Payments Systems with Random Matching and Private Information

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A model of dynamic risk-sharing is constructed where agents meet pairwise and at random, and there is private information about endowments. Risk sharing is accomplished through dynamic contracts involving credit transactions, and through monetary exchange. A Friedman rule is optimal, and solutions are computed. The welfare costs of inflation and the effects of inflation on the distribution of consumption and wealth are small for an economy calibrated to U.S. data. However, these effects are large when the credit system is relatively unsophisticated.

RECENT ADVANCES IN INFORMATION TECHNOLOGY have greatly improved our ability to move information across locations relative to our ability to move goods. Thus, while a particular transaction might be quite costly in terms of shipping goods or traveling to another location to inspect goods which are to change hands, the cost of transferring wealth from buyers to sellers has dropped dramatically. This decrease in transactions costs is reflected in the growth in the use of alternatives to currency in transactions. In the United States between 1991 and 1995, the nominal value of payments by credit card increased by 81 percent, and the total nominal value of transactions over CHIPS and FedWire (electronic interbank transactions mechanisms) increased by 30.1 percent (see Bank for International Settlements 1996).

In light of these developments, it would seem useful to develop models that allow us to study the use of alternative transactions media in environments where communication is sophisticated, but moving goods across locations is difficult. Such a model is considered here. We consider an environment where infinitely lived agents meet bilaterally and at random, much as in the monetary search models of Kiyotaki and Wright (1989, 1993), Williamson and Wright (1994), or Trejos and Wright (1995). Here, in contrast to the typical monetary search environment, information can be transmitted across locations, so that long-term contracts, interpreted as credit arrangements, are possible. However, there are imperfections in the transmission of information which create a role for currency, so that money and credit can coexist.

The motive for exchange in this model is risk sharing, as in the models studied in

Aiyagari and Williamson (1997a, 1997b, 1997c). In this previous work, we used developments in the literature on dynamic contracts under private information (Green 1987; Spear and Srivastava 1987; Phelan and Townsend 1991; Atkeson and Lucas 1992, 1995; and Aiyagari and Alvarez 1995) to analyze dynamic credit arrangements and to model money and credit. The model considered here has much in common with Aiyagari and Williamson (1997b, 1997c), but those setups have no random matching, that is, all trade takes place at a central location. Aiyagari and Williamson (1997a) is a pure credit version of the model considered here, but that other model has capital accumulation while this one does not. Our approach is related to work by Kocherlakota and Wallace (1997), which in turn builds on Kocherlakota (1996). The Kocherlakota and Wallace model primarily relies on a commitment friction rather than on private information, and they do not compute solutions, as is done here.

In the model, there are two types of agents, those who are risk averse, and those who are risk neutral. Risk-averse agents receive a random endowment each period that is private information. The endowments of risk-neutral agents are constant over time. Each period, a risk-averse agent is matched at random with a risk-neutral agent, and goods cannot be transported across locations. Transactions between a risk-averse and risk-neutral agent involve currency, and/or a centralized credit mechanism. Currency is useful because, at random, information cannot be perfectly transmitted through the credit mechanism. Further, risk sharing is limited by the fact that total consumption of any pair of risk-averse and risk-neutral agents is limited by the sum of their endowments.

A version of the Friedman rule is shown to hold in this environment. In particular, the transactions system works as well with an imperfect credit system and monetary exchange with deflation at the rate of time preference of the risk-neutral agent, as with a perfectly operating credit system. We are interested in computing steady state solutions, in order to study the effects of inflation and the effects of improvements in the credit system given suboptimal rates of inflation.

The main findings are the following. Inflation tends to increase the variability in consumption, conditional on the level of expected utility (which we might think of as a wealth variable). This effect occurs as higher inflation causes agents to hold less real cash balances, so that their ability to insure against income shocks, in instances where they cannot communicate through the credit system, is impaired. Higher inflation also tends to reduce the variability in expected utilities across the population. These effects are small for an economy calibrated to U.S. data. In fact, the cost for the average risk-averse agent of eliminating currency entirely (for example, by attempting to engineer an extremely high inflation rate) is about 2 percent of consumption. However, for an economy where the credit system is very inefficient (there is a parameter in the model that quantifies inefficiency), the quantitative effects of inflation on the distribution of consumption and wealth are substantial, as are the welfare effects.

The remainder of the paper is organized as follows. The model is constructed in section 1, while section 2 shows how efficient allocations are determined. In section 3 we study the properties of a benchmark "pure credit" or efficient allocation. We show that a Friedman rule always achieves the pure credit allocation. Section 4 discusses calibration and the computational exercises, while section 5 is a summary and conclusion.

1. THE MODEL

The population consists of a continuum of infinite-lived agents with unit mass. Half of these agents are risk averse, with preferences given by

$$E_0(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t),$$

where $0 < \beta < 1$, c_t is consumption, and $u(\cdot)$ is strictly increasing, strictly concave, and satisfies decreasing absolute risk aversion. Assume that $u(0) = 0$ and $u'(0) = \infty$. The remaining agents are risk neutral, with preferences

$$E_0(1-q)\sum_{t=0}^{\infty}q^t c_t,$$

where $0 < q < 1$, and c_t denotes consumption. Assume that risk-neutral agents are more patient than risk-averse agents, that is, $q > \beta$.[1]

Each period, each risk-averse agent is matched pairwise and at random with a risk-neutral agent. During the period, goods cannot move between locations (from one matched pair of agents to another matched pair), but information can move freely (subject to some restrictions which I elaborate below). The mechanism which governs the transfer of goods between the risk-averse and risk-neutral agents involves communication with a social planner at some centralized location. It may help to think of the social planner in this instance as a financial intermediary which performs commercial banking and central banking functions.

Two types of transactions can occur between risk-averse and risk-neutral agents: credit transactions and cash transactions. It will help to imagine that there is a "credit machine" and a "cash machine" at each location. The risk-neutral agent receives a fixed endowment x at the beginning of each period. This agent is required (by the planner) to deposit this endowment for use by the credit and cash machines. At the beginning of period t, the risk-averse agent receives a random endowment $y_t \in \{y_0, y_1\}$, where $0 \le y_0 < y_1$. Endowments are i.i.d. over time and across risk-averse agents. Also, the endowment of a risk-averse agent is private information. After receiving the endowment, the risk-averse agent first accesses the credit machine, which has been programmed to recognize her, to exchange information (the particulars of which will be specified below) with her, and to make transfers of consumption goods and currency contingent on this information. The risk-averse agent then accesses the cash machine, which has been programmed at the beginning of the period to exchange currency for consumption goods at the price p_t (that is, the price level, which is the same across all locations).[2] Both the cash machine and the credit machine have access to the same pool of goods, the quantity x that was deposited by the risk-neutral agent. We assume that any currency received by the credit or cash machines is destroyed,

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1. This is required to generate limiting distributions of wealth which exhibit mobility.
2. It is a technological requirement that prices be the same across cash machines in different locations. The machines simply cannot be programmed in any other way.

and that both machines have the capability of printing currency. Consumption goods cannot be moved across locations, and any consumption goods not consumed by the risk-averse agent are consumed by the risk-neutral agent. We assume that risk-neutral agents do not hold currency, but risk-averse agents can carry currency from one period to the next.

2. EFFICIENT ALLOCATIONS

The social planner is given $\psi_0(w_0, m_0)$, the distribution of date 0 expected utilities and real money balances (that is, nominal cash balances deflated by p_0) across risk-averse agents, and w^*, the initial level of expected utility for each risk-neutral agent (note that we suppose that all risk-neutral agents are treated the same). In the first and subsequent periods, risk-averse agents would like to carry out transactions, through the cash and credit machines, in order to share risk with the risk-neutral agents. Credit machines can always identify risk-averse agents, according to their initial (w_0, m_0), their current real money balances, and endowments that were reported to the machine at date 0 and subsequently. However, sometimes the credit machine malfunctions and cannot receive the buyer's current endowment report. A credit machine malfunction occurs with probability ρ, where $0 \leq \rho \leq 1$, and these malfunctions are i.i.d. across machines and over time. Cash machines, though more rudimentary than credit machines in that they cannot receive or transmit information (from/to the planner), do not malfunction. If the credit machine malfunctions, then the transfer to the risk-averse agent from the credit machine cannot be contingent on the risk-averse agent's current endowment. Whether or not the credit machine malfunctions, the risk-averse agent receives instructions (basically, a menu of choices) from the credit machine concerning how to transact with the cash machine. If these instructions are not followed, in which case this would be revealed by the risk-averse agent's money balances in the following period, then we assume that sufficient punishments are available to the social planner that the risk-averse agent will always follow instructions.

Currency can be injected and withdrawn from circulation through the cash and credit machines at each location. Ultimately, of course, standard accounting relationships must hold, and we can think of there being a monetary authority which must respect the constraint

$$\frac{(\overline{M}_t - \overline{M}_{t-1})}{p_t} = \frac{T_t}{p_t},$$

where $\overline{M}_t$ is the money supply per capita, and T_t is the net nominal transfer to private agents. Alternatively, letting $\overline{m}_t \equiv \overline{M}_t/p_t$, $\omega_t \equiv T_t/p_t$ and $\gamma_t \equiv p_t/p_{t-1}$, we have

$$\overline{m}_t - \frac{\overline{m}_{t-1}}{\gamma_t} = \omega_t. \tag{1}$$

That is, current per capita real balances minus per capita real balances in the previous period divided by the gross rate of return on money must equal the per capita money transfer to private agents in units of consumption goods.

Now, following the approach in Aiyagari and Williamson (1996) as well as Aiyagari and Williamson (1997a,b,c), which build on Atkeson and Lucas (1995), we can solve the planning problem recursively, and as if it were the optimization problem of an individual risk-neutral agent who meets a given risk-averse agent forever. To simplify the problem in this way it is necessary that the endowment shocks of the risk-averse agent be i.i.d., but it is not necessary that the "risk-neutral agent" be risk neutral. We confine attention to steady states, where $\gamma_t = \gamma$, and $\omega_t = \omega$ for all t, where γ and ω are constants. Also, the distribution of expected utilities and real money balances across risk-averse agents, $\psi(w, m)$, is constant in the steady state. We suppose that the social planner treats the price path $\{p_t\}_{t=0}^{\infty}$ as given, and confine attention, as indicated, to the constant inflation case. We will subsequently address how the planner chooses γ optimally.

As is common in dynamic contracting problems, the expected utility, w, of the risk-averse agent serves (though here only partially) as a summary statistic of the agent's period 0 level of expected utility and all subsequent reports (through the credit machine) of the agent's endowments. Here, we also need to include the risk-verse agent's real cash balances, m, as of the beginning of the period, in order to impound all the relevant information. We can think of (w,m) as summarizing the risk-averse agent's asset portfolio, where w can be interpreted as a credit balance. In a steady state, the planner's problem can be specified in recursive form in terms of the following Bellman equation, where the function $v(w,m)$ is the maximum level of expected utility of a risk-neutral agent who is paired with a risk-averse agent with an expected utility promise of w and real cash balances m, as if this pairing continued indefinitely. We have

$$v(w,m) = \max \left\{ \begin{array}{c} (1-q)[x - \rho\pi\tau_1(w,m) - \rho(1-\pi)\tau_0(w,m) - (1-\rho)\tau(w,m)] \\ +q \left[\begin{array}{c} \rho\pi v[w_{11}(w,m), m_{11}(w,m)] \\ +\rho(1-\pi)v[w_{10}(w,m), m_{10}(w,m)] \\ +(1-\rho)\pi v[w_{01}(w,m), m_{01}(w,m)] \\ +(1-\rho)(1-\pi)v[w_{00}(w,m), m_{00}(w,m)] \end{array} \right] \end{array} \right\} \quad (2)$$

subject to

$$w = (1-\beta) \left[\begin{array}{c} \rho\pi u(y_1 + m + \tau_1(w,m) - \gamma m_{11}(w,m)) \\ +\rho(1-\pi)u(y_0 + m + \tau_0(w,m) - \gamma m_{10}(w,m)) \\ +(1-\rho)\pi u(y_1 + m + \tau(w,m) - \gamma m_{01}(w,m)) \\ +(1-\rho)(1-\pi)u(y_0 + m + \tau(w,m) - \gamma m_{00}(w,m)) \end{array} \right]$$
$$+ \beta \left[\begin{array}{c} \rho\pi w_{11}(w,m) + \rho(1-\pi)w_{10}(w,m) \\ +(1-\rho)\pi w_{01}(w,m) + (1-\rho)(1-\pi)w_{00}(w,m) \end{array} \right] \quad (3)$$

$$(1-\beta)u(y_i + m + \tau_i(w,m) - \gamma m_{1i}(w,m)) + \beta w_{1i}(w,m)$$
$$\geq (1-\beta)u(y_i + m + \tau_j(w,m) - \gamma m_{1j}(w,m)) + \beta w_{1j}(w,m), \ (i,j) = (1,0), \ (0,1), \quad (4)$$

$$(1-\beta)u(y_i + m + \tau(w,m) - \gamma m_{0i}(w,m)) + \beta w_{0i}(w,m)$$
$$\geq (1-\beta)u(y_i + m + \tau(w,m) - \gamma m_{0j}(w,m)) + \beta w_{0j}(w,m), \ (i,j) = (1,0), \ (0,1), \quad (5)$$

$$\tau_i(w,m) \ \leq \ x, \ i = 0, \ 1,$$
$$\tau(w,m) \ \leq \ x \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (6)$$

$$x \geq m + \tau_i(w,m) - \gamma m_{1i}(w,m) \geq -y_i, \ i = 0, \ 1$$
$$x \geq m + \tau(w,m) - \gamma m_{0i}(w,m) \geq -y_i, \ i = 0, \ 1 \qquad\qquad\qquad (7)$$

$$m_{ij}(w,m) \geq 0, \ i,j = 0, \ 1 \qquad\qquad\qquad\qquad\qquad\qquad (8)$$

$$w, w_{ij}(w) \in [\underline{w}, \overline{w}], \ i,j = 0, \ 1 \qquad\qquad\qquad\qquad\qquad (9)$$

where $\underline{w} = \pi u(y_1 - y_0)$ and $\overline{w} = \pi u(y_1 + x) + (1 - \pi)u(y_0 + x)$. In the problem above, the planner chooses $\tau_i(w, m)$, the transfer the risk-averse agent receives from the credit machine when it does not malfunction, when the risk-averse agent's endowment is y_i; $\tau(w,m)$, the transfer from the credit machine when the machine malfunctions (note that this transfer cannot be contingent on the risk-averse agent's endowment); $w_{ij}(w,m)$, the risk-averse agent's expected utility in the next period when the credit machine is in mode i ($i = 0$ for malfunction) and the buyer's endowment is y_j; and $m_{ij}(w,m)$, the buyer's quantity of real cash balances at the beginning of the next period, with subscripts defined similarly. Here, (3) is a promise-keeping constraint, and (4) and (5) are incentive constraints. The resource constraints at the credit machine and cash machine are given by (6) and (7), respectively.[3] These are constraints on the net transfers from the credit and cash machines given by the nonnegativity constraints on the consumption of risk-averse and risk-neutral agents. The inequalities (8) are nonnegativity constraints on real cash balances, and (9) are constraints on expected utility promises, where $\underline{w}$ is the lowest level of expected utility which it is incentive compatible for the planner to force on any risk-averse agent, and $\overline{w}$ is the highest possible level of expected utility, which results from the risk-averse agent receiving the total endowment of the pair of agents in perpetuity.

Now that we have a recursive representation of the planner's problems we can proceed to an analysis of steady-state efficient allocations. We can think of solving for the steady state by first solving (2) subject to (3)–(9) to obtain $v(w,m)$, and optimal transfers, expected utility promises, and real cash balance recommendations. This then implies a dynamic stochastic path for (w,m) and we can accordingly solve for $\psi(w,m)$, the steady-state distribution of expected utility entitlements and real cash balances across buyers.

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3. Note that I have ignored the resource constraint at the credit machine that states that the risk-averse agent cannot transfer more than her endowment plus her money balances at the beginning of the period to the credit machine. This constraint would be redundant, as (7) implies that it holds.

As in Aiyagari and Williamson (1997b), it is straightforward to show that the problem (2) subject to (3)–(9) essentially collapses into a single-state-variable problem. That is, we can show that the value function takes the form

$$v(w,m) = -(1-q)m + \theta(w),$$

where $\theta(\cdot)$ is a function, and we can perform the change of variables $\tau_i^*(w) = \tau_i(w,m) - m$, $\tau^*(w) = \tau(w,m) - m$, $w_{ij}^*(w) = w_{ij}(w,m)$, and $m_{ij}^*(w) = m_{ij}(w,m)$. Then, the choice variables in the optimization problem (the variables with $*$ superscripts) are independent of m.[4] Therefore, the risk-neutral agent's current consumption allocation, future real cash balances, and future expected utility entitlement are determined only by the current expected utility entitlement and the current endowment. Clearly, this simplification will be important in computing solutions.

3. A PURE CREDIT ECONOMY

As a benchmark, it is useful to consider the special case where $\rho = 1$, so that cash machines are never used. That is, given that credit machines never malfunction, any transfers that can be achieved through the cash machine can be done at the credit machine, and with more information. Thus, cash machines are redundant, and we can think of this as a setup where money is not valued. The recursive problem for the planner then reduces to

$$z(w) = \max \begin{cases} (1-q)[x - \pi\tau_1(w) - (1-\pi)\tau_0(w)] \\ +q[\pi z(w_1(w)) + (1-\pi)z(w_0(w))] \end{cases} \tag{10}$$

subject to

$$w = (1-\beta)\big[\pi u(y_1 + \tau_1(w)) + (1-\pi)u(y_0 + \tau_0(w))\big]$$
$$+ \beta[\pi w_1(w) + (1-\pi)w_0(w)] \tag{11}$$

$$(1-\beta)u(y_1 + \tau_1(w)) + \beta w_1(w) \geq (1-\beta)u(y_1 + \tau_0(w)) + \beta w_0(w) \tag{12}$$

$$(1-\beta)u(y_0 + \tau_0(w)) + \beta w_0(w) \geq (1-\beta)u(y_0 + \tau_1(w)) + \beta w_1(w) \tag{13}$$

$$x \geq \tau_i(w) \geq -y_i, \quad i = 0, 1 \tag{14}$$

$$w, w_i(w) \in [\underline{w}, \overline{w}], \quad i = 0, 1. \tag{15}$$

Here, $z(w)$ is the value function, $\tau_i(w)$ is the transfer to the risk-averse agent through the credit machine when the buyer's endowment is y_i, and $w_i(w)$ is the future expected utility promise made to the risk-averse agent when her endowment is y_i. Equation

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4. Note that (6) will not bind at the optimum, since we will have $m_{ij}(w,m) = 0$ for $(i,j) = (0,0)$, $(1,0)$, $(1,1)$. If these constraints were binding, then the problem would not collapse to a single-state-variable problem.

(11) is the promise-keeping constraint, (12) and (13) are the incentive constraints, (14) are the resource constraints, and (15) describe the upper and lower bounds on expected utilities, with $\underline{w}$ and $\bar{w}$ defined as before.

The steady-state allocation defined above is identical to the one analyzed in Aiyagari and Williamson (1996). In that paper, we showed that the steady-state allocation was efficient, and that there exists a unique limiting distribution $\psi(w)$ which exhibits mobility. It is straightforward to show, following Aiyagari and Williamson (1997b), that when $\rho \neq 1$, $\gamma = q$ is optimal, that is, a Friedman rule that drives the nominal interest rate to zero achieves the efficient locations (the pure credit allocation). We can prove this by showing that, when $\rho \neq 1$, and $\gamma = q$, the problem (2) subject to (3)–(9) is equivalent to (10)–(15).

4. CALIBRATION AND COMPUTATION

From section 3, we know that a Friedman rule result holds in this economy so that, even if the credit system is imperfect (that is, $\rho < 1$) the correct monetary policy can make up for this completely. What we would now like to investigate is how wrong things can go when the monetary growth rate is suboptimal in the steady state, solving the planner's problems for a given suboptimal γ. In particular, in this section we will investigate the quantitative effects of inflation for a given value of ρ (that is, fixing the credit friction), and the effects of increasing ρ with the inflation rate fixed.

The first step is to calibrate the model by choosing functional forms and reasonable parameter values. The utility function I use is $u(c) = 1 - e^{-\alpha c}$, with $\alpha = 1$, which implies a coefficient of relative risk aversion of unity at the mean endowment (which will be unity). A constant absolute risk aversion utility function is convenient for computational purposes in this model as it is bounded. We interpret a period as one quarter, and set y_0, y_1, and π so as to match the variability in quarterly household income. Using PSID data, Aiyagari (1994) argues that a first-order autoregression closely matches the time series properties of annual earnings, with a range of .23 to .53 for the first-order serial correlation coefficient, and a coefficient of variation in unconditional earnings of 20 to 40 percent. Since it is not tractable to introduce serial correlation in endowments for risk-averse agents, we must do the best we can to fit an i.i.d. endowment shock in the model to the data. This is not too problematic, as the estimated serial correlation in annual data is low, and serial correlation for quarterly data would then be even lower. If we take the coefficient of variation to be 30 percent for annual data, then if quarterly income is i.i.d., the coefficient of variation for quarterly data would be 60 percent. Thus, we set $\pi = .5$, $y_0 = 1 - \varepsilon$, and $y_1 = 1 + \varepsilon$, with $\varepsilon = .6$. The remaining parameters are β, q, ρ, and ψ. We assumed at the outset that $q > \beta$, which is required for there to exist a limiting distribution of expected utilities with mobility.[5]

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5. In this model, $1/q - 1$ plays the role of an interest rate, in terms of how the allocation is computed. In related models with an endogenous interest rate, for example, Atkeson and Lucas (1995), and Aiyagari and Williamson (1997a,b,c), the interest rate is typically larger than the discount rate, as we have essentially assumed here.

We want the average discount factor across the population to be consistent with a quarterly interest rate of 1 percent, as in the real business cycle literature (Prescott 1986), so we then have $(q+\beta)/2 = .99$. Then, as a convenient benchmark, we let $q = .99 + v$ and $\beta = .99 - v$, and then find the value of v such that, in the steady state, the risk-neutral agent is indifferent between autarky and the steady-state allocation. That is, the risk-averse agents receive all the gains from trade in the steady state for the calibrated set of parameters. The parameter ρ is then set so that the model matches the available evidence on the use of currency relative to cash alternatives in transactions. A survey of households by the Federal Reserve (Avery, Elliehausen, Kennickell, and Spindt 1987), conducted in 1984, found that 24 percent of the value of transactions is carried out in currency. In the model, the steady-state quantity of currency transactions is

$$T_c = \frac{1}{2} \int \int \{[1 - (1-\rho)\pi]m + (1-\rho)\pi \mid m - \gamma m_{01}(m, w) \mid\} d\psi(m, w),$$

and the steady-state quantity of credit transactions is

$$T_{nc} = \frac{1}{2} \int \int \left[\begin{matrix} \rho\pi \mid \tau_1(m, w) \mid \\ + \rho(1-\pi) \mid \tau_0(m, w) \mid + (1-\rho) \mid \tau(m, w) \mid \end{matrix} \right] d\psi(m, w).$$

We want to set ρ so that $T_c/(T_c + T_{nc}) = .24$. When the Federal Reserve survey was done, the inflation rate was approximately 1 percent per quarter, so we set $\gamma = 1.01$ for calibration purposes.

Solutions were computed as follows. First, grids were chosen for the two state variables, w and m. The lower bound on the expected utility of the risk-averse agent is $\pi u(y_1 - y_0) + (1 - \pi)u(0)$, which is the minimum incentive compatible level of expected utility that can be imposed on a risk-averse agent, and the lower bound on m is zero. The upper bound on expected utility is $\pi u(y_1 + x) + (1 - \pi)u(y_0 + x)$. Since choice variables in the planner's problem are independent of m, it is only necessary to solve the problem at each point along the w grid, and for a single value for m, say $m = 0$, and then use this solution to determine what the solution is for all points on the grid. We make an initial guess for the function $\theta(w)$, and then use value iteration to arrive at the solution for $\theta(w)$. At each iteration, $\theta(w)$ is updated by fitting a third-order Chebychev polynomial (plus an additional term, $1/(1-w)$, which performed well in fitting the value function) to the values computed for the value function at points on the grid on the previous iteration. When convergence is achieved, then the decision rules are interpolated across a finer grid, and a matrix of Markov transition probabilities for the state w is constructed as an approximation using a lottery over the two closest grid points. A limiting distribution over w is then computed.

To match the real interest rate and the evidence from the Federal Reserve survey on household transactions, we set q and β very close to .99 (that is, v is very small, on the order of 10^{-5}, in the steady state to achieve the desired split of the surplus between risk-neutral and risk-averse agents), and $\rho = .81$.
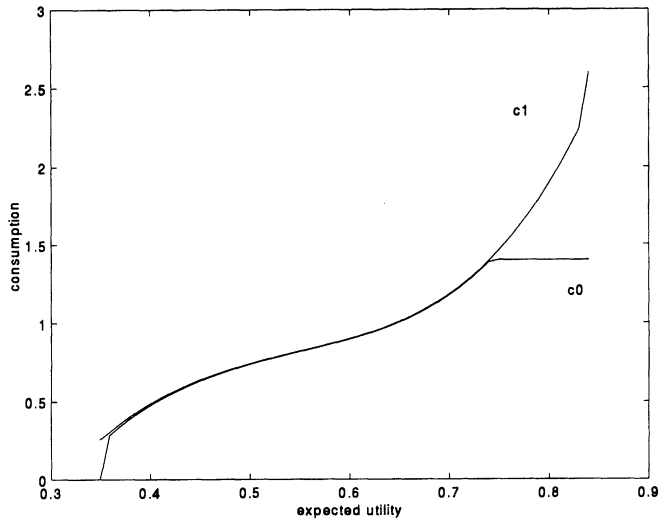
FIG. 1. Pure Credit Economy

Effects of Inflation with Parameters Set at Calibrated Values

We first consider the effects on the steady state for parameters calibrated to the U.S. economy, as described above. The results are in Figures 1 through 4 and Table 1. Figures 1 and 2 show results for the pure credit economy, which as we have shown is equivalent to the Friedman rule economy ($q = \gamma$) for any ρ.

In Figure 1, consumption profiles in the high- and low-income state for the risk-averse agent are plotted against w, expected utility. Note that, conditional on expected utility, there is very little variability in consumption, at least for the middle range of expected utilities. Near the upper bound on expected utilities, however, the pairwise resource constraint (14) binds; achieving something close to full insurance for the risk-averse agent would require more consumption goods in the low-income state than the risk-averse agent and the risk-neutral agent have between them. Also, near the lower bound on expected utilities, insurance against the income shock for the risk-averse agent is poor, as the lower bound on expected utilities (15) binds. That is, due to the fact that the lower bound on expected utilities restricts possible future penalties, there must be a larger gap between current consumption in the high-income and low-income states to induce truth-telling.

Figure 2 shows the limiting distribution of expected utilities in the pure credit economy. The bimodal feature of the distribution is associated with the binding pairwise resource constraints, a characteristic that makes the allocation problem here different from one in an environment where all agents meet in one location. The limiting distributions studied in Aiyagari and Williamson (1997b, c) tend to be unimodal. It is difficult to give clear intuition for why the binding resource constraints create a tendency toward bimodal limiting distributions other than to point out that the binding resource
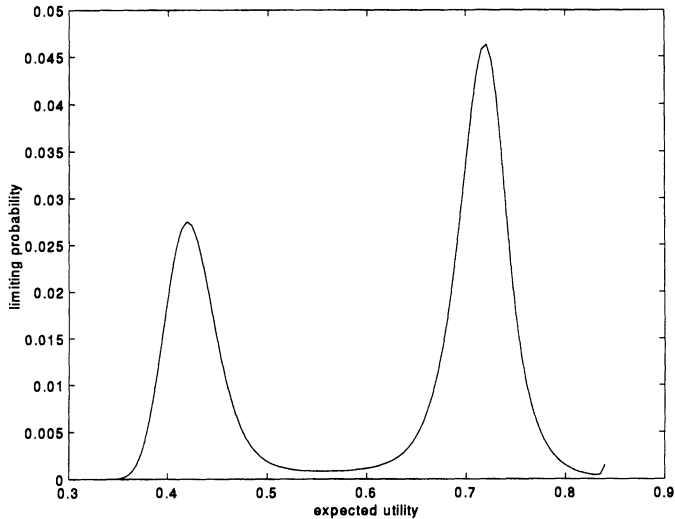
FIG. 2. Pure Credit Economy

constraint for high w works through the binding incentive constraint (for the agent with high income) to alter the law of motion for w in such a way that agents near the mean of the distribution will drift either up or down, while agents with high w tend to drift down and those with low w drift up. Note that, comparing Figures 1 and 2, a significant fraction of risk-averse agents will face binding resource constraints (and therefore poor consumption smoothing) for high w, but a negligible fraction of agents suffer poor consumption smoothing for low w.

Now we examine how inflation affects the steady state, fixing ρ at its calibrated value, $\rho = .81$. In Table 1, the entries in the rows are, respectively, annualized inflation rates, the mean expected utility in the steady state of risk-averse agents, the standard deviation of expected utility for risk-averse agents, the standard deviation of consumption for risk-averse agents, the fraction of the value of total transactions accounted for by currency, a measure of the distance between the Friedman rule (-3.94 percent annual inflation) steady-state distribution of expected utilities and the computed steady-state distribution of expected utilities for risk-averse agents (described in more detail below), the welfare cost of inflation as a fraction of consumption for the average risk-averse agent, and a similar measure for the risk-neutral agent. The welfare costs to each type of agent in rows 7 and 8 of Table 1 are measured in a conventional way. For the risk-averse agent, we want to compute what the average risk-averse agent would pay, in units of consumption, to live in the Friedman rule economy in row 1 of Table 1, rather than the economy with that particular inflation rate. We then express the welfare change in units of consumption relative to average consumption in the Friedman rule economy. That is, if U_F (U_S) is expected utility of the average risk-averse agent in the Friedman rule (suboptimal) economy, then given the constant absolute risk aversion utility function with $\alpha = 1$, the welfare cost is com-

TABLE 1

$\rho = .81$

Annual Inflation Rate	−3.94%	10%	100%	1500%	>non-monetary threshold
Mean Expected Utility	.6092	.6081	.6063	.5981	.5967
Standard Deviation of Expected Utility	.1411	.1415	.1418	.1408	.1406
Standard Deviation of Consumption	.3738	.3669	.3680	.4407	.4541
Currency Transactions	.2452	.2376	.2039	.0274	0
Welfare Cost	0	.0241	.3177	2.5734	3.0297
Cost to Risk-adverse Agents	0	.0018	.0048	.0184	.0207
Cost to Risk-neutral Agents	0	−.0027	−.0048	−.0023	−.0016

puted as $\ln (U_F/U_S)$. For the risk-neutral agent, the corresponding welfare measure (in row 8) is just the change in average consumption for risk-neutral agents, divided by consumption in the Friedman rule economy.

What is called a "welfare cost" in row 6 of Table 1 is a measure of the distance between the Friedman rule distribution and the relevant suboptimal distribution of expected utilities for risk-averse agents. It is useful to use this measure, as there can be effects of inflation on the distribution that will not show up in changes in the mean. Since for the distance measure we use it is necessary that limiting probabilities be bounded away from zero, we use a kernel estimation technique (Silverman 1986) to derive an estimated limiting density function for the expected utilities of risk-averse agents. That is, we estimate the density function $f(x)$ associated with the limiting distribution of expected utilities of risk averse agents by

$$f(x) = \frac{1}{h} \sum_{i=1}^{n} p_i K\left(\frac{x - w_i}{h}\right),$$

where h is the "window width," p_i is the limiting probability associated with expected utility level w_i, n is the number of grid points for expected utilities, and $K(\cdot)$ is a probability density function. For $K(\cdot)$, we used the standard normal density function. Choosing a grid for x, we let $f_j = f(x_j)$ for $j = 1, 2, \ldots, \ell$, and then compute the "welfare loss," $Z(g,f)$ associated with the limiting distribution $f(x)$, by

$$Z(g, f) = \sum_{j=1}^{\ell} h g_j (\log g_j - \log f_j),$$

where $g(x)$ denotes the limiting distribution under a Friedman rule. This distance measure is adapted from an approach in information theory used by Kullback (1959).

The primary effect of an increase in the inflation rate is to reduce the average quantity of real cash balances held by each risk-averse agent. As a result, these agents are less capable of smoothing consumption when they cannot use the credit machine to smooth consumption given their income shock. Thus, conditional on expected utility,

the variability of consumption should increase, but given that the incentive structure is also affected, there will be a change in the steady-state distribution of expected utilities. Table 1 shows that the mean level of expected utility for risk-averse agents decreases with inflation, while the standard deviation of expected utilities first increases and then falls. The standard deviation of consumption across risk-averse agents decreases and then increases with inflation. The effects on welfare are quite small, as shown by rows 7 and 8 of Table 1. In particular, the cost of driving money out of the system completely, which would occur if an inflation rate in excess of about 2,000 percent per annum were attempted, is only about 2 percent of consumption for the average risk-averse agent (see column 5 of Table 1). Thus, the costs of inflation here are an order of magnitude lower than the costs calculated by Cooley and Hansen (1989) for a real business cycle model with a cash-in-advance constraint. The difference in welfare costs is primarily due to the fact that there is an alternative in this model to making transactions with cash whereas there are no such alternatives in cash-in-advance or related representative-agent approaches to monetary analysis. Further, there are no effects of inflation on labor-leisure choice (Cooley and Hansen 1989), growth (Dotsey and Ireland 1996), or the costs of credit (Lacker and Schreft 1996). In Table 1, the fraction of the value of transactions using currency falls with inflation, but not markedly so for low rates of inflation. In moving from a Friedman rule to a 10 percent inflation, the percentage of transactions involving currency drops only a small amount, from 24.5 percent to 23.8 percent.

Figures 3 and 4 show consumption profiles given $\rho = .81$ and 10 percent annual inflation. Figure 3 shows consumption in the high ($c11$) and low ($c10$) income states when the risk-averse agent is able to smooth consumption through the credit system, while Figure 4 shows the corresponding consumption profiles when the agent smooths consumption by using money. Under the Friedman rule, Figures 3 and 4 would be identical, but here we can see the effect of a moderate inflation in increasing the variability of consumption, conditional on expected utility, in Figure 4. Figure 5 shows the difference between the Friedman rule limiting distribution and the limiting distribution when money is driven out of the economy. The "eyeball distance" between the two distributions seems small, which is consistent with the small welfare costs of inflation in rows 7 and 8 of Table 1. This tells us that, in terms of the "welfare cost" in row 6 of Table 1, three is a small number. This will be useful as a benchmark for the next section.

Inflation in an Economy with a Poor Credit System

In the previous section, with parameters calibrated to U.S. data, the welfare effects of inflation and the social value of currency were very small. Further, given that most evidence (see, for example, Bank for International Settlements 1996) indicates that noncash transactions media have seen increased use in the United States relative to currency recently, these costs should be viewed as upper bounds. However, it would be useful to know how inflation affects an economy where the payments system is less sophisticated. In the model, ρ measures the efficiency of the payments system, and in
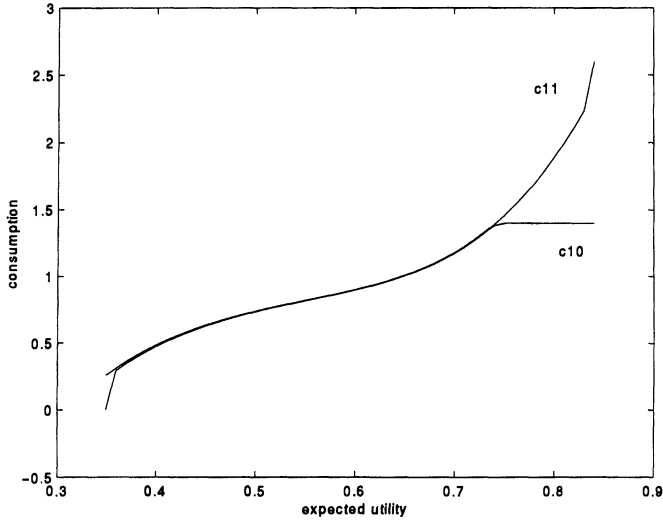
FIG. 3. Rho = .81, 10 Percent Inflation

this section we will replicate the computational exercises of the previous section for an economy identical in all respects, except that $\rho = .05$, an arbitrarily chosen number close to zero. Note that if $\rho = 1$, then the credit system works perfectly and money is not valued, while if $\rho = 0$, transactions using cash are the only means for smoothing consumption in the face of idiosyncratic shocks to income.

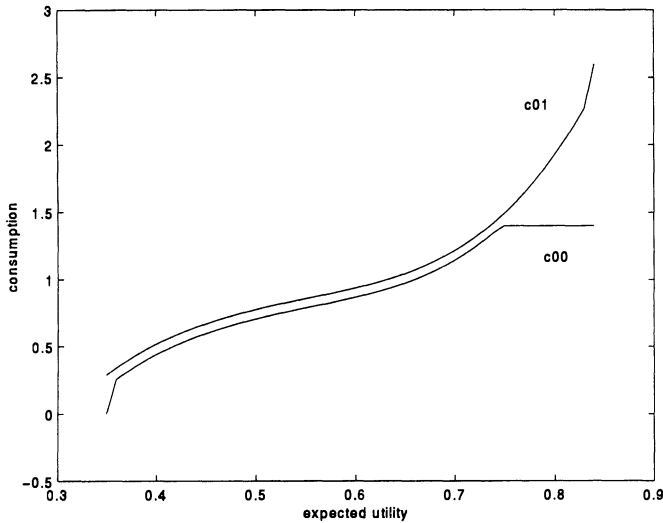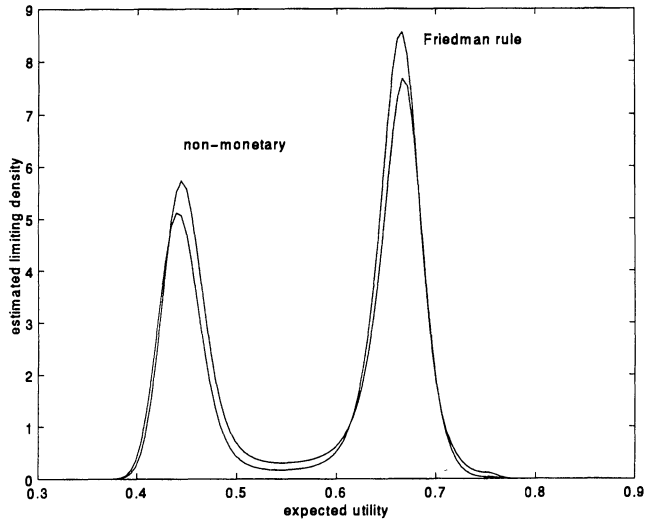The entries in Table 2 correspond to those in Table 1. Note in particular that the

FIG. 4. Rho = .81, 10 Percent Inflation

FIG. 5. Rho = .81

welfare costs of inflation, in terms of the effects on the risk-averse agent, are much larger in Table 2 than in Table 1. The much larger numbers in row 6 of Table 2 indicate that in this economy inflation induces much more pronounced effects on the distribution of expected utilities across risk-averse agents. Also, in row 7 of the table, the welfare effects on the average risk-averse agent are also much larger, to the point where the average risk-averse agent would be willing to give up almost 7 percent of average consumption to live in an economy with a Friedman rule rate of inflation, rather than an economy without currency (as compared to about 2 percent in Table 1).

Note also that, in row 3 of Table 2, the standard deviation of expected utilities for risk-averse agents drops dramatically at high rates of inflation. Figure 6 shows the distributions of expected utilities corresponding to the Friedman rule economy and the economy where money is not held. Here, when there is no currency (but the need for it is great), there is not only a large decrease in the dispersion of expected utilities

TABLE 2

$\rho = .05$

Annual Inflation Rate	−3.94%	10%	100%	1500%	>non-monetary threshold
Mean Expected Utility	.6092	.6064	.5843	.5689	.5685
Standard Deviation of Expected Utility	.1411	.1419	.1365	.0349	.0256
Standard Deviation of Consumption	.3738	.3566	.2091	.5270	.6014
Currency Transactions	.4930	.4978	.5128	.3151	0
Welfare Cost	0	.4300	14.92	1102	2006
Cost to Risk-adverse Agents	0	.0046	.0417	.0684	.0692
Cost to Risk-neutral Agents	0	−.0067	−.0564	−.0167	.0028
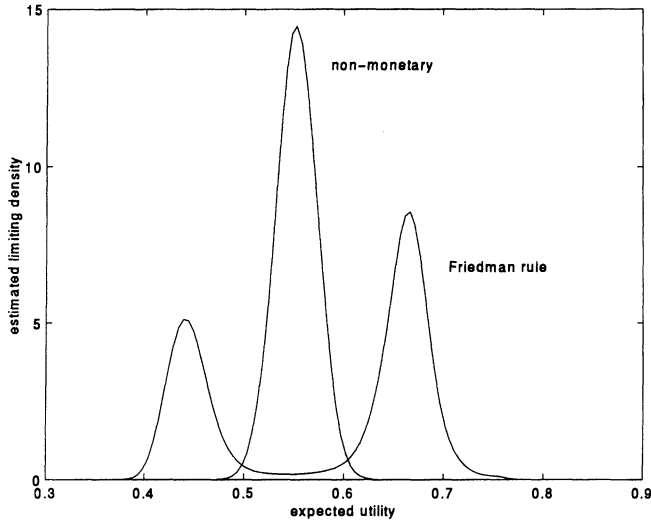
FIG. 6. Rho = .05

across risk-averse agents, but the distribution also becomes unimodal rather than bimodal. This is due mainly to the fact that the decrease in dispersion implies that the resource constraint in the planner's problem (7) does not bind. As was mentioned earlier, this constraint will tend to bind only for high values of w, and if it binds this tends to produce a bimodal distribution of expected utilities.

In row 4 of Table 2, the effect of inflation on the variability in consumption is nonmonotonic, reflecting the effects of two opposing forces. The first force is due to the fact that higher inflation tends to increase the variability in consumption given the level of expected utility, since risk-averse agents are less able to smooth consumption with lower cash balances. The second force is that the variability in consumption will tend to fall as the variability in expected utilities falls, since average (across income states) consumption increases monotonically with w. For low rates of inflation the second effect dominates, while for high rates of inflation the first does.

Perhaps surprisingly, the fraction of the value of transactions accounted for by currency (row 5 in Table 2) increases with the inflation rate at low levels of inflation, and then decreases. There are two opposing forces at work here. First, inflation has a tendency to decrease real cash balances conditional on expected utility, for each risk-averse agent. As a result, given the distribution of expected utilities across risk-averse agents, money is used less in transactions as the inflation rate increases. The second effect is related to the fact that the distribution of expected utilities changes with inflation. When w is high, the resource constraint (7) tends to bind, which implies that there is a sense in which risk-averse agents cannot purchase all the goods they want, given prices. They know this in advance, when they visit the credit machine, and therefore tend to get less currency than otherwise to make purchases from the cash machine.

This problem becomes more acute as w approaches its upper bound, where there will be no currency transactions even if the credit machine malfunctions. An increase in the inflation rate tends to reduce the dispersion in the distribution of expected utilities which then causes the fraction of risk-averse agents whose cash purchases are constrained to decrease. This then has the effect of tending to increase the average quantity of money held, and the quantity of monetary transactions for the population as a whole. At low rates of inflation, the second effect dominates, while the first effect dominates at high rates of inflation.

5. SUMMARY AND CONCLUSION

We have studied a random matching model with private information in which risk-averse agents insure against idiosyncratic income shocks through long-term credit arrangements and by holding money. The credit system is imperfect in that, at random, agents may not be able to communicate fully with the centralized credit agency. In spite of this imperfection, the transactions system will work perfectly; that is, the steady state allocation is efficient, if the quantity of money is governed by a Friedman rule, with deflation at the rate of time preference of the risk-neutral agents in the model. Solutions to the model were computed to determine the quantitative effects of sub-optimal money growth, first in an economy calibrated to U.S. data, and second in an economy with a relatively unsophisticated credit system. In general, the variability in consumption, conditional on the level of expected utility at the beginning of the period, tends to increase with inflation, and the variability in expected utilities across the population tends to decrease. In the first case, the welfare effects of inflation are quite small, but these welfare effects increase dramatically in the second case where cash transactions are much more important.

The primary difference between this model and the ones in Aiyagari and Williamson (1997b,c) is the random matching feature, so it would be useful at this point to discuss how random matching matters for the solution to the problem, and its implications for the results. In some random matching models, for example the absence-of-double-coincidence monetary environments studied by Kiyotaki and Wright (1989, 1993), random matching not only restricts how goods may move among economic agents, it makes more palatable the assumption that information does not flow between pairs of would-be trading partners, or over time. In our environment, as in models studied by Kocherlakota (1996), Aiyagari and Williamson (1997a), and Kocherlakota and Wallace (1997), information does flow across locations (though perhaps with some friction which provides a role for money), which permits a role for credit arrangements. Now, the fact that information is communicated to a central location in our model implies that the problem solved by a social planner here is much like the problems solved in Aiyagari and Williamson (1997b, c) in environments where all agents meet in a central location. There are two primary differences here implied by random matching. First, random matching implies a resource constraint on pairwise allocations. Clearly if the constraint binds, it matters, and solu-

tions were computed in the paper where this constraint was binding (and mattered in a big way for the resulting allocation) and where it was not. Second, solutions are computed as if there were a social planner associated with each level of expected utility of the risk-averse agent, with all these planners trading on a bond market at a fixed interest rate. This fixed interest rate is the discount rate for the risk-neutral agent. In Aiyagari and Williamson (1997b, c), we study economies with an endogenous interest rate, while Aiyagari and Williamson (1997a) is an endogenous interest rate economy with random matching and capital accumulation. We did not follow Aiyagari and Williamson (1997a) in allowing capital accumulation, as the approach here is somewhat less computationally taxing, and because the form of capital accumulation in Aiyagari and Williamson (1997a) would not add a great deal of additional insight.[6]

Dealing with some types of commitment issues in this random matching environment could be somewhat difficult, though it would be straightforward to permit reversion to autarky here, as in Kocherlakota and Wallace (1997). Perhaps more interesting are the commitment issues examined in Aiyagari and Williamson (1997b), where agents can abandon long-term credit arrangements but are still able to trade on a competitive money market. In that type of environment, a Friedman rule is no longer optimal, as higher inflation improves incentives by causing a decrease in the value of defecting.

This model might be successfully applied to other payments system issues, as spatial separation and the flow of information seem key to the analysis of such issues (see also Freeman 1996, 1997). For example, the random matching environment might lend itself to the study of the optimal design of payments systems subject to systemic risk. Consumers might write long-term contracts with financial intermediaries at spatially dispersed locations, with costly communication between these intermediaries and some central agency through which settlement might be made. Now, suppose there were some risk associated with each financial intermediary and its ability to meet its obligations through the centralized settlement mechanism. We might then ask how individual intermediaries would trade off the costs of communication and settlement against the costs associated with bearing system-wide risk, and whether they would choose to make this trade-off in a socially optimal way.

6. The additional mileage would be minimal as capital accumulation in Aiyagari and Williamson (1997a) is a device for achieving an endogenous interest rate with $q > \beta$, which then implies that there is a steady-state limiting distribution with mobility. The model is set up in such a way that capital accumulation decisions are independent of the problem determining the allocation of consumption goods at each location.

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