

# Banking Competition and Market Efficiency

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This paper analyses competition among financial intermediaries in a set-up where financial intermediaries economize on duplicated monitoring (Diamond (1984)). We analyse two different games in which both direct trade and indirect trade are available. We show that in general equilibrium outcomes are inefficient, so that Diamond's efficiency result is fragile. Intermediation may increase rather than decrease transaction costs. Disintermediation may also be an equilibrium. We discuss the role played by the nonconvexities of banks' technology and that played by competition for deposits and loans.

## I. INTRODUCTION

Many recent contributions to the theory of financial intermediation argue that banking intermediation is the most efficient way to solve incentive problems that typically affect the credit market. In the seminal papers by Diamond (1984) and Williamson (1986), intermediation emerges as a result of transaction-cost economies in monitoring the quality of projects. In these papers, the superiority of banks is established under the assumptions that banks make zero profits and fully exploit the returns to scale arising from transaction-cost economies. These assumptions are supposed to be satisfied whenever there is perfect competition in the financial intermediation sector. For this reason we refer to the corresponding solution as the competitive outcome. This paper raises the question of whether an explicit analysis of competition in the credit market yields the competitive outcome so that one can justify the Diamond and Williamson analyses. This is an important issue because any departure from the competitive solution may affect the **size** of the gains from intermediation as well as the distribution of these gains among the agents. Possibly, even the conclusion that intermediation is superior to market exchange may depend on the outcome of competition.

There are two reasons why the competitive outcome need not obtain. First, there are increasing returns to scale in intermediation and we know from the literature on imperfect competition that we should not expect perfectly competitive outcomes in this context. Second, competition among intermediaries takes place in two interrelated markets: the market for deposits and the market for loans. In such a situation, we know from Stahl (1988) and from Yanelle (1989) that competition may lead the intermediaries to corner one of the two markets in an attempt to achieve a monopoly outcome.

To test the validity of the perfect competition assumption, in this paper we explicitly analyse competition among intermediaries. We model it as a noncooperative game. As in Diamond's paper, the basic model allows for economies of scale that banks can best exploit. However, intermediaries now compete "à la Bertrand" both in deposit rates and in loan rates. Moreover, we allow borrowers and lenders to contract directly with each

other if they prefer to do so. We analyse two different versions of the game depending on whether intermediaries compete first for loans or for deposits. While it seems more natural to have banks competing first for deposits and then for loans, the reverse situation cannot be excluded, in particular when the quantity of loans is relatively scarce. The theoretical reason for looking at different games is that in general the outcome of “double-sided” Bertrand competition is sensitive to slight changes in the specification of the game (Stahl (1988), and Yanelle (1989)). Hence, it is important to know which conclusions are robust to these changes. This setting allows us to deal with three fundamental issues:

- (i) are the outcomes of the price-setting games among intermediaries competitive?
- (ii) when does intermediation emerge in equilibrium?
- (iii) what are the effects of intermediation on direct trade?

One can summarize the main results concerning these issues as follows. Both games have multiple subgame perfect equilibria which need not include the competitive outcome and also need not involve activity by banks. In almost all equilibria in which banks are active, their profits are strictly positive and the solutions are not efficient. The multiplicity of solutions is due to nonconvexities which are generated by transaction costs. In this environment, the usual Bertrand undercutting argument is not valid. Borrowers may be reluctant to raise funds at the lowest price if they anticipate that few other borrowers will apply to the same bank. Indeed, if only a few borrowers react to the price cut, the deviant bank goes into default and its clients are rationed. For this reason it may be unattractive for a borrower to ask for a low price. His best response depends not only on the price offers but also on the other clients' behaviour.<sup>1</sup>

The same type of coordination problem arises in both games and is responsible for the multiplicity of equilibria. However, the two equilibrium sets do not coincide. When banks compete for funds as well as for loans, they have an incentive to corner one of the two markets in an attempt to achieve a monopoly position. Competition becomes asymmetric, being much fiercer in the market banks try to corner. In Game 1, banks want to corner the market for loans, whereas in Game 2, they want to corner the market for funds. Consequently prices differ in the two games.

The multiplicity of equilibria raises the question of their robustness. The problem we face is one of equilibrium selection in coordination games with several strict Pareto-ranked Nash equilibria. At first glance, the Pareto-dominant equilibrium may look attractive as a focal point. However, it is contested by many recent experiments as well as by several new approaches to equilibrium selection. These analyses underline the relevance of strategic risk considerations—relative to payoff-dominance considerations—in coordination games with many agents.<sup>2</sup> Recent experiments, e.g. by Van Huyck *et al.* (1990, 1991), show that players often fail to coordinate on the Pareto-dominant equilibrium. These empirical results are corroborated by the theoretical results on global games (Carlsson and van Damme (1993)) and those on evolutionary processes. In  $2 \times 2$  games of common interest, Carlsson and van Damme as well as the evolutionary models select the risk-dominant equilibrium rather than the payoff-dominant one. While a generalization of this result to many-actions/many-agents games raises difficulties, the available results do not deny the important role played by risk considerations.

1. This is similar to the multiplicity of fulfilled expectations equilibria in Katz and Shapiro's (1985) analysis of positive network externalities.

2. “Strategic uncertainty” denotes the uncertainty that players face about how other players will respond to the multiplicity of equilibria.

In order to take these developments into account, we proceed as follows. For each game, we first characterize the whole set of Subgame Perfect Nash Equilibria (SPE). Then, we focus on “Coalition-Proof Nash Equilibria” (CPE).<sup>3</sup> This amounts to selecting Pareto-dominant equilibria in each subgame. Finally, we want to test the robustness of CPE to strategic-risk considerations. To that end, we ask whether the equilibrium strategies are evolutionary stable (ESS) or not.

Coalition-proofness solves the coordination problems described above, and restores the uniqueness of the equilibrium as well as the Bertrand undercutting argument. Hence, one might expect the CPE to be competitive. We show that this conjecture is not valid in general. The CPE may, but need not, correspond to the competitive equilibrium. The reason is that banks still have an incentive to corner one of the markets. In Game 2, the prospects of monopoly rents in the loan market give banks an incentive to offer deposit rates that exceed the competitive rate. Moreover, in Game 1 where the competitive outcome is a CPE, the strategies associated with it are not evolutionary stable (ESS).<sup>4</sup>

More precisely, our results are as follows. When banks compete first for loans and then for deposits, (Game 1), “coalition-proofness” does yield the competitive outcome. However, the equilibrium strategies are *not* ESS. Although attractive, the competitive solution is very “risky” for buyers. At the competitive price, buyers face the risk of being rationed whenever some buyer fails to coordinate on the right strategy. This risk decreases with the buying price, so that there is a real trade-off between the higher payoff associated with the competitive strategy and its greater risk. When banks compete first for deposits and then for loans, (Game 2), the CPE is *not* competitive: intermediaries’ buying and selling prices both exceed the market clearing price. Intermediation itself need not occur in equilibrium, and its occurrence need not result from its superiority. Moreover, the CPE is robust in the sense that it does not conflict with ESS.

These results show that intermediation affects the working of competition. Price competition among intermediaries does not in general yield the competitive outcome. Therefore it does not provide a justification for the perfect competition hypothesis that is common in the literature on financial markets. Moreover our results raise the question of whether competition among intermediaries is desirable or not. When the competitive pressure forces banks to offer high deposit rates, as in Game 2, transaction costs may *increase* rather than decrease with intermediation. (This is so because transaction costs are endogenous and increase with deposit rates.) This even occurs in an equilibrium where banks are inactive because their very presence in the market affects the terms of disintermediated relations. Hence, competition among intermediaries may eliminate the potential benefits from intermediation.

The rest of the paper is organized as follows. Section II presents the underlying model. Section III presents and analyses the game in which intermediaries compete first for projects and then for funds (Game 1). Section IV looks at the opposite move order (Game 2). Section V discusses the welfare implications of bank competition. Section VI concludes.

## II. THE MODEL

We consider a simplified credit market with three kinds of participants: borrowers, lenders and banks. There are  $N$  borrowers (or firms). Each borrower has an indivisible investment

3. Like the notion of strong Nash equilibrium, that of coalition-proof equilibrium requires stability against deviations by every conceivable coalition. The difference between the two notions is that coalition-proofness only considers self-enforcing deviations.

4. While ESS cannot select among strict Nash equilibria for infinite populations, it can discriminate among them when finite populations are considered.

project for which he needs outside funds of size one. The (gross) return of each project is nonstochastic and equal to  $\bar{Y}$ . There are  $(mL)$  lenders. Each lender has available wealth of size  $1/m$  which he seeks to invest. He can either deposit his money in a bank or co-finance the project of an entrepreneur. Moreover he has an outside option which yields a safe return of  $\bar{R}$  per dollar invested. Besides borrowers and lenders there are  $B$  banks. Banks act as intermediaries collecting funds from private lenders and investing these funds in the projects of entrepreneurs. Banks have no wealth of their own.<sup>5</sup>

We assume that enough funds are available in the market to finance all investment projects, i.e.  $L > N$ , and that all projects are worth financing, i.e.  $\bar{Y} > \bar{R}$ .

*Remark.* We adopt a deterministic, complete information framework because this allows us to concentrate on the credit/deposit-game.

### *The transaction costs*

We assume that trade is costly, and that two different transaction technologies are available. These technologies are intended to mimic different ways of overcoming incentive problems that typically arise in a world with uncertainty and asymmetric information. Therefore, in order to justify our assumptions on transaction costs and to describe the two transition technologies in more verbal terms, we assume in this paragraph that we are in a world “à la Diamond (1984)”, with uncertainty and asymmetric information. We will then turn back to our deterministic, complete information framework.

The first technology allows creditors to eliminate asymmetric information against a fixed positive monitoring cost “ $a$ ”. This technology has the important property that it involves duplication of costs. When different creditors co-finance the same debtor, they all need to spend “ $a$ ” on monitoring in order to obtain the information. Consequently this technology is very expensive when the number of co-financiers is large as is the case for direct trade when “ $m$ ” is big and for indirect trade when banks have many small depositors.

In contrast with the first technology, the second technology does not eliminate the information asymmetry. Rather, it allows creditors to offer incentive compatible contracts. In particular, it solves the incentive problems associated with debt contracts by imposing a per dollar penalty “ $b$ ” on the debtors who do not repay their debt entirely. The per dollar penalty “ $b$ ” is a function of borrowers’ indebtedness,  $D$ ,  $D = R\bar{L}$ , where  $R$  is the (gross) interest rate to creditors, and  $\bar{L}$ , the quantity borrowed. Hence, “ $b$ ” depends on the terms of trade. Moreover, “ $b$ ” is a function of  $\bar{X}$ , the number of projects that borrowers invest in. This assumption does not really concern firms, because firms cannot undertake more than one project. However, it is critical for banks.  $\bar{X}$  then corresponds to the number of firms that banks finance. This number is endogenous and can take any value between 0 and  $N$ . More formally, we assume that “ $b$ ” satisfies Assumption 1.

### *Assumption 1.*

$$\frac{\partial b(D, \bar{X})}{\partial D} > 0, \quad \frac{\partial^2 b(D, \bar{X})}{\partial D^2} > 0,$$

$$b(R1, 1) - b(R2, 2) > b(R2, 2) - b(R3, 3) > \dots > b(R(N-1), (N-1)) - b(RN, N) > 0,$$

5. In the sequel, we will use as synonymous debtors or borrowers, entrepreneurs or firms, and banks or intermediaries.

for all  $R$

$$\frac{\partial b(D, \bar{X})}{\partial (D/\bar{X})} > 0.$$

Assumption 1 is a natural assumption when firms' projects are stochastic, with identical and independent distributions and privately observed returns. Then, we know from Diamond (1984) that the optimal incentive compatible contract is a Modified Standard Debt Contract, i.e. a Standard Debt Contract with penalties that just cover the difference between the face value of the debt and the amount that the debtor actually pays back. The per dollar expected penalties associated with such contracts satisfy Assumption 1. They increase with the face value of the debt, i.e. with  $D$ : a higher face value makes a debtor more likely to fail, and increases the size of the penalties in the case of bankruptcy. The latter effect explains why the second-order partial derivative is positive. They also exhibit diversification economies. A bank which finances many i.i.d. projects is more able to pay the deposit rate  $R$  than a bank which finances only a few projects. (Its revenue is more concentrated.) It fails less often and therefore the penalties associated with each debt contract are lower. (We assume here that the diversification gains decrease with the "size" of the bank, i.e. with the number of projects it finances.) The per-dollar expected penalties associated with a Modified Standard Debt Contract also increase when banks' indebtedness increases more than the number of projects they finance. This is because the burden of the debt that each project supports increases.<sup>6</sup>

We can now compare the transaction costs associated with the two transaction technologies. When a firm or a bank borrows a dollar from (at least) " $m$ " different lenders, duplication costs are very high so that debt contracting is cheaper than monitoring Assumption 2(i). Consequently, direct finance as well as deposit contracts take the form of debt contracts. In contrast, when a firm borrows from a unique bank, information costs are not duplicated and monitoring is cheaper Assumption 2(ii). Consequently, banks monitor the firms they finance. Finally, diversification economies are large enough to make indirect trade "superior" to direct trade Assumption 2(iii).<sup>7</sup>

*Assumption 2.*

$$\forall R, \text{ s.t. } \bar{R} \leq R \leq \bar{Y}:$$

$$(i) \quad b(R, 1) < ma,$$

$$(ii) \quad a < b(R, 1),$$

$$(iii) \quad a + b(RN, N) < b(R, 1).$$

Note that banks' superiority crucially depends on two variables:  $R$ , the deposit rate of return they offer to lenders, and  $\bar{X}$ , the number of projects that they finance. In a competitive framework both  $R$  and  $\bar{X}$  are endogenous so that the superiority of indirect finance itself depends on the outcome of competition between banks, lenders and firms.

6. An alternative way of formalizing the same idea is as follows:  $b(D_2, \bar{X}_2) - b(D_1, \bar{X}_1) > 0$  if  $D_2/(D_1 > \bar{X}_2/(\bar{X}_1 > 1)$ .

7. For simplicity we assume here that  $\bar{L} = \bar{X}$ , rather than  $\bar{L} \geq \bar{X}$ .

The last assumption guarantees that direct finance is profitable:

*Assumption 3.*

$$\bar{R} + b(\bar{R}, 1) < \bar{Y}.$$

Having so justified the form of the two transaction technologies, we can turn back to our deterministic, complete information framework. By doing that, we take for granted that direct finance involves a cost " $b(R, 1)$ " per dollar traded, whereas indirect finance involves both a fixed cost " $a$ " per project that a bank finances and a cost " $b(R\bar{X}, \bar{X})$ " per deposit it borrows.

We now analyse **two** different games that model competition among banks, lenders and firms. In Game 1, banks first compete for investment projects, and then for the funds necessary to finance these projects. In Game 2, banks first compete for funds and then for investment projects. As we show below, the two games have different outcomes with different qualitative properties.

### III. COMPETITION FOR PROJECTS DETERMINES COMPETITION FOR DEPOSITS

#### III.1. *Specification of Game 1*

In this version of the credit game, banks first compete for entrepreneurs' projects and then for lenders' funds. When looking for funds, banks also face competition from the entrepreneurs who have rejected banks' credit offers at stage 1 and who try now to obtain credit directly from lenders. More precisely we consider the following game:

**At stage 1**, banks  $i=1, \dots, B$  compete with one another for entrepreneurs' projects. They each make a price offer  $p_i$  at which they are willing to fund any project.

**At stage 2**, entrepreneurs  $j=(B+1), \dots, (B+N)$  observe banks' proposals  $p=(p_1, \dots, p_B)$ . They either choose a bank at which to apply for credit or they "wait" for direct finance possibilities in stage 3. A behaviour strategy  $a_j^f$  of entrepreneur  $j$  at stage 2 is a function  $a_j^f: (p_1, \dots, p_B) \rightarrow (0, 1, \dots, B)$  such that:

$$a_j^f(p_1, \dots, p_B) = \begin{cases} 0, & \text{if } j \text{ waits for direct trade possibilities,} \\ i, 1 \leq i \leq B, & \text{if } j \text{ applies for credit to bank } i. \end{cases}$$

**At stage 3**, banks, as well as entrepreneurs who preferred to wait for direct trade at stage 2, compete with each other for lenders' funds. They offer two-part contracts that specify the price at which they are willing to borrow funds and the amount of funds they are willing to borrow in the aggregate. We denote by  $d_i=(R_i, C_i)$  and  $d_j^f=(R_j^f, C_j^f)$  the offers of bank  $i$  and firm  $j$  respectively.  $R_i$  and  $R_j^f$  stand for the monetary payments that  $i$  and  $j$  promise to make per unit of money borrowed.  $C_i$  and  $C_j^f$  are the maximum amounts of funds that  $i$  and  $j$  want to borrow from the market. On  $d_j^f$  we impose the restriction that  $R_j^f > 0$  only if  $a_j^f(p_1, \dots, p_B) = 0$ , i.e. firms cannot both borrow from banks in stage 2 and from individual lenders in stage 3.

**At stage 4**, lenders ( $l=1, \dots, L$ ) observe banks' and firms' offers. Then, they can choose to deposit their money within a bank, to finance the project of an entrepreneur, or to take their outside option. An action of lender  $l$  is a function  $a_l$  with domain

$(0, 1, \dots, (B + N))$ . The interpretation is as follows:

$$a_l = \begin{cases} 0, & \text{if lender } l \text{ takes his outside option,} \\ i, 1 \leq i \leq B, & \text{if lender } l \text{ deposits his funds within bank } i, \\ j, (B + 1) \leq j \leq (B + N), & \text{if lender } l \text{ finances firm } j. \end{cases}$$

This description of the game allows for situations in which there is no market clearing. In particular, banks may face a demand for funds that exceeds the amount of deposits they have. The following rationing rules then complete the description of the game by dealing with such situations.

*Rationing rules.* A bank cannot refuse to finance the project of an entrepreneur if it has the corresponding funds. However a bank cannot be forced to finance the project of an entrepreneur if it does not have the corresponding funds, because the demand of credit that it faces exceeds the amount of deposits that it obtains from the lenders. Similarly, a bank (firm) cannot reject the applications of lenders at stage 4 unless the total supply of funds exceeds its capacity limit  $C_i (C_j)$ .<sup>8</sup>

The selection process of the rejected entrepreneurs and lenders does not play any role here, so that we follow the tradition that the rejected agents are randomly chosen among those who apply to a given bank or firm. Moreover we assume that they cannot move to another bank or firm. Formally, let  $L_i^s$  and  $L_j^s$  denote the total amount of funds that are offered to bank  $i$  and firm  $j$  respectively at stage 4. Then, if  $\bar{L}_i (\bar{L}_j)$  denotes the amount of funds that bank  $i$  (firm  $j$ ) actually accepts, we have

$$\begin{aligned} \bar{L}_i &= \min (L_i^s, C_i), \\ \bar{L}_j &= \min (L_j^s, C_j^f). \end{aligned}$$

Furthermore, let  $X_i$  denote the number of entrepreneurs who apply to bank  $i$  at stage 2, and let  $\bar{X}_i$  denote the number of projects that bank  $i$  actually finances. We have

$$\bar{X}_i = \min ([\bar{L}_i], X_i),$$

where  $[\bar{L}_i]$  is the largest integer less than or equal to  $\bar{L}_i$ . Notice that we do not allow banks to finance a project only partially. Banks' technology would make this unprofitable in any case. An analogous assumption is also made concerning lenders, whose funds cannot be accepted only partially.

More importantly, notice that the above rationing rules will affect the behaviour of the agents. For example, suppose that very few firms apply to bank  $i$  at stage 2. Then bank  $i$  might prefer to withdraw from the market, because otherwise its transaction costs will exceed its revenue. The above rationing rule allows bank  $i$  to do this indirectly by choosing  $d_i = (0, 0)$  so that it gets no deposits and can reject firms' applications. In other words, the above rationing rule allows banks to accept or reject strategically firms' applications at stage 3.

*Lenders' payoffs.* Each lender  $l$  maximizes his revenue  $U_l$ , which is a function of the actions chosen by all participants in the market. Depending on lender  $l$ 's action at stage 4, there are three possible cases to consider:

8. Recall that  $C_i$  and  $C_j^f$  are strategic variables.

In the case where  $l$  takes his outside option, he obtains

$$U_l = \frac{1}{m} \bar{R}.$$

If  $l$  applies to bank  $i$  which offers a deposit contract  $d_i = (R_i, C_i)$ , he obtains<sup>9</sup>

$$U_l = \frac{1}{m} \min \left( R_i, \max \left( 0, \frac{G_i}{\bar{L}_i} \right) \right) \left( \frac{\bar{L}_i}{L_i^s} \right).$$

$G_i$  denotes the revenue that bank  $i$  can use to pay its depositors. We define  $G_i$  as being net of the transaction costs so that we cannot exclude the case that  $G_i$  is negative.  $(\bar{L}_i/L_i^s)$  is the probability that bank  $i$  accepts the deposit of  $l$  when it faces an excess demand of deposit contracts.

Finally, financing the project of entrepreneur  $j$  gives lender  $l$  a payoff of

$$U_l = \begin{cases} \frac{1}{m} R_j \frac{\bar{L}_j^f}{L_j^s}, & \text{if } \bar{L}_j^f \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

*Entrepreneurs' payoffs.* Entrepreneurs maximize their expected net profit  $\Pi_j$ , which is a function of the actions chosen by all participants in the market. Here we can distinguish between two possible cases depending on whether entrepreneurs apply to banks or look for direct finance.

First,  $j$  can apply to bank  $i$  at stage 2. His payoff is then given by

$$\Pi_j = \frac{\bar{X}_i}{X_i} (\bar{Y} - p_i).$$

Second,  $j$  can reject the banks' offers at stage 2 (i.e.  $a_j^f(p_1, \dots, p_B) = 0$ ) and offer a contract  $d_j^f = (R_j^f, C_j^f)$  to private lenders at stage 3. In this case  $j$ 's payoff amounts to

$$\Pi_j = \begin{cases} \bar{Y} - [R_j^f + b(R_j^f \bar{L}_j^f, 1)] \bar{L}_j^f, & \text{if } 1 \leq \bar{L}_j^f \leq C_j^f, \\ 0 - [R_j^f + b(R_j^f \bar{L}_j^f, 0)] \bar{L}_j^f, & \text{if } 0 \leq \bar{L}_j^f < 1. \end{cases}$$

*Banks' payoffs.* Banks maximize their expected net profit  $B_i$ , which is a function of the actions chosen by all participants in the market. When bank  $i$  finances  $\bar{X}_i$  projects at price  $p_i$  and takes an amount  $\bar{L}_i \leq C_i$  of deposits at price  $R_i$ , its payoff amounts to:

$$B_i = \bar{X}_i \left[ p_i - a - b(R_i \bar{L}_i, \bar{X}_i) \left( \frac{\bar{L}_i}{\bar{X}_i} \right) \right] - R_i \bar{L}_i,$$

$$B_i = G_i - R_i \bar{L}_i,$$

where  $G_i = \bar{X}_i [p_i - a - b(R_i \bar{L}_i, \bar{X}_i) (\bar{L}_i / \bar{X}_i)]$ .

### III.2. Analysis of Game 1

The game is solved backwards: we first analyse the competition for funds at stages 3 and 4 and then the competition for investment projects at stages 1 and 2.

9. Implicitly we assume that once rejected, a lender no longer has access to his outside option. This assumption is not essential to the results.



*Competition for funds: the deposit subgame.* When banks and firms compete for funds at stage 3, they already know from the previous stages of the game how much of these funds they can invest in: each firm can invest one unit in its own project; bank  $i$  can finance at most  $X_i$  projects. Consequently, the market demand for funds arising from banks and firms should not exceed  $N \leq L$  and the interest rate should not exceed the lenders' outside option  $\bar{R}$ . Proposition 1 confirms this intuition. It also establishes the possibility of underinvestment due to nonconvexities in banks' technologies.

**Proposition 3.1.** *Any equilibrium of the market for funds satisfies the following properties:*

(i) *Bank  $i$  offers a contract  $d_i^*$  which satisfies:*

$$d_i^* = \begin{cases} (\bar{R}, X_i), & \text{if } X_i \geq \underline{X}(p_i), \\ (0, 0), & \text{otherwise,} \end{cases}$$

where  $\underline{X}(p_i) := \{\min_{X \in N \cup \{0\}} X \mid [p_i - a - b(\bar{R}X, X) - \bar{R}] \geq 0\}$ .

(ii) *Firm  $j$  offers a contract  $d_j^*$  which satisfies:*

$$d_j^* = (\bar{R}, 1), \quad \text{for all } j \text{ such that } a_j^f(p_1, \dots, p_B) = 0.$$

*Proof of Proposition 3.1.* We proceed in several steps.

**Step 1.** In equilibrium, banks' and firms' demand for funds are bounded above by the amount they can invest, i.e.

$$\forall i, C_i^* \leq X_i, \quad \text{and} \quad \forall j, C_j^{*f} \leq 1.$$

*Proof of step 1.* At the beginning of stage 3, the maximum quantity of projects that a bank can invest in is given by  $X_i$ . Hence, by choosing  $C_i^* > X_i$ , bank  $i$  only builds up excess capacities which it must reward at rate  $\bar{R}$ . Clearly this cannot be profitable. The argument for firms is analogous.  $\parallel$

**Step 2.** In equilibrium, banks and firms need not pay more than  $\bar{R}$  to the lenders.

*Proof of step 2.* Indeed from step 1, the equilibrium market demand for funds  $(\sum_i C_i^* + \sum_j C_j^{*f})$  is bounded above by  $N$  which is less than  $L$ , the market supply of funds at any rate of return not smaller than  $\bar{R}$ . Step 2 follows directly.  $\parallel$

**Step 3.** In equilibrium, bank  $i$  offers:

$$d_i^* = \begin{cases} (\bar{R}, X_i), & \text{if } X_i \geq \underline{X}(p_i), \\ (0, 0), & \text{otherwise,} \end{cases}$$

where  $\underline{X}(p_i) := \{\min_{X \in N \cup \{0\}} X \mid [p_i - a - b(\bar{R}X, X) - \bar{R}] \geq 0\}$ .

*Proof of step 3.* At the beginning of stage 3,  $p_i$  and  $X_i$  are fixed and determine bank  $i$ 's maximum revenue:  $p_i X_i$ . From step 2, bank  $i$  need not pay more than  $\bar{R}$  to attract depositors. From Assumption 1,  $b(R\tilde{X}_i, \tilde{X}_i)$  is decreasing in  $\tilde{X}_i$ , and increasing in  $R$ . Hence, bank  $i$ 's profit is maximized when it chooses  $C_i = X_i$ ,  $R_i = \bar{R}$ , and  $L_i^s \geq C_i$ . However, if  $X_i < \underline{X}(p_i)$ , bank  $i$  is so small that it cannot break even because its transaction costs are

too high. In this case, it will prefer to withdraw<sup>10</sup> from the market:  $d_i^* = (0, 0)$ . Notice that from Assumption 2 the critical value  $\underline{X}(p_i)$  is strictly greater than one for at least some  $p_i$  where  $\bar{R} \leq p_i \leq \bar{Y}$ .  $\parallel$

**Step 4.** In equilibrium, firm  $j$  offers  $d_j^* = (\bar{R}, 1)$ .

*Proof of step 4.* Step 4 is a direct consequence of step 2 and Assumption 3.  $\parallel$

*Comments.* Proposition 3.1 says that competition for funds is not very fierce in Game 1. This results from two effects: first, there exists an excess supply of funds; second, banks cannot exploit any excess capacity they may build up at stage 3, because at that stage competition for loans has already taken place.

Indirect finance may generate underinvestment. Underinvestment occurs whenever  $X_i < \underline{X}(p_i)$  for some  $i$ .<sup>11</sup> “Small” banks face high transaction costs which make them unable to pay  $\bar{R}$  to the depositors. Notice however that banks’ required minimum size  $\underline{X}(p_i)$  is a decreasing function of the lending price  $p_i$ .

Finally, notice that banks do not influence the price of direct trade. Hence, in Game 1, banks can only be of benefit to firms.

*Competition for projects: solving the whole game.* We now turn to stages 1 and 2 which model banks’ competition for loans. Because they anticipate the outcome of the deposit subgame, firms reject any bank’s offer  $p_i$  that exceeds  $\bar{R} + b(\bar{R}, 1)$ , the total cost of raising funds directly. At the same time, because  $\underline{X}(p_i)$  is decreasing in  $p_i$ , firms will be cautious in trading with a bank that asks for a very low price  $p_i$ . In order to break even, such a bank needs to finance many firms. Therefore, the probability that it withdraws from the market and leaves its client firms without funds is not negligible. Hence, firms will trade-off prices and probabilities of being credit rationed. Clearly, such a trade-off only exists because of banks’ nonconvex technologies. One should expect that it influences banks’ competition and that the competitive outcome is not warranted any more. Proposition 3.2 confirms this intuition. It says that an equilibrium exists for any price below the price for direct finance,  $[\bar{R} + b(\bar{R}, 1)]$ , and above the “competitive” price  $\underline{p}(N)$ .

**Proposition 3.2.** Let  $\underline{p}(X)$  denote the lowest lending price at which a bank can break even when it finances  $X$  projects.

$$\underline{p}(X) := \min_p \{p \geq 0, [p - a - b(\bar{R}X, X)] \geq 0\}.$$

Then, for every  $p$  in the interval  $[\underline{p}(N), (\bar{R} + b(\bar{R}, 1))]$ , Game 1 has a subgame perfect equilibrium in which all banks offer  $p$  and all entrepreneurs apply to the same bank.

*Proof of Proposition 3.2.* First, there is no equilibrium in which the lending prices lie outside the interval  $[\underline{p}(N), (\bar{R} + b(\bar{R}, 1))]$ . No firm will ever accept a lending rate that exceeds the cost of raising funds directly. No bank can ever survive at a lending price below  $\underline{p}(N)$ .

10. Equivalently, bank  $i$  offers the highest deposit rate that is compatible with its revenue. This deposit rate is necessarily smaller than  $\bar{R}$  and discourages lenders from applying to bank  $i$ .

11. Incidentally, underinvestment can also occur when  $p_i$  is high enough to allow bank  $i$  to break even, say, with  $(X_i - 1)$  rather than with  $X_i$  projects:  $(X_i - 1)(p_i - a - b(\bar{R}(X_i - 1), X_i - 1)) \geq \bar{R}(X_i - 1)$ . In such a case, the market for funds has an equilibrium in which only  $m(X_i - 1)$  lenders apply to bank  $i$  and there is some firm  $j$  that bank  $i$  is unable to finance.

Second, let  $p^*$  be any price in the interval  $[p(N), (\bar{R} + b(\bar{R}, 1))]$ . We want to construct an equilibrium in which the lending price is equal to  $p^*$ . To that aim, consider the following strategy of firms at stage 2.

Firms divide banks' price offers into two categories: those belonging to the interval  $I_R = [p(N), p^*[$ , which they consider as "risky", and those in the interval  $I_S = [p^*, (\bar{R} + b(\bar{R}, 1))]$ , which they consider as "safe". They reject any price offer belonging to  $I_R$ , but choose the best price offer belonging to  $I_S$ . Moreover, they all apply to the same bank—say the bank with the smallest index—among those offering the lowest price in  $I_S$ . These strategies of the firms discourage banks to offer prices below  $p^*$ , but lead them to undercut any price above  $p^*$ . Hence, the Bertrand undercutting argument applies to the restricted price interval  $I_S = [p^*, (\bar{R} + b(\bar{R}, 1))]$ , and banks offer  $p_i = p^*$  in equilibrium.

We now verify that firms cannot profitably deviate. Suppose that firm  $\hat{j}$  deviates from the above strategy and applies to bank  $i'$  which offers  $p_i < p^*$ .<sup>12</sup> Because firm  $\hat{j}$  is the only deviant, bank  $i'$  will finance at most 1 project. From Assumption 2, this bank cannot break even

$$B_i < [\bar{R} + b(\bar{R}, 1)] - a - b(\bar{R}, 1) - \bar{R} < 0.$$

Its withdrawal from the market at stage 3 leaves firm  $\hat{j}$  without credit. This "confirms" that prices in the interval  $I_R$  are too "risky" and therefore not profitable.  $\parallel$

The profits of the active bank,  $B_1$ , are strictly positive whenever  $p^* > p(N)$ , and zero otherwise.

Proposition 3.2 characterizes the set of equilibria with only one active bank. Non-convexities reduce banks' competition, so that a monopoly bank can stay in the market at a price above the competitive one. We now show that nonconvexities also place an upper bound on the number of banks that can coexist in the market.

To simplify notation, we denote by  $p^{DF}$  the cost at which a firm can raise credit directly from lenders, i.e.  $p^{DF} := \bar{R} + b(\bar{R}, 1)$ .

**Proposition 3.3.** (i) *If  $\underline{X}(p^{DF}) > N/2$ , all equilibria involve a unique active bank.*

(ii) *If  $N/(k + 1) < \underline{X}(p^{DF}) \leq N/k$ ,  $k \geq 2$ ,  $k \in N$ , equilibria display up to  $k$  active banks. In equilibrium, all active banks offer the same—possibly noncompetitive—price. However, they need not be of the same size, and direct finance can coexist with indirect finance.*

*Proof of Proposition 3.3.* Because  $\underline{X}(p)$  is decreasing in  $p$ , no bank will stay in the market if it finances less than  $\underline{X}(p^{DF})$ . When this number is greater than  $N/2$ , no market sharing is possible, because banks' profitability requires that they finance more than half of the firms.

The same argument applies to show that when  $N/(k + 1) < \underline{X}(p^{DF})$ ,  $(k + 1)$ -banks cannot profitably share the market. However, when  $\underline{X}(p^{DF}) \leq N/k$ , there exist equilibria in which up to  $k$  banks split the market. Indeed, suppose that  $\underline{X}(p^{DF}) < N/k$ ,  $k \geq 2$ ,  $k \in N$ . Let  $t \leq k$ . We want to construct an equilibrium in which  $t$  banks are active and offer the same price  $\hat{p} \in [p(N/t), p^{DF}[$ . The construction is very similar to that in the proof of Proposition 3.2. We consider the following strategy of firms. Firms reject any price offer strictly below  $\hat{p}$ , but choose the best offer in  $[\hat{p}, p^{DF}[$ . When  $z$ ,  $z \leq t \leq k$ , banks offer this price, firms split their demand of credit (almost) equally among the  $z$  banks. Firms 1 to

12. Recall that we are interested in subgame perfect equilibria so that we have to consider firm  $\hat{j}$ 's reaction to any price offer  $(p_1, \dots, p_B)$ , whether it lies on the equilibrium path or not.

$[N/z]$  apply to the bank whose index is smallest, firms  $[N/z] + 1$  to  $2[N/z]$  apply to the next bank etc. . . Finally, last  $N - (z - 1)[N/z]$  firms go to the remaining bank.

In equilibrium,  $z = t$  banks are active and offer the same price  $\hat{p}$ . Suppose otherwise. In equilibrium, the active bank with the lowest price must break even. Hence, there is no "risk" to be rationed by this bank. All firms will buy from it, in contradiction to the hypothesis that  $t > 1$  banks are active.

Still, banks' equilibrium payoffs may differ because banks' size need not be equal.

When  $\underline{X}(p^{DF}) < N$ , there is an equilibrium in which direct finance and indirect finance coexist. In equilibrium, all banks offer  $p^{DF}$ , all active banks finance  $\underline{X}(p^{DF}) < N$ , and some firms are financed directly at the same price  $p^{DF}$ . No such equilibrium exists with prices below  $p^{DF}$  because firms would then prefer to apply to banks. ||

*Comments.* Propositions 3.2 and 3.3 say that in general price competition under increasing returns to scale does not yield the "competitive" outcome and allows for strictly positive profits. Banks' nonconvex technologies create a coordination problem among firms. The incentive for bank  $i$  to undercut its competitors depends on how many firms it attracts. Even a very low price is no guarantee for attracting firms. Firms also care about the probability of being rationed which depends on other firms' behaviour. When many firms apply to bank  $i$ , bank  $i$  finds it profitable to finance all of them because its transaction costs are sufficiently low. However, when only few firms apply to bank  $i$ , bank  $i$  does not find it profitable to finance them because its transaction costs are too high. All firms applying to bank  $i$  are rationed and would better pay a higher price in order to get credit. At the same time, bank  $i$  would be better to choose a higher price.

The multiplicity of equilibria is entirely due to a coordination problem. At stage 2, firms face a multiplicity of Pareto-ranked equilibria, among which the Pareto dominant equilibrium may appear as a focal point. And indeed, the Pareto-dominant equilibrium will be selected if we require the equilibrium to be coalition-proof. This concept captures the notion of an efficient self-enforcing agreement when unlimited, but nonbinding, pre-play communication is possible. In particular, coalition-proofness requires that the equilibria are robust against (self-enforcing) deviations by any set of players moving at the same time. In the present case, it is like allowing firms to coordinate to the best price offer, so that banks are forced to offer the competitive price  $\underline{p}(N)$ .

**Proposition 3.4.** *The competitive price  $\underline{p}(N)$  is the only coalition-proof equilibrium of Game 1.*

*Proof of Proposition 3.4.* Indeed, suppose that firms face different price offers,  $p_1 < p_2 < \dots < p_B$ , in the interval  $[\underline{p}(N), (\bar{R} + b(\bar{R}, 1))]$ . Suppose that some of the firms do not choose bank 1. Then, there exists a deviation that is profitable to all firms. If all firms applying to banks 2, . . . ,  $N$  deviate and apply to bank 1, they make bank 1's offer "viable", so that they all get financed at the lowest possible price. Firms' behaviour gives banks the incentive to undercut any price above the competitive one,  $\underline{p}(N)$ , so that Proposition 3 holds. ||

At this point, note that coalition-proof strategies are "risky" for all players. Banks offering the "competitive" price  $\underline{p}(N)$  are very fragile.  $\underline{X}(\underline{p}(N)) = N$ , so that if firms do not coordinate perfectly, banks will not find it profitable to stay in the market and their clients will be rationed. Because  $\underline{X}(p_i)$  is decreasing in  $p_i$ , higher prices act as a guarantee against coordination failures. Firms may choose to buy credit at a higher rate if by so

doing they increase their probability of getting credit. High rates compensate banks for “unexpectedly” high transactions costs in the case where firms do not coordinate well. Therefore they reduce the number of situations in which banks prefer to withdraw from the market.

Many recent experiments (e.g. by Van Huyck *et al.* (1990, 1991)) have demonstrated the relevance of strategic risk considerations—relative to payoff-dominance considerations—in coordination games with many agents. “Strategic uncertainty” denotes the uncertainty that players face about how other players will respond to the multiplicity of equilibria. Players often fail to coordinate on the Pareto-dominant equilibrium. In such situations, “the higher payoffs when all players choose (the payoff dominant strategy) must be traded off against its greater risk of lower payoffs if they do not” (Crawford (1991)).

These experimental results are corroborated by those of different theoretical approaches which address the selection problem in a new way. Two different approaches are worth mentioning. One is static and analyses the consequences of perturbing the original game with uncertainty about the players information structure (Carlsson and van Damme (1993)). The other is dynamic and analyses the consequences of (persistent) noise or mutations on the set of “long-run equilibria” in an evolutionary model. Carlsson and van Damme show that a small departure from common knowledge forces the players to coordinate on the risk-dominant equilibrium in  $2 \times 2$  games. Taking a different approach, Foster and Young (FY) (1990), Kandori, Mailath and Rob (KMR) (1993), Matsui and Matsuyama (MM) (1995), Young (Y) (1993), and Ellison (E) (1995) show that the risk-dominant equilibrium is the unique long-run (absorbing in MM) equilibrium in  $2 \times 2$  games of common interest. Kim (1995) extends the preceding literature to  $n$ -person binary action coordination games with two strict Pareto-ranked Nash equilibria. He shows that unless the payoff at the Pareto inferior equilibrium is extremely small, CvD, KMR and FY models still select the risk-dominant equilibrium. Ellison considers the more general case of many actions—many agents games. He shows that the selection of the risk-dominant equilibrium in  $2 \times 2$  games generalizes to the selection of the  $\frac{1}{2}$ -dominant equilibrium<sup>13</sup> of Morris, Rob and Shin (1995) in arbitrary games.

It is beyond the scope of this paper to apply these theoretical developments to determine the whole set of evolutionary stable equilibria. However, it seems interesting to test the robustness of “coalition-proof” equilibria (CPE) to strategic risk concerns. A simple way to do that, while keeping the spirit of the above literature, is to raise the rather restricted question of whether CPE are evolutionary stable (ESS) or not. A strategy is evolutionary stable if it is robust to deviations by some (small) part of the population. That is, ESS-players must have greater payoffs than any mutant entering the population with sufficiently low frequency.<sup>14</sup> This requirement therefore expresses to a certain extent the cautious behaviour of players in coordination games. Although ESS cannot select among strict Nash equilibria in infinite populations,<sup>15</sup> it can discriminate among them when finite populations are considered. Below, we show that the coalition-proof equilibrium of Game 1 (which yields the competitive outcome) is not ESS.<sup>16</sup>

13.  $\frac{1}{2}$ -dominance is stronger than risk-dominance as it compares the candidate action pair to *all* other actions, not just to an alternative Nash equilibrium action pair.

14. A formal definition of ESS is given in the Appendix.

15. ESS is robust to local *one-time* shock only.

16. This result may be surprising because the coalition-proof equilibrium is a strict equilibrium. However, strict equilibria are ESS only in an infinite population framework.

**Proposition 3.5.** *The coalition-proof equilibrium of Game 1 is not ESS. ESS limits the set of possible lending prices to the interval  $[\underline{p}(N-1), (\bar{R} + b(\bar{R}, 1))]$ . When this interval is empty, all firms are financed directly, i.e. disintermediation is the only ESS.*

*Proof of Proposition 3.5*

See appendix. ||

Following Schaffer (1989) one can even go further and ask for more stability. An ESS is said to be “ $Y$ -stable if, for a population with a total of anywhere from 2 to  $Y$  deviants with any deviant strategies, the payoff of an evolutionary equilibrium player is still greater than the payoffs of all the deviants” (Schaffer, 1989). Imposing  $Y$ -stability in our framework further restricts the interval of equilibrium lending prices to  $[\underline{p}(N-Y), (\bar{R} + b(\bar{R}, 1))]$ .

Proposition 3.5 shows that even though the competitive outcome belongs to the solutions of Game 1, it may be difficult to obtain. The cost of enforcing the competitive solution is not specific to the game we are looking at. Suppose, for example, that banks’ offers at stage 1 were binding so that banks could not ration firms. This would be another way to eliminate the coordination problem among firms and to get the competitive solution. The problem with such a specification is that it incorporates an element of involuntary finance: in order to fulfil their contracts with firms, banks may be forced to borrow funds from depositors even though it is not profitable.

#### IV. COMPETITION FOR DEPOSITS DETERMINES COMPETITION FOR PROJECTS

In this version of the game, banks first compete for funds and then for entrepreneurs’ projects. When looking for funds, banks also face competition from firms which prefer to raise credit directly from lenders. We consider the following game, in which the same notation as in Game 1 is used.

##### IV.1. Specification of Game 2

**At stage 1**, banks and firms compete with one another for lenders’ funds. They offer two-part contracts:  $d_i = (R_i, C_i)$  and  $d_j^f = (R_j^f, C_j^f)$ .

**At stage 2**, lenders observe banks’ and firms’ offers. They either deposit their funds in a bank, or finance a firm, They also have an outside option. Formally,

$$a_l = \begin{cases} 0, & \text{if lender } l \text{ takes his outside option,} \\ i, 1 \leq i \leq B, & \text{if lender } l \text{ deposits his funds within bank } i, \\ j, (B+1) \leq j \leq (B+N), & \text{if lender } l \text{ finances firm } j. \end{cases}$$

**At stage 3**, banks compete with each other for entrepreneurs’ projects. They each offer a price  $p_i$  at which they are willing to fund projects.

**At stage 4**, firms observe banks’ proposals  $p = (p_1, \dots, p_B)$ . If their projects have not already been funded, they choose a bank to which they apply for credit or withdraw from

the market. A strategy  $a_j^f$  of entrepreneur  $j$  at stage 4 is a function:

$$a_j^f(p_1, \dots, p_B) = \begin{cases} 0, & \text{if } j \text{ abandons his project,} \\ i, 1 \leq i \leq B, & \text{if } j \text{ applies for credit to bank } i. \end{cases}$$

*Rationing rules.* We make essentially the same assumptions as in game 1

$$\begin{aligned} \bar{L}_i &= \min(L_i^s, C_i) \\ \bar{L}_j^f &= \begin{cases} \min(L_j^s, C_j^f), & \text{if } L_j^s \geq C_j^f \geq 1, \\ 0, & \text{if } L_j^s < 1. \end{cases} \end{aligned}$$

As in Game 1, we ignore the possibility that a firm is financed with both direct and indirect funds. Firms that face a supply of direct funds less than one reject lenders' applications. They can still raise credit from banks at state 4.<sup>17</sup> Further we have:

$$\bar{X}_i = \min([\bar{L}_i], X_i).$$

*Remark.* In contrast to Game 1, the selection process of lenders here plays a role. In Game 2 the competition for funds will be strong so that banks and firms want to offer high interest rates. However, at these rates they cannot take an arbitrary amount of deposits. In particular, firms may not want to buy more than the dollar they need for their project. Then, a random selection process like that of Game 1 would reduce the attractiveness of firms' price offers because lenders take their high rejection probability into account. Other, less anonymous, selection processes do not have this drawback of penalizing small agents. Among all possible rules, we choose the following one (the same for all banks and firms): "accept with probability 1 the funds of the lenders with the smallest indices".

*Payoffs.* Banks' and firms' payoffs are the same as in Game 1. Lender  $l$ 's payoff is given by

$$U_l = \frac{1}{m} \min \left( R_l, \max \left( 0, \frac{G_l}{\bar{L}_l} \right) \right) \gamma_{li},$$

or by

$$U_l = \frac{1}{m} R_j \gamma_{lj},$$

depending on whether lender  $l$  applies to bank  $i$  or to firm  $j$ .  $\gamma_{li}$  and  $\gamma_{lj}$  denote the probabilities with which  $l$ 's funds are accepted by bank  $i$  and firm  $j$  respectively. These probabilities are either zero or one, depending both on the index of the lender considered and on the behaviour of the lenders with lower indices.

#### IV.2. Analysis of Game 2

The game is solved backwards: we first analyse the competition for investment projects at stages 3 and 4 and then the competition for funds at stages 1 and 2.

17. This assumption simplifies the analysis.

*Competition for projects: the investment subgame.* When banks compete for investment projects at stage 3, the amounts of deposits ( $\bar{L}_1, \dots, \bar{L}_B$ ) that they can dispose of are given. These amounts determine the maximum number of projects  $(K_1, \dots, K_B) = ([\bar{L}_1], \dots, [\bar{L}_B])$  that they can finance. The investment subgame looks like a Bertrand–Edgeworth game where the  $K_i$ 's are banks' capacities. Banks compete in the prices  $p_i$  at which they offer to finance firms' projects. Let  $\hat{N}(p_*)$  denote the market demand of credit by firms. We have

$$\hat{N}(p_*) = \begin{cases} N - \sum_{j=1}^N \{j: L_j^s \geq 1\}, & \text{if } p_* \leq \bar{Y}, \\ 0, & \text{otherwise.} \end{cases}$$

At stage 4, some firms may already be financed directly. For the other firms, the alternative is between getting a credit from the banking sector and remaining inactive. Notice that firms choose the **banks** to which they apply for credit and not only the **price** at which they want to borrow their funds. This amounts to endogenizing the splitting rules. When banks have increasing returns to scale, the splitting rules affect the sum as well as the distribution of their payoff. Usual splitting rules like "equal splitting" have the drawback that the sum of banks' payoffs is not upper-semi-continuous so that one of the conditions of Dasgupta and Maskin (1986) theorem is violated and we cannot apply it to show existence of equilibrium. Endogenizing the splitting rule eliminates this problem because for any given capacity constellation there is a distribution of lenders' funds such that the sum of banks' payoffs is upper semi-continuous.<sup>18</sup>

To summarize, stages 3 and 4 look like a Bertrand–Edgeworth game with increasing returns to scale and endogenous splitting rules. Below we provide an existence proposition and characterize the equilibrium of the investment subgame for all possible capacity constellations. We restrict our attention to the case where there are two banks in the market.

**Proposition 4.1.** *Existence: For every possible capacity constellation  $(K_1, K_2)$ , the investment subgame has an equilibrium in pure or mixed strategies.*

*Uniqueness: Whenever banks' profits are nonnegative, the equilibrium of the investment subgame is unique.*

*Proof of Proposition 4.1.* Depending on  $K_1, K_2$ , and  $\hat{N}$ , there are three cases that must be considered.

**Case 1: The Monopoly Case.** Suppose that  $K_1, K_2$ , and  $\hat{N}$  satisfy the following condition:

$$(K_1 + K_2) \leq \hat{N}, \text{ or } (K_1 + K_2) > \hat{N}, \text{ with } \min(K_1, K_2) = 0.$$

Then, clearly, there will be no real competition for projects. The two banks charge the monopoly price:  $p^* = \bar{Y}$ .

**Case 2: The Competitive Case.** Suppose that  $K_1, K_2$ , and  $\hat{N}$  satisfy the following condition:

$$\min(K_1, K_2) > \hat{N}.$$

Each bank can satisfy the total demand for credit, so that competition for projects is very strong. Below, we show that banks' equilibrium price offers must even lie below the constant monitoring cost  $a$ .

18. See also Maskin (1986).



**Step 1.** Suppose that  $\min(K_1, K_2) > \hat{N}$  holds. Then, if the investment subgame has an equilibrium in pure strategy, the equilibrium price offer  $(p_1^*, p_2^*)$  associated with  $(K_1, K_2)$  must satisfy:

$$p_1^* = p_2^* < a.$$

Indeed, at stage 3, bank  $i$  maximizes the expression:

$$\bar{X}_i(p_i - a) - b(R_i \bar{L}_i, \bar{X}_i) \bar{L}_i$$

where  $R_i$ , and  $\bar{L}_i$  are given and  $\bar{L}_i \geq \bar{X}_i$ .

In equilibrium, both banks must offer the same price:  $p_1^* = p_2^* = p^*$ . Otherwise, the bank which offers the higher price gets no demand and its payoff is negative. Its revenue is null while it bears a penalty equal to  $-b(R_i \bar{L}_i, 0) \bar{L}_i$ , for being unable to honour its contract with depositors. Now suppose that  $p^* > a$ . At least one bank finances less than  $\hat{N}$  projects. This bank should deviate and offer  $p^* - \varepsilon$ . Indeed, for  $\varepsilon$  small enough, and  $\bar{X}_i < \hat{N}$ , this increases its revenue:  $((p^* - \varepsilon) - a) \hat{N} > (p^* - a) \bar{X}_i$  and reduces its cost:  $b(R_i \bar{L}_i, \hat{N}) \bar{L}_i < b(R_i \bar{L}_i, \bar{X}_i) \bar{L}_i$  for  $\bar{X}_i < \hat{N}$ . The ‘‘cost reduction’’ aspect linked to an increase in bank sales explains why  $p^* = a$  cannot be an equilibrium either.

**Step 2.** Suppose that  $\min(K_1, K_2) > \hat{N}$  holds. Then, the investment subgame has at least one equilibrium in pure strategies.

Indeed, from step 1, we already know that  $p_1^* = p_2^* < a$ . Suppose that, when  $\min(K_1, K_2) > \hat{N}$  and  $p_1^* = p_2^*$ , all firms go to bank 1. Then bank 2 has an incentive to undercut bank 1 until  $p^*$  satisfies:

$$\bar{L}_2 [b(R_2 \bar{L}_2, 0) - b(R_2 \bar{L}_2, \hat{N})] = (a - p^*) \hat{N}.$$

At this price, bank 2’s revenue losses equal the cost reduction due to an increase in bank 2’s sales from 0 to  $\hat{N}$ . In equilibrium both banks offer this price.<sup>19</sup>

**Case 3: The Oligopoly Case.** Suppose that  $K_1, K_2$ , and  $\hat{N}$  satisfy the following condition:

$$\min(K_1, K_2) < \hat{N}, \quad \text{with } (K_1 + K_2) > \hat{N}.$$

Then, the investment subgame has an equilibrium in either pure or mixed strategies.

Indeed, suppose, w.l.o.g. that  $b(R_1 \bar{L}_1, K_1) \bar{L}_1 \geq b(R_2 \bar{L}_2, K_2) \bar{L}_2$ . Suppose that whenever  $p_1 = p_2$ , firms apply to bank 1 first, so that bank 1 finances  $K_1$  firms and bank 2,  $(\hat{N} - K_1)$ . Then, it is easy to see that the investment subgame fulfills the ‘‘Dasgupta and Maskin’’ (sufficient) conditions<sup>20</sup> for the existence of an equilibrium. In particular, the splitting rule specified above guarantees that the sum of banks’ transaction costs do not increase when there are ties, so that the sum of bank payoffs is upper semi-continuous. ||

*Remarks.*

(1) On the existence of pure strategy equilibria in case 3:

Define  $\hat{p}_i$  as follows:

$$(\hat{N} - K_i)(\bar{Y} - a) - b(R_i \bar{L}_i, (\hat{N} - K_i)) \bar{L}_i = K_i(\hat{p}_i - a) - b(R_i \bar{L}_i, K_i) \bar{L}_i.$$

19. On the Multiplicity of Equilibria in Case 2. The cost reduction associated with undercutting depends on the way firms allocate themselves between the banks offering the same price, i.e. it depends on the splitting rule. For example, ‘‘equal splitting’’ makes this gain smaller than that in the text and yields an equilibrium price which is higher. (The proof is available upon request.)

20. The set of pure strategies for bank  $i$  is non-empty and compact. Its payoff is continuous except when its action coincides with the action of another bank. It is also bounded and lower semi-continuous in its own action. Finally, the sum of banks’ payoffs is upper semi-continuous.

Equivalently,

$$\hat{p}_i = a + (\hat{Y} - a) \frac{(\hat{N} - K_i)}{K_i} \frac{\bar{L}_i(b(R_i \bar{L}_i, (\hat{N} - K_i)) - b(R_i \bar{L}_i, K_i))}{K_i},$$

$\hat{p}_i$  is so defined that when  $p_j$  is above  $\hat{p}_i$ , bank  $i$  prefers to undercut bank  $j$  rather than to charge the monopoly price  $\bar{Y}$ . The converse holds for prices below  $\hat{p}_i$ , where the absence of rationing is not sufficient to counterbalance the low price. Moreover:  $\hat{p}_i \leq \hat{p}_j$  when  $b(R_j \bar{L}_j, K_j) \bar{L}_j \leq b(R_i \bar{L}_i, K_i) \bar{L}_i$  and the pair  $(\hat{p}_i, \hat{p}_j)$  does not depend on the splitting rule.

Then,  $p_i^* = p_j^* = p^* = \min(\hat{p}_i, \hat{p}_j)$  is a natural candidate for a price equilibrium. Whether it is an equilibrium or not depends on the parameters of the investment subgame. Suppose that, at the beginning of the subgame, the two banks are identical:  $K_1 = K_2$  with  $\bar{L}_1 = \bar{L}_2$ . Then,  $(p^*, p^*)$  is an equilibrium. On the other hand, if  $K_1 = \bar{L}_1 > K_2 = \bar{L}_2$ , then  $\hat{p}_1 > \hat{p}_2$  and there is no equilibrium in pure strategies.

(2) When the equilibrium is in mixed strategies, the expected profits of the bigger bank<sup>21</sup> are higher than those of the smaller bank.

(3) Whether the equilibrium is in pure strategies or not, it is unique in cases 1 and 3. Hence, the splitting rule only affects banks' payoffs in case 2. However, in that case, banks' payoffs are necessarily negative and this cannot happen in an equilibrium of the whole game.

Taking these results into account we can now solve the whole game. First, we show that the game has a continuum of subgame perfect equilibria. Second, we restrict our attention to those subgame perfect equilibria which are coalition proof. Again there exists a unique coalition-proof equilibrium. However, in contrast to that of Game 1, it is not competitive. Further, disintermediation may be a coalition-proof equilibrium and competitive forces may increase rather than decrease total transaction costs.

*Competition for funds: Solving the whole game.* We now turn to stages 1 and 2 in which banks and firms compete for lenders' funds. Competition is in prices and everybody realises that the outcome of stages 1 and 2 determines the outcome of the investment subgame. The Bertrand character of stages 1 and 2 suggests that banks and firms will overbid each other. But this is not the case here. The point is analogous to that discussed for Game 1: banks' and firms' offers do not entirely determine lenders' payoffs. Lenders must also take banks' and firms' ability to pay into account, and this ability itself depends on lenders' behaviour.

There are three main situations in which a lender should refuse a high interest rate: First, the bank which offers a high rate might get involved in a price war at stage 3, in which case its receipts will not allow it to reward its depositors according to its promises; Second, the bank with the high rate offer may be so small that its transaction costs take up all its revenues, leaving nothing for the depositors; Third, the firm offering a high rate may attract less than  $(m-1)$  other lenders and switch to indirect funding.

In all these cases, our lender does better to disregard the offer and invest his funds elsewhere. But notice the role played by lenders' behaviour at stage 2. Other allocations of their funds would make the same offers attractive. That is, we have a pure coordination problem among lenders, and one should expect that it will weaken competition among banks and firms for funds.

21. To be precise: the bank for which the  $b$ -cost  $b(R_i \bar{L}_i, K_i) \bar{L}_i$  is the smallest.

Proposition 4.2 confirms this intuition. Game 2 has a continuum of equilibria that can be ranked according to the interest rate that banks and firms grant to lenders. The range goes from  $\bar{R}$  to  $\hat{R}$ , which is the highest deposit rate that a bank can ever pay to its depositors. An equilibrium may involve the activity of several banks. Some (or even all) firms may not raise their finance from banks.

**Proposition 4.2.** *Define  $\hat{R}$  as follows:*

$$\hat{R} = \bar{Y} - a - b(\hat{R}N, N).$$

- (1) *For every rate  $R$  in the interval  $[\bar{R}, \hat{R}]$ , game 2 has a subgame perfect equilibrium with  $R$  as the equilibrium rate.*
- (2) *Some equilibria may involve the activity of several banks.*
- (3) *Some equilibria may involve direct trade.*

*Proof of Proposition 4.2.* First, there can be no equilibrium in which the rate  $R$  lies outside  $[\bar{R}, \hat{R}]$ . No lender will ever accept an interest rate below its outside option  $\bar{R}$ . By definition,  $\hat{R}$  is the highest interest rate that a monopoly bank can pay its depositors without making losses. Hence, no bank will ever offer a rate above  $\hat{R}$ .

The rest of the proof is very similar to that of Proposition 3.2 and Proposition 3.3 and is therefore omitted.  $\parallel$

*Coalition-proof equilibria.* Coalition-proofness entirely solves the lenders' coordination problem. Hence, the equilibrium is unique. However, the solution is far from being competitive. Competition for funds leads to a price war between banks and firms which drives the interest rate up to a rate that exceeds the competitive rate  $\bar{R}$  and may even end in disintermediation. Moreover, such high deposit rates unduly raise transaction costs so that banks' ability to reduce transaction costs must be questioned.

**Proposition 4.3.** (1) *Game 2 has a unique subgame perfect coalition-proof equilibrium.*  
 (2) *There exists  $T \geq 0$  such that if  $(L - N) \leq T$ , all exchanges are intermediated by a single bank which makes zero profits. In equilibrium,  $p^* = \bar{Y}$  and  $R^* > \bar{R}$ . If  $(L - N) > T$  there is no active bank, all trades are direct trades between lenders and firms.  $R^* > \bar{R}$  as long as  $(L - N) < \bar{T}$ , where  $\bar{T} > T$ .*

*Proof of Proposition 4.3.* See the appendix.  $\parallel$

*Interpretation.* Coalition-proofness allows lenders to coordinate on the best, highest price offer and to prevent banks' competition for projects by applying to a unique bank. The resulting competition for deposits is very fierce (despite the excess supply of funds) because by cornering the market for funds, banks can obtain a monopoly position in the market for projects.<sup>22</sup> Anticipating the high price of bank loans, firms try to escape banks' market power by financing themselves directly. This exacerbates competition for funds which may turn to firms' advantage as Proposition 4.3 says. This is rather surprising, because banks benefit from a cost advantage<sup>23</sup> that should enable them to offer better rates. However, competition by firms forces banks to buy all available funds in the market. Otherwise, some firms will finance themselves directly with the remaining funds. Financing

22. This effect does not exist in Game 1.

23.  $a + b(\bar{R}N, N)$  is less than  $b(\bar{R}, 1)$ .

less projects, banks become unprofitable as their revenue falls and their cost increases. To avoid this, banks must buy all available funds in the market and build up excess capacities. In consequence they have to cut down the rate they offer to each depositor. At some point, firms become competitive, and disintermediation follows. It is interesting to note that the disintermediation equilibrium is different from the equilibrium that is obtained when banks are not present in the market. The competition between direct trade and indirect trade raises the price for funds.

*Coalition-proof equilibria and ESS.* The concept of ESS has been defined for symmetric games only, and the coordination game among lenders is not symmetric. However, it is still possible to ask whether the coalition-proof equilibria are robust against deviations by a small part of the population of lenders. With some abuse in the definition of ESS, we can show.

**Proposition 4.4.** *When  $(L - N) \leq T$ , the coalition-proof equilibrium fulfills the condition for ESS under very weak assumptions.*

*When  $(L - N) > T$ , a deviant strategy may reveal superior to only a very small part of the lenders' population.*

*Proof of Proposition 4.4.* See the appendix. ||

In comparison to that of Game 1, Game 2's coalition-proof equilibria are more stable. This can be explained by the degree of interdependency that intermediation creates among the agents of a given category. In Game 1, intermediation creates a link between firms so that firms influence each other and the possibility of "spiteful" behaviour exists: "a firm may forgo profit-maximization and lower its profits and even its survival chances, but if the profits of its competitors are lowered still further, the "spiteful" firm will be the more likely survivor" (Schaffer, 1989). By contrast, in Game 2, intermediation reduces the link between lenders so that no "spiteful" behaviour is possible.

## V. INTERMEDIATION AND WELFARE

At this point we would like to question the efficiency of intermediation that has been underlined by several authors, such as Diamond (1984) or Brainard and Tobin (in Hester and Tobin, 1971). From the above analysis of the two games, we know that, in general, indirect trade is not Pareto superior.<sup>24</sup> The question remains whether it necessarily reduces transaction costs. The answer depends on the game that is played. In Game 1, intermediation minimizes transaction costs as long as a single bank is active in equilibrium. First, there is no real competition for funds so that banks' total indebtedness is low. Second, the size of the active bank, i.e. the number of projects it finances, is at a maximum.

In Game 2, however, things change dramatically. Transaction costs are never at a minimum. Competition for funds drives the interest rate up beyond  $\bar{R}$ . Banks' and firms' indebtedness increases, and this makes transaction costs go up. Intermediation may increase rather than decrease transaction costs. This is immediate in the case where banks are inactive in equilibrium, because in order to escape competition from banks, some firms have to pay interest rates that exceed  $\bar{R}$ . When banks are active in equilibrium, the same

24. In contradiction to the Brainard and Tobin conjecture.

result obtains when the cost reduction due to the size of active banks is more than compensated by the cost increase due to the higher deposit rate:  $aN + \tilde{L}b(\hat{R}^B \tilde{L}, N) > Nb(\bar{R}, 1)$ .

VI. CONCLUSION

In this paper we show that price competition among banks need not yield or converge to the Walrasian outcome. Two reasons explain this phenomenon. First, banks' technology involves nonconvexities. Second, banks compete for inputs as well as for outputs—for funds as well as for projects. Both aspects distort price competition. The nonconvexities create coordination problems. The double-sidedness of competition introduces an asymmetry in the markets by inducing banks to corner one market, in an attempt to achieve a monopoly position in the other market. The cornering of one market may require that banks offer noncompetitive prices. This effect is more robust than it looks like. It does not depend on the sequential character of the game. In Yanelle (1989), we show that the same results as in Game 1 and Game 2 obtain when borrowers and lenders move simultaneously. (What matters in a simultaneous move game is which side of the market banks ration when the demand they face is not equal to the supply.) The cornering effect is also robust to the introduction of frictions as we show in Gottardi and Yanelle (1996). Maybe at this point it is worth mentioning that most results of this paper are in harmony with Stewart's intuitions.

APPENDIX

*Proof of Proposition 3.5.* We adopt the notations and definitions of Crawford (1991). We denote by  $E(r|s)$  the payoff of a player playing  $r$  when the population frequencies are given by  $s$ . A finite-population ESS is a pure strategy,  $p$ , such that for each pure strategy  $q \neq p$ .

$$E\left(p \left| \frac{N-2}{N-1} p + \frac{1}{N-1} q \right. \right) > E(q|p).$$

First, suppose that the interval  $[p(N-1), (\bar{R} + b(\bar{R}, 1))]$  is not empty. Suppose, w.l.g., that bank 1 offers  $p_1$  such that  $X(p_1) = N-1$ , whereas bank 2 and the other banks offer the competitive price  $p(N)$ . Let  $p$  denote the strategy that consists for a firm to apply to bank 2. We denote by  $q$  firms' outside option, i.e. the strategy that consists to wait until stage 3 for direct finance. Then,

$$E\left(p \left| \frac{N-2}{N-1} p + \frac{1}{N-1} q \right. \right) = 0 \quad \text{and} \quad E(q|p) = \bar{Y} - \bar{R} - b(\bar{R}, 1) > 0.$$

Hence, the competitive equilibrium is not ESS.

We denote by  $p'$  the strategy that consists for a firm to apply to bank 1, and by  $q'$  any mutant strategy. We still use  $q$  to represent firms' outside option. We want to show that  $p'$  is ESS. For any possible strategy  $q'$ , ESS-players obtain

$$E\left(p' \left| \frac{N-2}{N-1} p' + \frac{1}{N-1} q' \right. \right) = \bar{Y} - p'.$$

When  $p' < \bar{R} + b(\bar{R}, 1)$ , the latter expression exceeds what any mutant entering the population with frequency  $1/N$  can obtain. Indeed

$$E(q|p') = \bar{Y} - \bar{R} - b(\bar{R}, 1) > E(q'|p') \text{ for all } q'.$$

The third part of Proposition 3.5 directly follows from the observation that, when  $[p(N-1), (\bar{R} + b(\bar{R}, 1))]$  is empty, there exists no ESS-price  $p$  that banks can profitably offer.  $\parallel$

*Proof of Proposition 4.3.* Let  $\hat{R}^F$  denote the highest interest rate that a firm can pay lenders

$$\hat{R}^F = \bar{Y} - b(\hat{R}^F, 1).$$

Similarly, let  $\hat{R}^B$  denote the highest deposit rate that a bank can pay its depositors if it leaves less than one dollar on the market

$$\hat{R}^B = \frac{1}{\tilde{L}} (N\bar{Y} - aN - b(\hat{R}^B \tilde{L}, N) \tilde{L})$$

where

$$\tilde{L} := \begin{cases} N, & \text{if } ((L-N) < 1, \\ \left(L - \frac{m-1}{m}\right), & \text{otherwise.} \end{cases}$$

$\hat{R}^B$  results when the bank finances all projects at a price  $p = \bar{Y}$ . It is the highest rate that a bank can pay while making sure that no trade takes place directly. Notice that  $\hat{R}^B$  is decreasing in  $(L-N)$ .

Define  $T$  as the amount of excess funds such that  $\hat{R}^B = \hat{R}^F$ .

**Case 1.**  $(L-N) \leq T$ .

Let us first analyse the situation where the quantity of excess funds is small. When  $(L-N) = 0$ ,  $\tilde{L} = N$  and, from Assumption 2(iii),  $\hat{R}^B > \hat{R}^F$ . This inequality holds as long as  $(L-N)$  is less than  $T$ . It means that when  $(L-N) < T$ , banks can overbid firms in the competition for funds. We now show that the following constitutes a coalition-proof equilibrium.

At stage 1, banks offer  $d_i = (\hat{R}^B, \tilde{L})$  and firms offer  $d_i^f = (R', 1)$  where  $\bar{R} < R' < \hat{R}^F$ .

At stage 2, lenders  $i = 1, \dots, (m\tilde{L})$  offer their funds to bank 1.<sup>25</sup> Other lenders take their outside option.

At stage 3, bank 1 chooses  $p = \bar{Y}$ .

At stage 4, firms apply to bank 1.

Given stages 1 and 2, it is clear that the above actions at stages 3 and 4 constitute an equilibrium of the investment subgame. So let us concentrate on stages 1 and 2. Suppose that banks 1 and 2 offer the same contract  $d_i = (R, \tilde{L})$ ,  $i = 1, 2$  with  $\hat{R}^F < R < \hat{R}^B$ . In that case,  $(m\tilde{L})$  lenders go to bank 1 whose profits are strictly positive. Then, bank 2 has an incentive to deviate<sup>26</sup> by offering  $d_2 = ((R + \varepsilon), \tilde{L})$  with  $R + \varepsilon < \hat{R}^B$ . At this rate bank 2 makes positive profits if it gets a quantity  $\tilde{L}$  of deposits. Coalition-proofness guarantees that  $(m\tilde{L})$  lenders will actually go to bank 2. A lender prefers to go to bank 2 rather than to bank 1, when  $(m\tilde{L} - 1)$  other lenders do the same. When  $(m\tilde{L})$  lenders go to bank 2, bank 2 is able to honour its promise to pay a higher deposit rate. Allowing group deviations makes the competition for funds very strong. The same argument as above shows that in equilibrium, the two banks must offer  $\hat{R}^B$ . Moreover, given that  $\hat{R}^B > \hat{R}^F$ , and  $(\tilde{L} - N) < 1$ , firms cannot raise funds directly.

Until now, we have only considered price deviations. However, in the above candidate equilibrium,  $\tilde{L}$  is greater than  $N$  so that banks build excess capacities. We now show that banks cannot profitably decrease the quantities they buy.

Suppose that banks accept less than  $\tilde{L}$  funds, and suppose as before that bank 1 is the only active bank. Then, at least one firm will be financed directly from lenders at a rate  $R'$  below  $\hat{R}^B$ . Indeed, when  $R'$  is above  $\bar{R}$ , firms face a residual supply of funds of at least 1. Consequently, bank 1 finances at most  $(N-1)$  firms. Its increased transaction costs make it unable to pay  $\hat{R}^B$  to its depositors. Then bank 2 can profitably deviate by offering a rate just above bank 1's *actual* rate together with a capacity equal to  $\tilde{L}$ . At this rate bank 2 attracts all lenders and make profits.

**Case 2.**  $(L-N) > T$ .

Next, consider the situation where  $(L-N) > T$  so that  $\hat{R}^B < \hat{R}^F$ . Again Game 2 has a "single" subgame perfect coalition-proof equilibrium. But now all banks are inactive in equilibrium. More precisely, the equilibrium looks as follows.

At stage 1, at least one bank—say bank 1—offers  $(\hat{R}^B, \tilde{L})$ . In order to specify firms' equilibrium strategy at stage 1, we need some more notation. Let  $\bar{R}(\hat{R}^B, (\tilde{L}-k), (N-k))$  denote the *actual* deposit rate that a bank

25. When indifferent, we suppose that lenders apply to the bank with the smallest index.

26. The profitability of the deviation does not depend on our assumption that bank 2 gets no funds when it offers the same deposit rate as bank 1.

pays its depositors when it announces  $\hat{R}^B$ , gets  $(\tilde{L}-k)$  deposits and finances  $(N-k)$  projects. Let  $\tilde{k}$  denote the smallest  $k \in N$  for which the following inequality holds

$$\tilde{R}(\hat{R}^B, (\tilde{L}-k), (N-k)) < \bar{R}.$$

Then, in equilibrium, firm  $j$  chooses  $d_j^f = (R_j^f, 1)$  where  $R_j^f$  is such that

$$R_j^f = \begin{cases} \tilde{R}(\hat{R}^B, (\tilde{L}-(k-1)), (N-(k-1))), & \text{if } 1 \leq j \leq (\tilde{k}-1), \\ \bar{R}, & \text{otherwise.} \end{cases}$$

At stage 2, lenders  $1, \dots, m$  finance firm 1; lenders  $(m+1), \dots, 2m$  finance firm 2, etc. . . . Stages 3 and 4 are not reached in equilibrium.

Given the strategies of firms at stage 1, banks cannot profitably deviate. An increase in the deposit rate (leaving the capacity limit  $\tilde{L}$  unchanged), does not change the *actual* rate offered by banks. A decrease in the capacity limit (leaving the deposit rate unchanged) can only raise the transaction costs and is therefore useless. So, let us show that simultaneous changes in  $R_i$  and  $C_i$  cannot be profitable either.

Suppose that bank  $i$  tries to finance  $(N-k)$  firms. First, the capacity limit it chooses must be equal to  $\tilde{L}' = L - (k+1) + 1/m$ .<sup>27</sup> Then, it should be clear that bank  $i$  cannot offer a better deposit rate than before. The maximum rate that  $i$  can pay its lenders is:

$$\hat{R}^B = \frac{(N-k)}{\tilde{L}'} (\bar{Y} - a) - b(\hat{R}^B \tilde{L}', (N-k))$$

$(N-k)/\tilde{L}'$  is smaller than  $N/\tilde{L}$ , so that the first term is smaller than in the definition of  $\hat{R}^B$ .  $b(\hat{R}^B \tilde{L}', (N-k)) > b(\hat{R}^B \tilde{L}, N)$  for any  $R$ , since  $(N-k) < N$  and

$$\frac{R\tilde{L}'}{(N-k)} > \frac{R\tilde{L}}{N}$$

Hence, given the strategies of firms, banks cannot profitably deviate.

Next we show that, given banks' strategies, firms cannot profitably deviate either. It is clear that firms  $j = (\tilde{k}, \dots, N)$  cannot profitably deviate as they already pay the minimum rate  $\bar{R}$ . So, let us concentrate on the situation of firms  $j = (1, \dots, (\tilde{k}-1))$ . Suppose that firm  $j=1$  offers  $R_1^f < \hat{R}^B$ . Then, all but  $(m-1)$  lenders prefer to go to bank 1 which pays a higher rate. Consequently, firm 1 gets no funds directly and must raise credit from a bank at the price  $\bar{Y}$ . At this price, firm 1 makes zero profit, whereas its profits are positive when it raises funds directly from the lenders at the rate  $R_1^f = \hat{R}^B (< \hat{R}^f)$ . Hence, this deviation is not profitable.

Next, consider the case of firm 2. Suppose that firm 2 offers  $R_2^f < \tilde{R}(\hat{R}^B, (\tilde{L}-1), (N-1))$ . Then, all lenders but the  $m$  ones who finance firm 1 prefer to go to bank 1 which pays  $\tilde{R}(\hat{R}^B, (\tilde{L}-1), (N-1))$ . Hence, firm 2 gets no funds directly from lenders and must pay  $\bar{Y}$  to bank 1. Profits thereby vanish so that this deviation is not profitable. The same argument applies to the remaining firms.

Finally, it is easy to see that lenders cannot profitably deviate. ||

*Proof of Proposition 4.4.* First, consider the case where  $(L-N) \leq T$ . Then, there are two possibilities: either  $(L-N) \geq 1$ ; or  $(L-N) < 1$ . In the former case, the coalition-proof equilibrium is ESS. In the latter case, it is ESS provided that the difference  $(\hat{R}^B - \bar{R})$  is not too small.

Suppose that  $(L-N) \geq 1$ . Suppose that a depositor of bank 1 deviates and takes his outside option (this is actually the best alternative). This does not alter the choice of other depositors because bank 1 is still able to finance all projects and to pay the promised deposit rate  $\hat{R}^B$ . Given that  $\hat{R}^B$  is greater than  $\bar{R}$ , applying to bank 1 is evolutionary stable.

By contrast, when  $(L-N) < 1$  and a depositor of bank 1 deviates, bank 1 can finance only  $(N-1)$  projects. Then, its depositors obtain:

$$R = \frac{1}{\tilde{L}} ((N-1)\bar{Y} - a(N-1) - b(\hat{R}^B \hat{L}, (N-1)\hat{L}))$$

where  $\hat{L} := N - 1/m$ . If  $R$  is still bigger than  $\bar{R}$ , applying to bank 1 is evolutionary stable.

Next, consider the case where  $(L-N) > T$ . In equilibrium all banks are inactive. If one of the financiers of firm 1 deviates and takes its outside option (again the best alternative), firm 1 can no longer be financed directly. Its other  $(m-1)$  financiers get 0 and prefer to adopt the strategy of the deviant. However, the situation

27. Otherwise, more than  $k$  firms are able to find direct funding.

of all other lenders remains unchanged, so that the deviation actually affects only a small part of the population of lenders. ||

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## REFERENCES

- BAUMOL, W. J., PANZAR, J. C. and WILLIG, R. D. (1982) *Contestable Markets and the Theory of Industry Structure* (New York: Harcourt, Brace Jovanovich, Inc.)
- BERNHEIM, B. D., PELEG, B. and WHINSTON, M. D. (1987), "Coalition Proof Nash Equilibria, I: Concepts", *Journal of Economic Theory*, **42**, 1–12.
- BRAINARD, W. C. and TOBIN, J. (1971), "Financial Intermediaries and the Effectiveness of Monetary Controls", in Hester, D. D. and Tobin, J. (eds.) *Financial Markets and Economic Activity*, Cowles Foundation Monograph 21.
- CARLSSON, H. and VAN DAMME, E. (1993), "Global Games and Equilibrium Selection", *Econometrica*, **61**, 989–1018.
- CRAWFORD, V. P. (1991), "An "Evolutionary" Interpretation of Van Huyck, Battalio, and Beil's Experimental Results on Coordination", *Games and Economic Behavior*, **3**, 25–59.
- DASGUPTA, P. and MASKIN, E. (1986), "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory", *Review of Economic Studies*, **53**, 1–26.
- DAVIDSON, C. and DENECKERE, R. (1982), "The Influence of Consumer Behaviour on Duopoly Power in Two Stage Games", Michigan State University, mimeo.
- DIAMOND, D. W. (1984), "Financial Intermediation and Delegated Monitoring", *Review of Economic Studies*, **51**, 393–414.
- DIXON, H. (1984), "Existence of Mixed Strategy Equilibria in a Price-Setting Oligopoly with Convex Costs", *Economic Letters*, **16**, 205–212.
- ELLISON, G. (1995), "Basins of Attraction, Long Run Equilibria, and the Speed of Step-by-Step Evolution", *MIT Discussion Paper 96-4*.
- FOSTER, D. and YOUNG, H. P. (1990), "Stochastic Evolutionary Game Dynamics", *Theoretical Population Biology*, **38**, 219–232.
- GOTTARDI, P. and YANELLE, M. O. (1996), "Financial Innovation and Competition Among Intermediaries", *DELTA Discussion Paper*.
- GURLEY, J. G. and SHAW, E. S. (1960) *Money in a Theory of Finance* (Washington: The Brookings Institution).
- HANSEN, R. G. and SAMUELSON, W. F. (1987), "Evolution in Economic Games", *Journal of Economic Behavior and Organization*, **10**, 315–338.
- HARSANYI, J. C. and SELTEN, R. (1988) *A General Theory of Equilibrium Selection in Games* (Cambridge: MIT Press).
- KATZ, M. L. and SHAPIRO, C. (1985), "Network Externalities, Competition, and Compatibility", *American Economic Review*, **75**, 424–440.
- KIM, Y. (1995), "Equilibrium Selection in  $n$ -Person Coordination Games", *Games and Economic Behavior*, (forthcoming).
- KANDORI, M., MAILATH, G. and ROB, R. (1993), "Learning, Mutation, and Long Run Equilibria in Games", *Econometrica*, **61**, 29–56.
- KREPS, D. M. and SCHEINKMAN, J. A. (1983), "Cournot Pre-Commitment and Bertrand Competition Yield Competitive Outcomes", *Bell Journal of Economics*, **14**, 326–338.
- MASKIN, E. (1986), "The Existence of Equilibrium with Price-Setting Firms", *American Economic Review, Paper and Proceedings*, **76**, 382–386.
- MATSUI, A. and MATSUYAMA, K. (1995), "An Approach to Equilibrium Selection", *Journal of Economic Theory*, **65**, 415–434.
- MORRIS, S., ROB, R. and SHIN, H. S. (1995), " $p$ -Dominance and Belief Potential", *Econometrica*, **63**, 145–157.
- RUBINSTEIN, A. and WOLINSKY, A. (1990) "Decentralized Trading, Strategic Behavior and the Walrasian Outcome", *Review of Economic Studies*, **57**, 63–78.
- RUBINSTEIN, A. and WOLINSKY, A. (1987), "Middlemen", *Quarterly Journal of Economics*, **102**, 581–593.
- SCHAFFER, M. E. (1989), "Are Profit-Maximizers the Best Survivors?", *Journal of Economic Behavior and Organization*, **12**, 29–45.
- SELTEN, R. (1975), "Re-Examination of the Perfectness Concept for Equilibrium Points in Extensive Games", *International Journal of Game Theory*, **4**, 25–55.
- STAHL, D. O. (1988), "Bertrand Competition for Inputs and Walrasian Outcomes", *American Economic Review*, **78**, 189–201.
- STEWART, J. (1770) *An Inquiry into the Principles of Political Economy: Being an Essay on the Science of Domestic Policy in Free Nations*.



- STIGLITZ, J. E. and WEISS, A. (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, **71**, 393–410.
- VARIAN, H. (1980), "A Model of Sales", *American Economic Review*, **70**, 651–659.
- VAN HUYCK, J. B., BATTALIO, R. C. and BEIL, R. O. (1990), "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure", *American Economic Review*, **80**, 234–248.
- VAN HUYCK, J. B., BATTALIO, R. C. and BEIL, R. O. (1991), "Strategic Uncertainty, Equilibrium Selection, and Coordination Failure in Average Opinion Games", *Quarterly Journal of Economics*, **106**, 885–910.
- WILLIAMSON, S. D. (1986), "Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing", *Journal of Monetary Economics*, **18**, 159–179.
- YANELLE, M. O. (1989), "The Strategic Analysis of Intermediation", *European Economic Review*, **33**, 294–301.
- YANELLE, M. O. (1988), "On the Theory of Intermediation" (Unpublished Dissertation, Bonn University).
- YOUNG, H. P. (1993), "The Evolution of Conventions", *Econometrica*, **61**, 57–84.