -0.13 for High Endowment. This indicates that consumption increases after the price of capital falls (consumption in period t occurs after the market closes for period t). Each of the four correlations is significantly different from zero at the 5-percent level of significance.

C. *Coordination of Production and Consumption Activity Among Agents*

The ability of the economy to attain the optimal steady state is all the more impressive when one considers that for the economy to produce along its production possibility frontier, a nontrivial coordinating function has to be performed by the economy. To attain the frontier, at the end of trading in the market and the

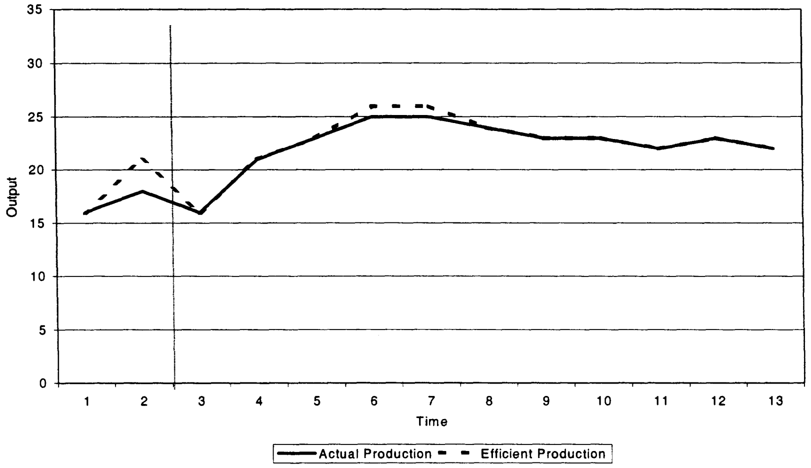

FIGURE 6. ACTUAL PRODUCTION VS. EFFICIENT PRODUCTION FOR COHORT MktL2

consumption phase, the capital stock must be held by those agents who have the highest marginal product of capital. A typical time series of Actual Production vs. Efficient Production is shown in Figure 6, which illustrates the two time series for the data from cohort MktL2. The Efficient Production is the production that would result if the economy's units of capital were reallocated to the agents who had the highest marginal product of capital, so that the economy would achieve the highest feasible level of output given its current stock of capital. The first horizon, which lasted only two periods, shows some inefficiency, as in period 2 the capital stock in the economy could have produced 21 units of output $C_t + K_{t+1}$ if the appropriate agents held it. However only 18 units were produced. In the second horizon of the session, the actual production was one unit below the frontier in the fourth and fifth periods of the horizon, but was along the frontier in all later periods.

The actual production was a very high percentage of the optimal level for all seven cohorts, indicating that the economies tended to produce along their frontiers. The actual production averaged 98.8, 98.3, and 99.2 percent of the efficient production level for the three Low Endowment cohorts. It averaged 97.2, 99.7, 100, and 99.5 percent for the four High Endowment cohorts, for an overall cohort average of 99.0. One percent of potential production was lost from being at the interior of the production possibilities frontier. In 10.3 percent of the periods of High Endowment, production was suboptimal in the sense that a reallocation of capital could have increased output. Under Low Endowment, the corresponding figure was 19.6 percent.

From the individual consumption data, the *Consumption Efficiency* (a measure of welfare of the economy) can be calculated. In the optimal steady state, the total earnings from consumption for the five agents in the economy are 2,940 units of experimental currency. We measure the efficiency of the economy by calculating the realized earnings from consumption each period and dividing them by 2,940. In the optimal steady state the level of efficiency is 1 (or 100 percent). It is an imperfect measure of welfare for our economies, because efficiency can be greater than 1 if suboptimal overconsumption occurs. However, consumption efficiency less than 1 late in a horizon indicates clear suboptimality.

The average efficiency levels in periods 6 and later were 99.1 percent under High and 96.0 percent under Low Endowment. Using the convergence model of equation (10), we can estimate the consumption efficiency level to which the economy is converging. The estimates, shown in Tables 1 and 2, in the row labeled Realized $u(C_t)$ as percent of optimum, indicate that the data are converging to 0.979 and 0.989

in the High and Low Endowment treatments respectively. These levels not significantly different from 1. The economies are converging to full consumption efficiency. Not only does output tend to be produced by those with the highest marginal products, and total output tend to converge to the optimal steady-state quantity, but individual-level consumption tends to be realized by those with the highest marginal utilities. The efficiency loss from suboptimal consumption of actual production, which could be eliminated by the transfer of units to the agents with the highest marginal products, averaged 3.6 percent of the total realized $u(C_t)$.

IV. The Social Planner Treatment

In this section we consider the role that the departures from the literal formulation of the theoretical model that were included in the Market treatment played in guiding the economy to its optimum. We compare the outcomes generated in the Market treatment to the outcomes that result when individual subjects are placed in the role of the social planner, and are given a monetary incentive to maximize the objective function given in equation (1), subject to the constraints (2) and (3). In this treatment, called the *Social Planner* treatment, we try to reproduce the literal formulation of the model as closely as possible.

In the Social Planner treatment, individual subjects were endowed with either 5 (for subjects with Low Endowment) or 20 (for subjects with High Endowment) units of capital stock at the beginning of each time horizon. Each individual was endowed with the entire economy's production technology $f(K_t)$ and the economy's entire utility function $u(C_t)$. The actual discrete values used for production and consumption were identical to those shown in Figure 1 and listed in footnote 7. There were eight subjects in the Social Planner treatment, four under High and four under Low Endowment.[17] At no time did any of these subjects interact with or observe decisions made by any other participants.

The sequence of activities within each period was similar to what has been described in Section II, subsection D. However, since there was no market for exchanging capital between subjects, the procedure was simplified and did not require computerization. At the beginning of period t, production took place mapping current capital stock, K_t, into output, $C_t + K_{t+1}$. Subjects produced by filling out a form with the value of $f(K_t)$, which they could determine from their Production Schedules. This had the effect of guaranteeing that the economy in the Social Planner treatment always produced along its production possibility frontier because it forced constraint (2) to bind. The experimenter then circulated among the subjects and verified that they had written down the correct quantity of output. Subjects then had three minutes to decide how to allocate the output between consumption C_t and end-of-period capital stock K_{t+1}. A subject's period earnings were equal to the cash award that he received from consumption; that is, earnings were proportional to $u(C_t)$. Under High (Low) Endowment the conversion rate from experimental currency to U.S. dollars was 3,500 (2,500) = 1 dollar. The period ended after the three minutes had elapsed. To determine if the horizon would continue, we used the random ending rule described in Section II, subsection E.

In sessions that were the first in which the subjects participated, the sequence of activity in a session was the following: (a) The instructions for the experiment were handed out to the subjects. The experimenter read through the instructions. Subjects were permitted to ask questions as the instructions were being read. (b) The experimenter transformed the subject's initial capital stock into output. (c) The subject was given three minutes to allocate his output between consumption and end-of-period capital stock for a practice period (period 0), which did not count toward his final earnings. (d) After the end of period 0, the inventories of capital stock were reinitialized to the starting values of either

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[17] At first glance, using four subjects might appear to be too small a sample size. However, unlike in the Market treatment, each subject is an independent economy in the Social Planner treatment, so that the number of independent observations equals the number of agents. Furthermore, the data conformed to our priors, which were based on the results reported by Noussair and Matheny (2000), who studied 65 similar economies. After eight observations, we had confidence that the patterns we were observing would be confirmed had we gathered more data.

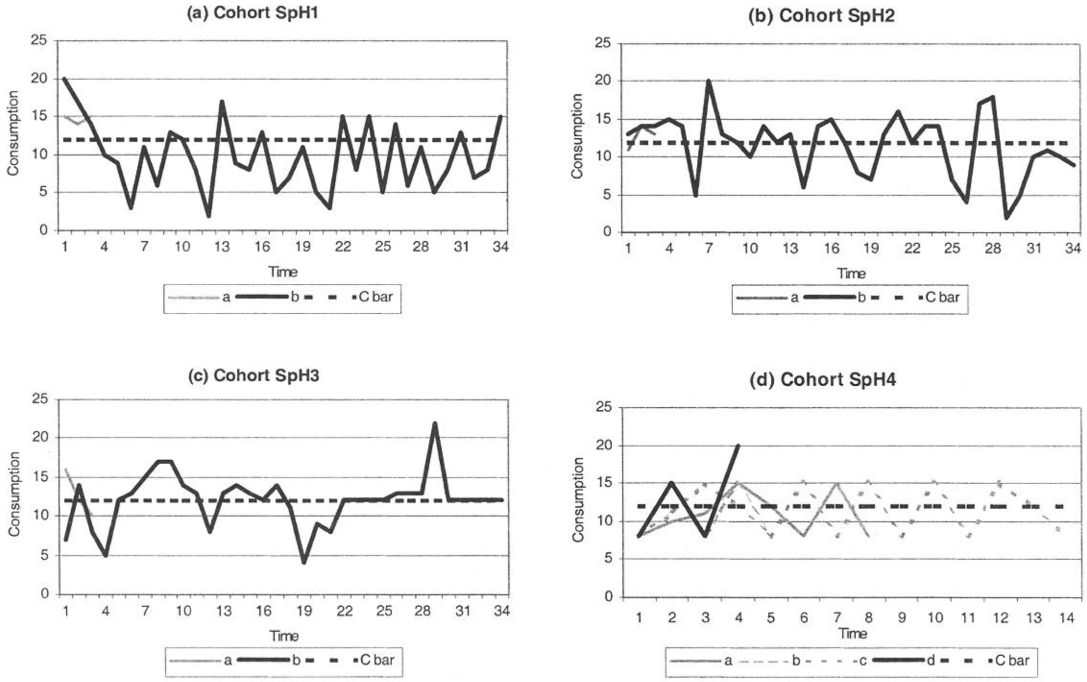

FIGURE 7. TIME SERIES OF CONSUMPTION: SOCIAL PLANNER TREATMENT, ALL HORIZONS, HIGH ENDOWMENT

Notes: (a), (b), (c), and (d) denote the first, second, third, and fourth horizons that a cohort participated in. C bar denotes the consumption level in the steady-state equilibrium.

5 or 20. (e) Period 1 and subsequent periods proceeded in a similar manner as period 0, except that the subject's earnings starting from period 1 did count toward his final earnings, and that the capital stock was not reinitialized for the remainder of the horizon.

Figures 7(a)–(d) show the consumption data from all horizons for subjects in the Social Planner treatment with High Endowment. Figures 8(a)–(d) graph analogous data for Low Endowment. From the figures one gains the impression that, with the exception of one subject in the Low Endowment treatment, subjects' consumption decisions tend to exhibit greater absolute deviations from the optimal steady-state level of consumption than under the Market treatment. There are frequent large changes in consumption from period to period. In general, the data resemble those reported by Noussair and Matheny (2000).[18] The difference between the observed consumption and the optimal steady-state level suggests that the overall level of welfare in the Social Planner treatment

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[18] In the Noussair and Matheny study, subjects were given the role of social planners in a similar manner to the Social Planner treatment. The production functions used in the study were $f(K_t) = 25.23K_t^{0.2}$ and $f(K_t) = 0.884K_t^{0.9}$. Under the first production function, predicted convergence to the optimal steady state is faster than under the second. δ was equal to 0.5. Subjects make decisions for 20 "infinite" horizons, but were not required to spend a minimum amount of time on each decision. They were required to spend a minimum of 75 minutes on the 20 horizons. Subjects averaged about 25 seconds per decision. There was no tendency to smooth out consumption. Rather, consumption was characterized by bouts of overconsumption followed by bouts of underconsumption, as in the Social Planner treatment data given here, and as illustrated in Figures 7 and 8. Efficiency averaged 83.6 percent in treatments comparable to those of this paper. We consider as comparable treatments those using the same subject pool (Purdue undergraduates) and using the same random ending rule to implement the infinite horizon. In the Noussair and Matheny study, the results were similar if a fixed ending rule, in which the horizon was certain to terminate after ten periods, was used. The results also replicated in a different subject pool, undergraduate students at Waseda University in Tokyo, Japan (see Fumihiko Hiruma and Noussair [1998] for a detailed analysis).

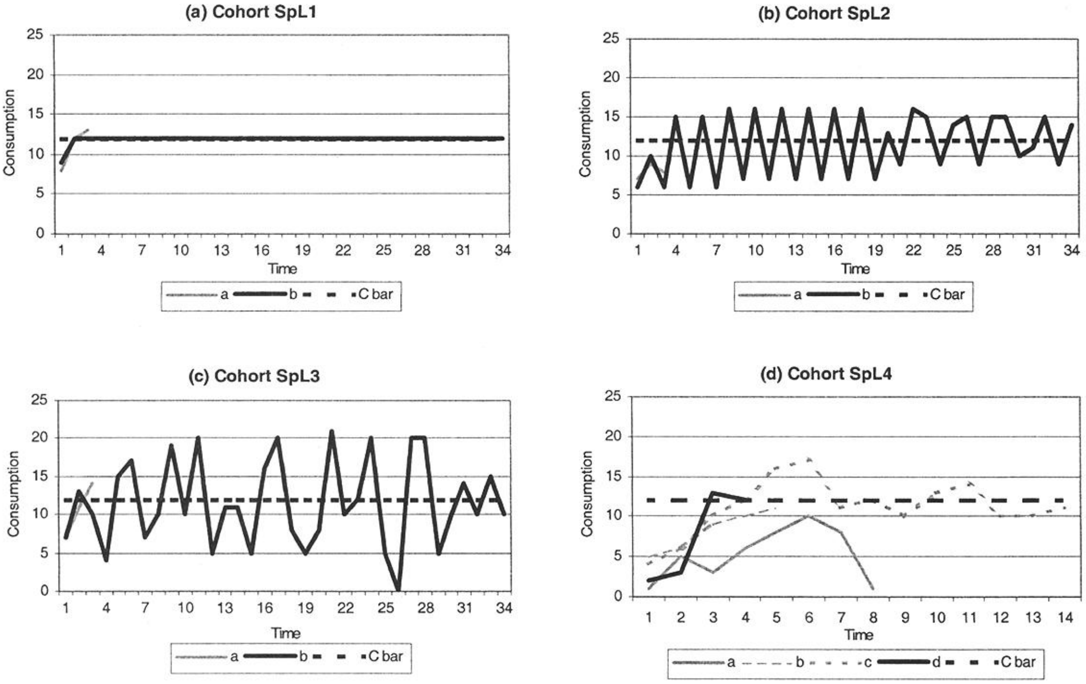


FIGURE 8. TIME SERIES OF CONSUMPTION: SOCIAL PLANNER TREATMENT, ALL HORIZONS, LOW ENDOWMENT

Notes: (a), (b), (c), and (d) denote the first, second, third, and fourth horizons that a cohort participated in. C bar denotes the consumption level in the steady-state equilibrium.

is lower than in the data from the Market treatment.

Tables 3 and 4 show the averages of consumption, absolute deviation of consumption from the optimum, capital stock, and efficiency for the Social Planner treatment under both High and Low Endowment. The data in the tables confirm the impression that the Social Planner data is more volatile than the Market data. The last column of the tables, which indicates the overall average value of the variable from period 6 onward, shows a larger standard deviation for every comparable variable in both treatments (eight of eight variables). The volatility is also apparent in the average and the estimated asymptote of $|C_t - \bar{C}|$, which is much larger than in Market, indicating greater variance of consumption under both High and Low Endowment, even asymptotically, in Social Planner.

The estimated coefficients of the model of convergence are given in Tables 3 and 4. As before, β_{1j} equals the initial value for each economy, and the β_2 terms are the estimated asymptotes of the time series. In Table 3, the estimates for High Endowment are given. Three of the four variables converge to values significantly different from the predicted levels. The absolute deviations of consumption from the optimum are significantly different from 0 and much larger than in the Market treatment, and the realized consumption efficiency of the economies is substantially and significantly below 100 percent. The estimates suggest that, even though the economy had on average a capital stock equal to the optimal steady-state level, average consumption could not be sustained at the optimal level, due to the large fluctuations in consumption and investment from period to period. The low and variable consumption is reflected in low efficiency estimates.

The estimates from the Low Endowment data, displayed in Table 4, exhibit a somewhat different pattern. Consumption is estimated to be converging to close to the optimal steady-state level but does so on a level of capital stock

TABLE 3—ESTIMATES OF MODEL OF CONVERGENCE, SOCIAL PLANNER TREATMENT, HIGH ENDOWMENT

	β_{1SpH1}	β_{1SpH2}	β_{1SpH3}	β_{1SpH4}	β_2	Model prediction	Rho	Average (periods 6 and later)
Consumption	16.12	14.24	12.39	10.21	10.95	12	−0.0430	10.96
(C_t)	(2.18)	(2.18)	(2.18)	(1.54)	(0.39)			(4.05)
$\lvert C_t - \bar{C}\rvert$	5.48	1.17	3.21	3.57	3.04	0	0.1167	3.20
	(1.51)	(1.51)	(1.51)	(1.06)	(0.28)			(2.66)
Capital stock	13.86	17.18	18.44	16.29	11.84	10	0.7062	10.29
(K_{t+1})	(3.17)	(3.17)	(3.17)	(2.02)	(0.98)			(6.54)
Realized $u(C_t)$ as	1.196	1.111	0.997	1.122	0.913	1	0.0883	0.905
percent of optimum	(0.170)	(0.170)	(0.170)	(0.119)	(0.031)			(0.275)

Notes: Standard errors are in parentheses. $N = 142$.

TABLE 4—ESTIMATES OF MODEL OF CONVERGENCE, SOCIAL PLANNER TREATMENT, LOW ENDOWMENT

	β_{1SpL1}	β_{1SpL2}	β_{1SpL3}	β_{1SpL4}	β_2	Model prediction	Rho	Average (periods 6 and later)
Consumption	9.86	5.71	8.38	1.68	12.19	12	−0.1981	11.79
(C_t)	(1.83)	(1.83)	(1.83)	(1.30)	(0.32)			(3.99)
$\lvert C_t - \bar{C}\rvert$	2.67	6.03	6.14	8.26	2.35	0	0.4531	2.82
	(1.80)	(1.80)	(1.80)	(1.19)	(0.39)			(2.82)
Capital stock	7.66	9.94	10.50	8.53	14.80	10	0.5747	13.06
(K_{t+1})	(3.65)	(3.65)	(3.65)	(2.36)	(0.90)			(6.35)
Realized $u(C_t)$ as	0.876	0.559	0.719	0.227	0.989	1	−0.1617	0.960
percent of optimum	(0.125)	(0.125)	(0.125)	(0.089)	(0.022)			(0.260)

Notes: Standard errors are in parentheses. $N = 142$.

that is too high. The absolute deviations of consumption from the optimal steady-state level are large and significant. The fluctuations in consumption mean that to sustain an average level of consumption at the optimal steady-state level, the amount of capital required is greater than the optimal level.

In the Market treatment, the β_2 estimate is closer to the optimal steady-state level than all but one β_{1j} estimate for all four dependent variables, C_t, K_{t+1}, $u(C_t)$, and $\lvert C_t - \bar{C}\rvert$ (27 out of 28 estimates, 7 β_{1k} terms * 4 dependent variables). However, in the Social Planner treatment, the β_2 estimate is closer than the corresponding β_{1j} in 25 of 32 cases. Convergence is more reliable in the Market treatment than in Social Planner. This is consistent with a comparison of Figures 4 and 5 with Figures 7 and 8. Figures 4 and 5 give the impression of smooth convergence to a greater extent than Figures 7 and 8.

V. A Planning Agency

One explanation for the differences between the Market and Social Planner treatments is that the Market treatment is more likely to find the optimum simply because there are five agents rather than one, so that there is on average five times the cognitive capacity present. Furthermore, since the Market treatment allows agents to observe some of the actions of others, good decisions can be imitated by other agents.

To study the effect of the addition of multiple decision makers to the Social Planner treatment we include a third treatment that we call the Planning Agency treatment. This treatment closely resembles the Social Planner treatment except that a group of five people has the role of the planner. Each of the five participants is informed of the aggregate utility and production functions of the economy and receives a dollar payment proportional to the realized value of

$u(C_t)$, the economywide utility level. Thus, the incentives are perfectly aligned; each of the five members has an incentive to maximize total welfare.

There were six new cohorts recruited for this treatment. Three cohorts participated in Low and three participated in High Endowment conditions. The procedures were identical to the Social Planner treatment except for the following differences. In each period, four of the five members of the economy were required to propose an allocation of resources to investment and consumption, given the current capital stock level. The proposals were collected by the experimenter who delivered them to the fifth participant, who was designated as the "spokesperson" for that period. This participant chose C_t and K_{t+1} on behalf of all members of the economy, by writing the choice on a form and submitting it to the experimenter. She was free to incorporate or ignore the proposals of the other agents. After consumption and investment were chosen, the experimenter recorded the choice on the blackboard, so that the spokesperson could record the earnings of the group. Afterwards, four subjects were invited to propose consumption and investment for the following period. The role of decision maker rotated among the five agents so that a given agent played that role in every fifth period. When making decisions, subjects could observe on the blackboard the history of previous consumption and capital stock levels in the economy. Sessions lasted between two and three hours.

This particular information structure was chosen over alternatives where there was more or less explicit communication so that the interaction between subjects was at a similar level of depth as in the Market treatment. The nature of the institutions does not allow us to hold the level of communication constant between the Market and the Planning Agency treatments, but the structure we chose appeared to be reasonably close to the level of communication that occurs through market prices. For example, allowing unrestricted communication would be excessive, while rotating the role of planner and allowing him only to observe decisions of the previous periods' planners would be insufficient.

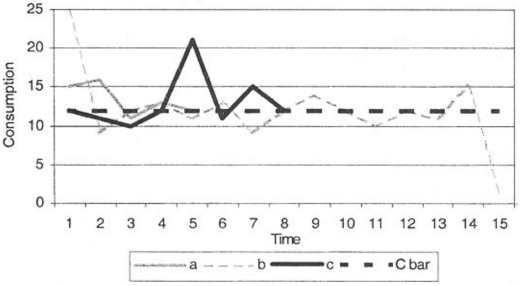
The only differences between the Planning Agency and the Social Planner treatment is the presence of multiple decision makers, and any differences between the outcome variables in the two treatments is due to having one versus five agents. Differences between the Market and the Planning Agency treatments may be due to several factors, but not to a different number of agents. The Planning Agency treatment has the same advantages that the Social Planner has over the Market treatment: alignment of incentives of all agents, constraints prohibiting production inside the frontier, constraints guaranteeing consumption by the highest valued agents, and the absence of private information.

Figures 9 and 10 show the time series of consumption in the Planning Agency treatment. The choices appear to be on average close to the optimal steady-state level of 12, but show more variance than in the Market treatment. The estimates from the regression model (10) are shown in Tables 5 and 6. The estimated β_2's are 11.97 for High and 12.34 for Low Endowment. Neither is significantly different from 12. In both equations, β_2 is closer to 12 than any of the β_{1j} terms. Convergence occurs from the predicted direction for all six cohorts.

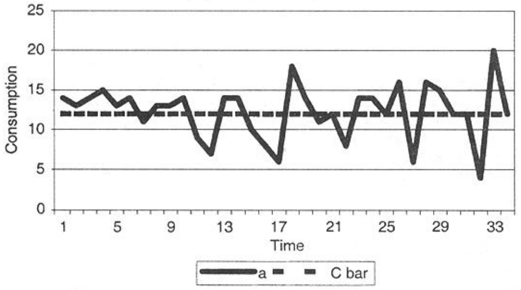
Though the quantity of consumption is converging toward the optimal steady state, there is more variance in consumption than in the Market treatment. The estimated asymptotic values of $|C_t - \bar{C}|$ equal 2.53 and 1.52, greater than in the corresponding values in Market, but less than those in Social Planner. Overall, under Planning Agency, the standard deviations of all eight variables in periods 6 and greater are lower than in Social Planner. In seven of eight instances they are higher in Planning Agency than in Market. Planning Agency exhibits smaller fluctuations than Social Planner but larger fluctuations than Market.

The tables also show that the capital stock and efficiency levels are also converging to the optimal steady-state values. Whatever inefficiency that is present is due to the fluctuations of consumption and capital stock around the optimal steady state. In contrast, the Market treatment exhibits less efficiency loss due to fluctuations in overall consumption and capital stock, but instead has losses due to suboptimal allocation of production and consumption among agents.

(a) Cohort PaH1



(b) Cohort PaH2



(c) Cohort PaH3

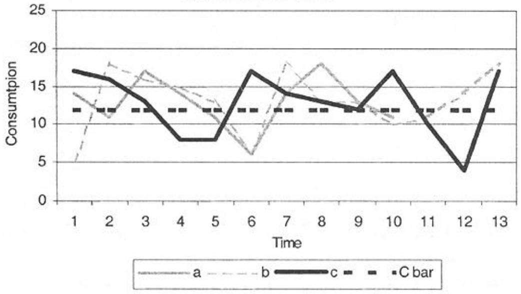
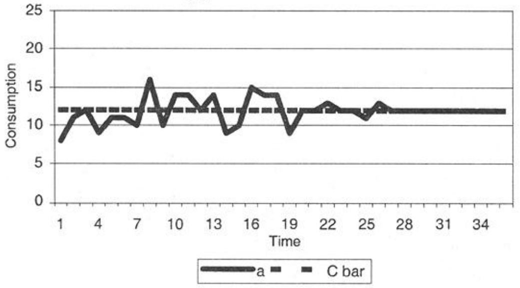


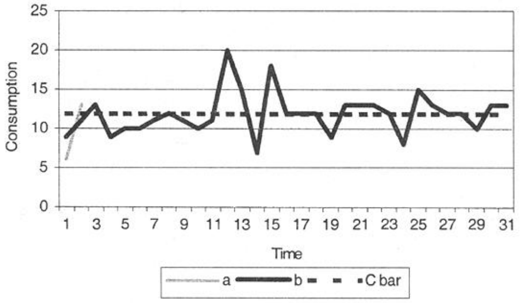
FIGURE 9. TIME SERIES OF CONSUMPTION: PLANNING AGENCY TREATMENT, ALL HORIZONS, HIGH ENDOWMENT

Notes: (a), (b), and (c) denote the first, second, and third horizons that a cohort participated in. C bar denotes the consumption level in the steady-state equilibrium.

(a) Cohort PaL1



(b) Cohort PaL2



(c) Cohort PaL3

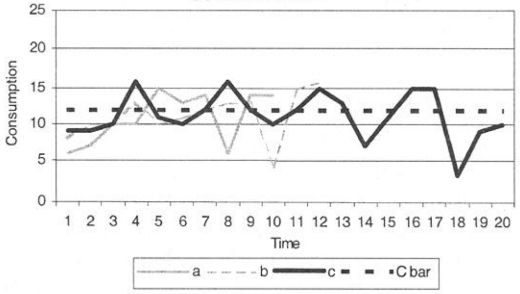


FIGURE 10. TIME SERIES OF CONSUMPTION: PLANNING AGENCY TREATMENT, ALL HORIZONS, LOW ENDOWMENT

Notes: (a), (b), and (c) denote the first, second, and third horizons that a cohort participated in. C bar denotes the consumption level in the steady-state equilibrium.

VI. Discussion

In the Market treatment, we observe a strong tendency for the variables in the economy—consumption, capital stock, the price of capital, and the realized utility of consumption—to evolve to the optimal steady-state levels. There is some variation in these variables from period to period, as one might expect in an economy in which five agents must coordinate their decisions every period. However, the model performs remarkably well in describing the state toward which the economy is converging over time.

The experiment provides an example of the role that institutions, particularly market institutions, can play in enabling an economy to allocate its resources efficiently. Welfare is higher and departures from the optimal steady state are smaller in the Market treatment than in the Social Planner treatment. The difference exists despite some inherent advantages for the Social Planner. The planner is aware of the

TABLE 5—ESTIMATES OF MODEL OF CONVERGENCE, PLANNING AGENCY TREATMENT, HIGH ENDOWMENT

	β_{1PaH1}	β_{1PaH2}	β_{1PaH3}	β_2	Model prediction	Rho	Average (periods 6 and later)
Consumption	15.16	14.70	14.32	11.97	12	−0.1367	12.14
(C_t)	(1.62)	(2.76)	(1.63)	(0.45)			(3.77)
$\lvert C_t - \bar{C} \rvert$	3.65	1.50	4.86	2.53	0	0.0402	2.81
	(1.24)	(2.09)	(1.23)	(0.35)			(2.49)
Capital stock	11.90	16.66	20.29	9.70	10	0.4940	10.03
(K_{t+1})	(1.96)	(3.54)	(1.92)	(0.71)			(3.48)
Realized $u(C_t)$ as	1.134	1.172	1.101	0.979	1	−0.1245	0.985
percent of optimum	(0.104)	(0.176)	(0.104)	(0.029)			(0.252)

Notes: Standard errors are in parentheses. $N = 98$.

TABLE 6—ESTIMATES OF MODEL OF CONVERGENCE, PLANNING AGENCY TREATMENT, LOW ENDOWMENT

	β_{1PaL1}	β_{1PaL2}	β_{1PaL3}	β_2	Model prediction	Rho	Average (periods 6 and later)
Consumption	8.33	8.49	7.54	12.34	12	−0.1264	12.01
(C_t)	(1.85)	(1.36)	(1.09)	(0.27)			(2.65)
$\lvert C_t - \bar{C} \rvert$	3.13	4.29	4.36	1.52	0	0.2505	1.77
	(1.64)	(1.25)	(0.95)	(0.26)			(1.95)
Capital stock	7.92	9.05	5.10	10.45	10	0.7682	9.71
(K_{t+1})	(3.18)	(2.33)	(1.57)	(0.89)			(3.89)
Realized $u(C_t)$ as	0.760	0.755	0.687	1.012	1	−0.1415	0.989
percent of optimum	(0.121)	(0.088)	(0.071)	(0.017)			(0.174)

Notes: Standard errors are in parentheses. $N = 111$.

aggregate structure of the economy, whereas the market treatment has only market activity to reveal the information. The planner is exogenously constrained to produce on its frontier and allocate consumption efficiently, whereas the market must coordinate production and consumption in a decentralized manner.

The Planning Agency treatment also leads to a substantial improvement over the Social Planner. The economies in the Planning Agency treatment converge to the optimal steady state in terms of average consumption, capital stock, and efficiency. However, these variables exhibit more variance than in the Market treatment. While there is some efficiency loss because of this volatility, the Planning Agency generally attains as high a level of welfare as Market, because it has the same inherent advantages over the Market treatment as the Social Planner does.

How do the economies of the Market treatment manage to allocate resources efficiently between consumption and investment, in a decentralized setting in which each agent knows only his own production and utility functions? It appears that the existence of a price for capital encourages agents to make better trade-offs between consumption and investment. The market price converges to a level at which the marginal utility of using capital for consumption and for investment is equated. The market price appears to serve as an informative signal of scarcity. When the price is higher (lower) than the optimal steady-state level, capital stock tends to rise (fall).

The heterogeneity of agents may also enhance the operation of the capital market, because it implies potential gains from trading capital in each period. This creates an incentive to use the market for capital, and leads to the establishment of a competitive equilibrium market price, which in turn facilitates optimal decision making on the part of agents. Had our agents all been identical, the incentive to use the market would have been weaker, much of the activity in the markets might have been due to mistakes on the part of subjects, and prices might have failed to stabilize at the competitive level. In that case we may not have observed convergence to the optimal steady state.[19]

Another feature of our economies that may promote convergence is the global concavity of the production function. This ensures that convergence toward the optimal steady state is always predicted, for any positive level of current capital stock. Early errors in decision-making do not prevent the economy from converging to the optimum later on. Suppose that there is an initial stage of the experiment in which subjects make mistakes as they learn about the decision environment, but that as individuals acquire more experience in the experiment, they make better decisions.[20] If subjects begin to make optimal decisions at any time, the economy is predicted to converge to the optimal steady-state level from that point on, regardless of previous history.

Future experiments can be conducted that relax the concavity assumption on the production function. In particular, if the production function includes a region in which increasing returns are present, multiple locally optimal steady states can exist, with different basins of attraction. The current paper introduces a type of experimental economy that will converge to an optimal steady state when it is unique. However, to which, if any, steady states will the economy converge if it has multiple locally optimal steady states? Will it converge to the predicted steady state given its initial endowment? Will it always converge to the steady state with the highest levels of consumption and capital stock? Experimental work has shown that there are normal form games where play converges to Pareto-dominated equilibria (see, for example, Russell W. Cooper et al. [1990] or John B. Van Huyck et al. [1990]). It is possible that economies organized as Social Planners or Planning Agencies may actually be more conducive to optimal equilibrium selection than those organized like our Market treatment. It may be easier for a planner or team of planners to switch from a suboptimal equilibrium to a better one, than it would be for multiple agents in a decentralized economy to recoordinate on the better equilibrium.

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[19] See Alan P. Kirman (1992, p. 134) for a discussion of interpreting a representative agent as an aggregation of heterogeneous individuals. He suggests that assuming that multiple heterogeneous agents populate the economy circumvents many of the theoretical contradictions that arise under the representative-agent assumption, and at the same time is intuitively more appealing as a descriptive model. He writes "Given the arguments presented here (...) it is clear that the representative agent should have no future. Indeed, contrary to what current macroeconomic practice seems to suggest, requiring heterogeneity of agents within the competitive general-equilibrium model may help to recover aggregate properties, which may be useful for macroeconomic analysis."

[20] See Plott (1996) for a detailed discussion of stages of rationality in economic experiments.

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