Barter and Monetary Exchange Under Private Information

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We develop a model of production and exchange with uncertainty concerning the quality of commodities and study the role of fiat money in ameliorating frictions caused by private information. The model is specified so that, without private information, only high-quality commodities are produced, and there is no welfare gain from using money. With private information, there can be equilibria (and sometimes multiple equilibria) where low-quality commodities are produced, and money can increase welfare. Money works by promoting useful production and exchange. In efficient monetary equilibria, agents adopt strategies that increase the probability of acquiring high-quality output. (*JEL* E40, D82, D83)

> "To barter is to exchange goods for other goods rather than money. This was common in early days. Presumably, however, the deal was not always fair. Barter is from the old French *barater*—to cheat!"
>
> [Mike Atchison, 1991 p. 23]

This paper develops a model of production and exchange under private information concerning the quality of commodities. Qualitative uncertainty and the impediments it presents to exchange have been important elements of the economics of information at least since the contribution of George A. Akerlof (1970). In the spirit of his model, we assume that inferior-quality commodities—"lemons"—are cheaper to produce but less desirable for consumption. Since it is not always possible for consumers to discern the quality of every commodity they may have occasion to desire, there is an incentive for sellers to produce low-quality output and attempt to cheat uninformed buyers. Buyers therefore face a risky decision every time they cannot identify a commodity's quality. We construct a model that captures these ideas and use it to study exchange both with and without fiat money, defined as an intrinsically worthless but universally recognizable medium of exchange.

Many traditional discussions of money have emphasized its function as a medium of exchange and especially its role in overcoming the "double coincidence of wants problem" with pure barter (W. Stanley Jevons, 1875). The focus is often on the intrinsic properties of objects that make them more or less natural media of exchange, including properties such as a relatively low storage or exchange cost and a relatively high cost of producing the object privately, such as counterfeiting or digging precious metals out of the ground (Karl Menger, 1892). Recently, some of these ideas have been formalized using search-theoretic equilibrium models of the exchange process (see e.g., Nobuhiro Kiyotaki and Wright, 1989, 1991, 1993) (see Joseph M. Ostroy and Ross M. Starr [1990] for a

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survey of earlier related work). In these models money can arise endogenously as a medium of exchange, leading to reductions in the search and transactions costs associated with direct barter.

Traditional discussions have also argued that, in addition to helping to reduce the double-coincidence problem, money is important in mitigating frictions associated with moral hazard or adverse selection. These frictions can be impediments to exchange when agents have limited opportunities for enforcing contracts and there is private information concerning the quality of goods or the intentions of agents to honor private claims.[1] Armen A. Alchian (1977 p. 139) has gone so far as to argue that overcoming the double-coincidence problem is at most a minor part of what money accomplishes, and private information is the principal friction underlying the institution of monetary exchange:

> "It is not the absence of double coincidence of wants, nor the costs of searching out the market of potential buyers and sellers of various goods nor of record keeping, but the costliness of information about the attributes of goods available for exchange that induces the use of money in an exchange economy."

The basic framework adopted here is related to the search-theoretic literature, in the sense that agents meet randomly in pairs over time, they carry unit inventories (consisting here of either good-quality commodities, bad-quality commodities, or fiat currency), and they trade if and only if it is mutually agreeable. However, we abstract from the double-coincidence problem by assuming that all commodities of a given quality provide the same utility to all consumers. This makes pure barter easy and means that there is no welfare-improving role for money without private information. We then add the assumption that sometimes an individual may be unable to recognize the quality of commodities that are for sale. As compared to models based on the double-coincidence friction, now buyers have to decide whether or not to purchase commodities of unknown quality, and sellers have to choose the quality of their output. This leads to some different implications and different interpretations of the role of fiat money.[2]

We begin by studying nonmonetary equilibria, where fiat currency does not circulate. In this case, private information may or may not lead to agents producing low-quality commodities and trying to pass them off on uninformed consumers. If the private-information problem is not too severe, there will exist an equilibrium where only high-quality output is produced; if it is more severe, some low-quality output will necessarily be produced; and if it is severe

[1] General discussions are contained in Karl Brunner and Allan H. Meltzer (1971) and Armen A. Alchian (1977). Models where money helps ameliorate lemons problems in the exchange of goods are constructed by Robert G. King and Charles I. Plosser (1986) and Dan Bernhardt and Merwan Engineer (1987). Scott Freeman (1985) and S. Rao Aiyagari (1989) consider models with private information concerning the quality of assets. Other papers in which private information in credit markets expands the role for fiat currency include Bruce Smith (1986), Saquib Jafarey and Peter Rupert (1992), and Williamson (1992). Robert M. Townsend (1989) studies a model in which private information leads to a role for money as a record-keeping device. These models are all very different from what will be presented here. Abhijit V. Banerjee and Eric S. Maskin (1991) have written a paper with qualitative uncertainty that is similar to ours in spirit. However, they try to keep markets as close to Walrasian as possible, and they impose informational frictions by assuming that there are some commodities that certain agents are simply not allowed to acquire. In contrast, we adopt a search-theoretic approach with bilateral exchange and allow our traders to attempt to acquire all commodities, even though they might face the possibility of getting "ripped off." In combination with some other technical differences, this leads us to focus on different issues and to derive different results.

[2] According to one commentator, the double-coincidence problem occurs when you need a taxi, all you have is one of your recent research papers, and you cannot find a cabby who wants it. In the model to be presented here, all cabbies want research paper, but unfortunately, some of them cannot recognize good monetary theory. On the other hand, they can all recognize money.

enough, the only equilibrium is one where no high-quality output is produced. For given parameter values, sometimes multiple Pareto-ranked equilibria coexist. This illustrates an externality or strategic complementarity inherent in the exchange process that, to our knowledge, has not been discussed in the previous literature: the production of better-quality output on average increases gains from trade, which then encourages every individual to produce better-quality output.

We then introduce generally recognizable fiat currency. It is shown that it is possible for an active (nondegenerate) monetary equilibrium to exist when the only nonmonetary equilibrium is degenerate. Furthermore, even when active nonmonetary equilibria exist, there may also exist monetary equilibria that entail higher welfare. Since use of money does not improve welfare without private information, given that we assume away the double-coincidence problem, it must therefore help to ameliorate the information problem. In fact, the use of money leads to agents adopting trading strategies that increase the probability of high-quality output being exchanged. In a monetary economy, the sellers of high-quality commodities have the luxury of demanding payment in cash, for example, which can generate positive incentive effects on other traders and producers.

Although there can exist several qualitatively different types of monetary equilibria, the ones that tend to do best in terms of welfare are those in which agents refuse to trade good-quality commodities for commodities whose quality they cannot recognize. This effectively imposes a "cash-in-advance" constraint on the producers of bad commodities, in that they must sell their output for money before trading for a consumption good. Given this, individuals are more willing to bear the cost of producing high quality in order to avoid cash-in-advance. The reason that agents buying with money are more willing to take a chance on something they cannot recognize than agents buying with their own high-quality commodities is that the former have a greater chance of getting high quality in return. That is, because money is universally recognizable, while commodities are not, sellers of high-quality output are more willing to trade if payment is in cash than if one tries to barter.

From another perspective, the fact that agents are willing to give up high-quality commodities for a universally recognizable money means that it can have value in exchange even though it has no value in consumption. In a monetary equilibrium, the probability of executing a trade is greater and the expected time until a trade is executed is shorter when one has currency rather than a commodity for sale. Although we make assumptions that essentially fix the price in any exchange to be either 1 or 0 (i.e., agents either swap inventories one-for-one or do not trade at all), the exchange value of various objects is determined by the expected time it takes to execute a successful transaction starting with these objects. Our emphasis on the time it takes to trade an object at a given price (its *liquidity*), rather than the relative price at which it can be sold instantaneously, is shared with many search-theoretic monetary models.

In practice, qualitative uncertainty in a market can be reflected in both the amount of time required to sell a particular good and in the price one receives for it. In Akerlof (1970), low average quality is reflected in a low market price, but his model completely abstracts from the idea that uncertainty makes goods more difficult to buy and sell and thereby increases search and other transaction costs. The model considered here is a polar opposite to Akerlof's, in the sense that price is not affected at all by the presence of differentially informed agents, but the amount of time it takes to buy and sell is. The fact that we abstract from price effects does not mean that they are uninteresting or unimportant, only that we wish to focus on another aspect of the frictions created by qualitative uncertainty. The model presented below is well suited to the study of how uncertainty affects the time required to buy and sell goods and is

also a natural model within which to consider the role of fiat money.[3]

The paper is organized as follows. Section I presents the basic framework, and Section II analyzes the complete-information case as a benchmark. Section III considers nonmonetary equilibria with private information, and Section IV considers monetary equilibria. Section V concludes with some general comments and possible extensions.

I. The Basic Model

Time is discrete and continues forever. There is a continuum of homogeneous, infinite-lived agents, whose population is normalized to 1. There are three objects that may potentially be traded in this economy: a good-quality commodity, a bad-quality commodity, and money.[4] Both good and bad commodities can be produced by all agents, with the cost in terms of disutility to producing one unit of the good commodity equal to $\gamma > 0$ and the cost to producing the bad commodity equal to 0. All objects (money and commodities) are indivisible, freely disposable, and storable at zero cost, but only one unit at a time. This implies that agents' inventories always consist of at most one unit of one object—either a good commodity, a bad commodity, or money. Consumption of either money or a bad commodity yields zero utility. Consumption of one unit of a good commodity yields utility $u > 0$ if it was produced by someone else, while consumption of one's own output yields zero utility.[5]

Money cannot be produced by any private individual. At the beginning of the initial period, a fraction M of the agents in the economy are chosen at random, and each is endowed with one unit of cash. They may either keep it or dispose of it. Then, any agent without money can produce. In each succeeding period agents meet pairwise and at random and decide whether or not to trade. Trade always entails a one-for-one swap of inventories, since objects are indivisible (which allows us to ignore bargaining), and takes place if and only if it is mutually agreeable. There are no private credit arrangements, since agents who meet in one period will meet again in another period with probability zero. Agents holding (good or bad) commodities are called *commodity traders*, while agents holding money are called *money traders*. Let p be the proportion of commodity traders holding commodities that are good (and $1-p$ the proportion holding commodities that are bad).

Money is always identifiable. In any meeting between an agent and a commodity trader, however, there is a probability $\theta \leq 1$ that the former recognizes the quality of the latter's commodity.[6] This probability is independent across commodity traders when they meet. Agents do not know whether

[3]Although the first-generation search-theoretic monetary models neglected prices almost entirely, bilateral bargaining has been introduced explicitly into the framework in recent papers by Shouyong Shi (1992) and Alberto Trejos and Wright (1992). We think this model, with prices fixed and all of the effect of qualitative uncertainty occurring via changes in the time required to execute a trade, is complicated and interesting enough that we have not yet attempted to combine bargaining, private information, and money.

[4]To motivate the analysis of private information, it may help to imagine a large number of differentiated commodities. However, since consumers will derive the same utility from every commodity, holding quality fixed, we can proceed as though there is a single consumption good.

[5]The assumption that agents derive no utility from the consumption of their own produce is an easy way to generate gains from trade and is common in search-based models of exchange (see e.g., Peter A. Diamond, 1982, 1984; Kiyotaki and Wright, 1991, 1993); it can be relaxed in a generalized version of the model. The assumption that agents cannot consume their own output or endowment is also used in many cash-in-advance models (see e.g., Robert E. Lucas, 1980).

[6]As suggested above, this is meant to capture the fact that in modern economies with many commodities it is typically not possible to identify the quality of everything one may need. For example, one consumer may be well informed concerning clothing but ignorant when it comes to electronics, while another is an expert in electronics but cannot tell Armani from K-Mart. Nonetheless, there are times when the former needs to purchase a stereo or the latter a suit of clothes.

other agents recognize their commodities and do not know anything about other agents' histories. When two traders meet, they simply inspect each other's inventories and simultaneously announce whether or not they wish to trade. If both announce "trade" then they swap inventories and separate, whereupon the quality of each object is revealed if it was previously unknown. A trader then has the option of consuming the object, disposing of it, or storing it until the next period. If the object is consumed or disposed the agent can instantaneously produce a new commodity of either good or bad quality at the associated cost.

Agents choose production, consumption, disposal, and trading strategies in order to maximize the expected discounted utility of consumption net of production costs. In doing so, they take as given the strategies of others and the probabilities of meeting agents holding particular inventories. We look for stationary Nash equilibria in which the strategies and meeting probabilities are time-invariant and expectations are rational. The meeting probabilities are summarized by m, the probability of meeting a money trader, and p, the fraction of commodity traders holding high-quality commodities. If no one disposes of money then $m = M$, where M is the initial endowment of the stuff; if some agents dispose of money then we can have $m < M$. We confine attention mainly to what we call *active* (or nondegenerate) equilibria, where at least some good commodities are produced and consumed, and utility is strictly positive.[7]

II. Complete Information

As a benchmark, in this section we consider the case of $\theta = 1$ so that there is no private information. Let V_j denote the *payoff* or *value function* at the end of a period for an agent holding object j, where $j = g$, b, or m denotes a good commodity, a bad commodity, or money, respectively. Let $r > 0$ be the (common) discount rate. Let $W = \max(V_g - \gamma, V_b)$ represent the value function for an agent with nothing in inventory, who is deciding which quality commodity to produce. By definition, in any active equilibrium at least some production of the good commodity must occur, which implies $p > 0$ and $W = V_g - \gamma \geq V_b$. We begin with nonmonetary equilibria, in which money is never accepted in trade. In an active nonmonetary equilibrium, anyone initially endowed with fiat money disposes of it, which means $m = 0$.

Suppose an active equilibrium exists, so that $V_g - \gamma > 0$. Then it is not hard to see that when a trader meets someone with a good commodity he will want to trade, and when he meets someone with a bad commodity he will not want to trade. Hence, traders with bad commodities can never trade for good commodities, which implies that no bad commodities are produced and $p = 1$ in any active equilibrium without private information. Thus, if an active nonmonetary equilibrium exists, every agent produces a good commodity in the initial period, and each period thereafter he meets, trades, consumes, and produces another good commodity. To verify that this in fact constitutes an equilibrium we need to check that an agent has no incentive to deviate from this strategy when everyone else is following it; in other words, we need to check that it is a fixed point of the best-response correspondence.

We make extensive use of a fundamental principle of dynamic programming known as the *unimprovability criterion*.[8] To apply this, first note that the payoff to an individual from producing a good commodity, given

[7]There will always exist degenerate equilibria; for example, if no trades are ever accepted then agents may as well not produce any good commodities, which is an equilibrium generating zero utility.

[8]A candidate policy is called *unimprovable* (more accurately, unimprovable in a single step) if the payoff from using it cannot be increased by deviating to a different decision at a single date and then reverting back to the candidate policy for the rest of time. Obviously a payoff-maximizing policy is unimprovable; it is also true that an unimprovable policy is payoff-maximizing as long as the instantaneous return function is bounded below (see David M. Kreps, 1990).

the candidate equilibrium strategy, is $V_g - \gamma$, where

$$rV_g = u - \gamma. \tag{1}$$

This and the more complicated expressions for payoff functions that appear later are special cases of general results in Appendix A. If the individual deviates by producing a bad commodity, tries to trade (without success, since $\theta = 1$), and then reverts next period to producing a good commodity, his payoff is V_b, where

$$rV_b = V_g - \gamma - V_b. \tag{2}$$

Equations (1) and (2) imply that $V_g - \gamma \geq V_b$ if and only if

$$u \geq (1+r)\gamma. \tag{3}$$

We conclude that when others are producing only good commodities the individual strategy of producing only good commodities is unimprovable, and therefore a best response and therefore an equilibrium, if and only if (3) holds.[9] Let Z denote welfare, defined as the expected utility of the representative agent in the initial period *before* the endowment of money is distributed and production takes place. In the unique active nonmonetary equilibrium $Z = Z^*$, where Z^* satisfies

$$rZ^* = u - (1+r)\gamma. \tag{4}$$

Except for the borderline case where (3) holds at equality, (4) implies $Z^* > 0$ whenever this equilibrium exists, and it strictly Pareto-dominates an inactive equilibrium.

We now consider monetary equilibria, in which fiat currency is accepted in at least some exchanges. In this economy, one can show that money is either always accepted or never accepted in exchange (see below), and so we concentrate on the case of *pure* monetary equilibria in which money is accepted with probability 1. For money to be accepted in exchange for a good commodity we require $V_m \geq V_g$, which implies $V_m > V_g - \gamma$; therefore no one ever disposes of currency, and $m = M$. As above, for an active equilibrium we require $V_g - \gamma \geq V_b$, and no agent with a good commodity trades for a bad commodity. The fact that no trader ever accepts bad commodities implies $p = 1$, exactly as in the nonmonetary case. It only remains to check that if others are accepting money and producing good commodities then this strategy is a best response for an individual.

To this end, note that the payoff functions for agents with either good commodities or money, given the candidate strategy, satisfy the following conditions:

$$rV_g = (1-M)(u-\gamma) + M(V_m - V_g) \tag{5}$$

$$rV_m = (1-M)(u-\gamma) + (1-M)(V_g - V_m). \tag{6}$$

These imply that $V_m = V_g$, and so a deviation from the candidate strategy by not accepting money does not improve one's payoff. Further, a deviation by producing a bad commodity yields V_b, where V_b again satisfies (2). This does not improve one's payoff if and only if $V_g - \gamma \geq V_b$, which holds if and only if

$$M \leq 1 - r\gamma/(u-\gamma). \tag{7}$$

By the unimprovability criterion, accepting money and producing only good commodities is a best response and therefore an equilibrium if and only if (7) is satisfied.

This equilibrium exists for some $M > 0$ if and only if $u > (1+r)\gamma$. By (4), this is the same as the condition for the existence of the active nonmonetary equilibrium. Hence, whenever the active monetary equilibrium exists, so does the active nonmonetary equilibrium, and we claim the latter is Pareto superior. In the monetary equilibrium, expected utility is $Z = MV_m + (1-M)(V_g - \gamma)$,

[9] This is the unique active equilibrium, where an active equilibrium is defined as one generating positive utility. There is also an equilibrium where $0 < p < 1$ and $V_g - \gamma = V_b = 0$, but we call it degenerate.

which simplifies to

$$(8) \qquad rZ = (1-M)[u-(1+r)\gamma].$$

This is decreasing in M, and $M=0$ implies $Z=Z^*$, where Z^* is welfare in the nonmonetary equilibrium. Thus, the welfare-maximizing value of M is zero here. This should not be too surprising—things have been rigged so that money has no role without private information.[10]

It may be helpful to compare the present model with the one in Kiyotaki and Wright (1993), which is very similar when $\theta=1$, except that it includes a double-coincidence problem. In particular, there is a number x, with $0<x<1$, such that x equals the probability that an agent selected at random accepts a given commodity and also equals the probability that a commodity selected at random is accepted by a given agent. Smaller values of x make barter more difficult. In the present model $x=1$, so that (at least when $\theta=1$) barter is trivial. For $x\in(0,1)$ there exist exactly three active equilibria: a nonmonetary equilibrium where money is never accepted, a pure monetary equilibrium where money is accepted with probability 1, and a mixed monetary equilibrium where money is accepted with probability x. We set $x=1$ here, so the latter two cases coalesce, and there is no equilibrium where money is accepted with a probability strictly between 0 and 1.

For a general value of $x\in[0,1]$, the payoff functions in pure monetary equilibrium satisfy

$$(9) \qquad rV_g = (1-M)x^2(u-\gamma) + Mx(V_m - V_g)$$

$$(10) \qquad rV_m = (1-M)x(u-\gamma) + (1-M)x(V_g - V_m)$$

which should be compared with (5) and (6). Note that barter requires a double coincidence, which occurs with probability x^2, while a cash purchase requires a single coincidence, which occurs with probability x. Welfare is given by

$$(11) \qquad rZ = (1-M)[Mx+(1-M)x^2](u-\gamma) - (1-M)\gamma r.$$

If x is small enough, then Z is increasing in M at $M=0$, and consequently the welfare-maximizing value of M is strictly positive.

When barter is sufficiently difficult, there is a role for fiat money even without private information. We set $x=1$ in this paper in order to remove the double-coincidence friction entirely and focus on qualitative uncertainty. If money is going to have a welfare-improving role in what follows, it will be exclusively due to the presence of private information.

III. Private Information: Nonmonetary Equilibria

We now assume that $\theta<1$, so that in some meetings agents are not able to identify the quality of a commodity in a potential trading partner's inventory. Agents may try to take advantage of a lower cost to producing bad commodities with the hope of being able to pass them off on uninformed consumers, which means p may potentially be less than 1. However, p cannot be zero in any active equilibrium, by definition, since this would yield zero utility. We also restrict attention for now to nonmonetary equilibria, where $m=0$.

As in the previous section, two commodity traders who meet and recognize each other's inventories as good quality will al-

[10] As a technical aside, we could have followed Aiyagari and Neil Wallace (1991, 1992) and considered a slightly different setup in which agents cannot produce until they consume. This implies that those initially endowed with money never produce in nonmonetary equilibrium, although they are still a part of the meeting technology (i.e., agents with commodities still have a probability M of meeting a money trader every period). Under these assumptions, money traders are better off in the monetary equilibrium, and commodity traders are indifferent, and so *ex ante* welfare is higher. However, it remains the case that Z is decreasing in M.

ways want to trade. Also, an agent with a bad commodity is willing to trade at every opportunity, since at worst he gets another bad commodity in return. The only nontrivial decisions concern whether to produce good- or bad-quality output, and whether to accept or reject a commodity of unrecognized quality when you are currently holding a good commodity yourself. Let Σ denote the probability with which an agent believes that other traders with good commodities will accept commodities of unrecognized quality, and let σ be an individual's best response. Observe that we must have $\Sigma > 0$ in any active equilibrium.[11]

The best-response problem is now described by the following equations (again, formal derivations are contained in Appendix A):

$$(12)\quad rV_g = \theta p[\theta + (1-\theta)\Sigma](u + W - V_g) + (1-\theta)\max_{\sigma}\sigma\{p[\theta + (1-\theta)\Sigma](u + W - V_g) + (1-p)(W - V_g)\}$$

$$(13)\quad rV_b = p(1-\theta)\Sigma(u + W - V_b).$$

Equation (12) sets the flow return to holding a good commodity, rV_g, equal to the sum of two terms. The first term is the probability that the individual meets someone with a good commodity he can identify, θp, multiplied by the probability the other agent is willing to trade, $\theta + (1-\theta)\Sigma$, multiplied by the gain from trading, $u + W - V_g$. The second term is the probability that the individual meets someone with something he cannot identify, $1-\theta$, multiplied by the gain from choosing the acceptance probability σ. Similarly, (13) sets the return to holding a bad commodity, rV_b, equal to the probability that the individual meets someone with a good commodity who is uniformed but still willing to trade, $p(1-\theta)\Sigma$, multiplied by the gain from trading.

An equilibrium now consists of values for p and Σ with the following properties. First, $V_g - \gamma > V_b$ implies $p = 1$ and $p < 1$ implies $V_g - \gamma = V_b$ (recall that p can be less than 1 but cannot be 0). Second, given p, $\sigma = \Sigma$ must solve the maximization problem in (12). There are potentially three types of equilibria. A type-a equilibrium has $p = 1$, which implies $\Sigma = 1$; in this case, no bad commodities are ever produced, and therefore traders always accept commodities even when they cannot recognize them. A type-b equilibrium has $0 < p < 1$ and $\Sigma = 1$; in this case, some bad commodities are produced, but traders still always accept commodities even when they cannot recognize them. A type-c equilibrium has $0 < p < 1$ and $0 < \Sigma < 1$; in this case some bad commodities are produced, and traders randomize between accepting and rejecting commodities they cannot recognize. We take up each case in turn.

First consider a type-a equilibrium, with $p = \Sigma = 1$. In such an equilibrium, if it exists, private information is not a problem in the sense that the outcome is the same as when $\theta = 1$. This might still be an equilibrium, even if $\theta < 1$, because agents might be disciplined into producing only good commodities by the possibility of having bad commodities rejected; that is, the reduction in cost may not be worth the required increase in the time it takes to sell low-quality output. This requires $V_g - \gamma \geq V_b$. Using the unimprovability criterion, we insert $p = \Sigma = \sigma = 1$ into (12) and (13) and rearrange to find that $V_g - \gamma \geq V_b$ if and only if $\theta u \geq (1+r)\gamma$. We conclude that a type-a equilibrium exists if and only if $\theta \geq \theta_1$, where

$$(14)\quad \theta_1 = (1+r)\gamma/u.$$

Now consider a type-b equilibrium with $0 < p < 1$ and $\Sigma = 1$. If $V_g - \gamma = V_b$, then unimprovability implies that it is a best response to produce a good commodity with

[11] The proof is immediate: if $\Sigma = 0$ then unrecognized commodities are never accepted, and so bad commodities are never produced; but then consumers should accept unrecognized commodities, contradicting $\Sigma = 0$.

an arbitrary probability.[12] After substitution of $\sigma = \Sigma = 1$ into (12) and (13), $V_g - \gamma = V_b$ can be solved for

$$(15) \qquad p = \frac{\gamma(1-\theta+r)}{\theta(u-\gamma)} = p_b.$$

Notice that $p_b > 0$, and $p_b < 1$ if and only if $\theta > \theta_1$, where θ_1 is defined in (14). We therefore need only check that $\sigma = 1$ is also a best response. If we insert $W = V_g - \gamma$ and $\Sigma = 1$ into (12), $\sigma = 1$ is a best response if and only if $pu - \gamma \geq 0$. By (15), this holds if and only if $\theta \leq \theta_2$, where

$$(16) \qquad \theta_2 = (1+r)u/(2u-\gamma).$$

Hence, equilibrium b exists if and only if $\theta_1 < \theta \leq \theta_2$.

Finally, consider a type-c equilibrium, with $0 < p < 1$ and $0 < \sigma < 1$. Any $p \in [0,1]$ is a best response if $V_g - \gamma = V_b$, and any $\sigma \in [0,1]$ is a best response if an individual is indifferent between accepting and rejecting an unrecognized commodity. By virtue of (12), the latter requires

$$(17) \quad p = \frac{\gamma}{\gamma + [\theta + (1-\theta)\Sigma](u-\gamma)} = \pi(\Sigma).$$

Notice that $0 < \pi(\Sigma) < 1$ for all $\Sigma \geq 0$. Using $p = \pi(\Sigma)$, we now can solve for the value of Σ that yields $V_g - \gamma = V_b$:

$$(18) \quad \Sigma_c = \frac{\theta(u-\gamma) + (1-\theta)\gamma - (1-\theta+r)[\gamma + \theta(u-\gamma)]}{(1-\theta)[\gamma + (1-\theta+r)(u-\gamma)]}.$$

For future reference, let $p_c = \pi(\Sigma_c)$. One can show that $0 < \Sigma_c < 1$, and therefore equilibrium c exists, if and only if $\theta_3 < \theta < \theta_2$, where θ_2 is defined in (16) and θ_3 is defined by

$$(19) \quad \theta_3 = 0.5r + \frac{0.5}{u-\gamma}\sqrt{r^2(u-\gamma)^2 + 4r\gamma(u-\gamma)}.$$

The above analysis indicates that the set of equilibria depends on θ relative to three critical values, θ_1, θ_2, and θ_3, which themselves depend on the other parameters. Figure 1 graphs θ_1, θ_2, and θ_3 as functions of r for given but arbitrary values of u and γ. Notice that there exist three values of r, with $0 < r_1 < r_2 < r_3$, such that the following is true. For $r < r_1$ we have $\theta_3 < \theta_1 < \theta_2$; in this case, equilibrium c is the unique active equilibrium for $\theta \in (\theta_3, \theta_1)$, equilibria a, b, and c coexist for $\theta \in (\theta_1, \theta_2)$, and equilibrium a is the unique active equilibrium for $\theta \in (\theta_2, 1)$. For $r_1 < r < r_2$ we have $\theta_1 < \theta_3 < \theta_2$; in this case, equilibria a and b exist for $\theta \in (\theta_1, \theta_3)$, all three coexist for $\theta \in (\theta_3, \theta_2)$, and equilibrium a is the unique active equilibrium for $\theta \in (\theta_2, 1)$. Finally, for $r_2 < r < r_3$ we have $\theta_1 < 1 < \theta_2 < \theta_3$; in this case, equilibria a and b coexist for $\theta \in (\theta_1, 1)$, while equilibrium c does not exist for any θ. We always assume $r < r_3$ from now on, which is equivalent to assuming that the active equilibrium exists when $\theta = 1$.

The following general observations can be made. First, there are always values of θ less than 1 such that equilibrium a exists. This means that a little private information can always be introduced and the "first best" outcome will still be an equilibrium. Second, there are always values of θ close to 0 such that no active equilibrium exists. This means that enough private information can be introduced so that all economic activity shuts down. Third, there are always values of θ such that multiple equilibria coexist. Finally, notice that equilibrium c is the only possible equilibrium where θ is very low, and its existence depends on a low value of r. Equilibrium c has the greatest chance of surviving when the private-information problem is severe because $\Sigma < 1$ imposes the greatest discipline on producers of bad commodities: not only do they have to meet uninformed agents, the latter also have to be willing to take a chance, which

[12] By use of an appropriate law of large numbers for continuum economies (see Harold Uhlig, 1987), the proportion of agents with good commodities is equal to the probability with which the average agent produces a good commodity. Alternatively, we can simply impose that a fraction p of the agents always produce good commodities while the rest always produce bad commodities, since both are best responses.

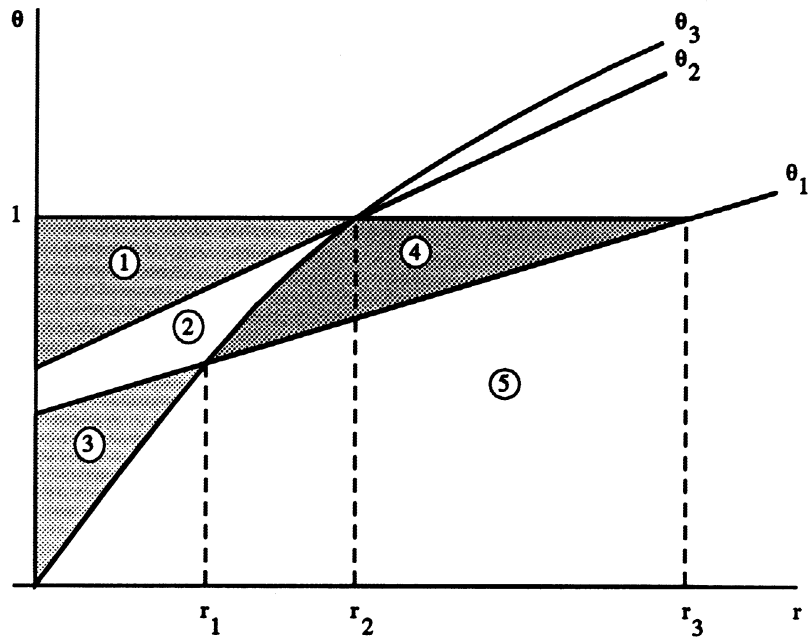

FIGURE 1. θ_1, θ_2, AND θ_3 AS FUNCTIONS OF r

Notes: Region 1, equilibrium a exists; region 2, equilibria a, b, and c exist; region 3, equilibrium c exists; region 4, equilibria a and b exist; region 5, no active equilibrium exists.

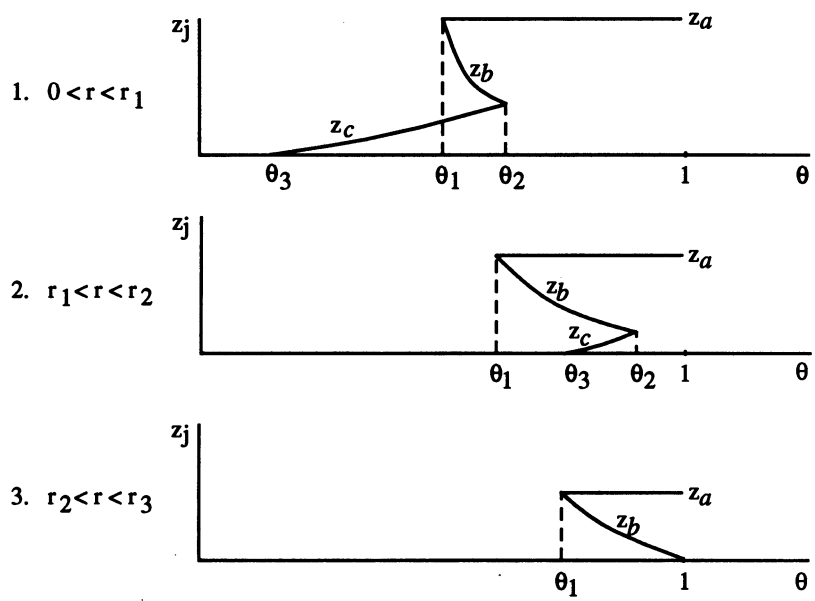

FIGURE 2. WELFARE IN EQUILIBRIA a, b, AND c AS FUNCTIONS OF θ

occurs with probability Σ. Low values of Σ reduce the incentive to produce bad commodities, which would otherwise be great when θ is small.[13]

We now demonstrate that when multiple equilibria coexist they can be Pareto-ranked. Let Z_j be welfare in equilibrium j, where $j = $ a, b, or c. Straightforward algebra implies that Z_j can be written as

$$(20)\quad rZ_j = p_j\left[\theta + (1-\theta)\Sigma_j\right](u-\gamma) - (1-p_j)(1-\theta)\gamma - r\gamma.$$

As $p_a = \Sigma_a = 1$, welfare is greatest in equilibrium a and, of course, $Z_a = Z^*$ where Z^* is welfare in the nonmonetary equilibrium with $\theta = 1$. Since $\Sigma_c < \Sigma_b = 1$, and since it is also possible to show that $p_c < p_b < 1$, it is immediate from (20) that $Z_c < Z_b < Z_a$. In Figure 2 we plot welfare in equilibria a, b, and c as functions of θ, for all values of θ for which they exist. Three cases are shown, corresponding to values of r in each of the three intervals described by Figure 1.

The multiplicity of Pareto-ranked equilibria is due to a *strategic complementarity* (see Russell Cooper and Andrew John, 1988). When more agents produce high-quality output, there is a direct effect that increases the value of producing both good and bad commodities, with the value of producing a good commodity increasing by more provided that the probability of "getting caught" with a bad commodity, θ, is in the appropriate range. There is also an indirect effect. If more good commodities are produced, then an unidentified commodity is more likely to be acceptable, which encourages the production of lemons. If θ is in the appropriate range, given the other parameters, the direct effect dominates; then a strategic complementarity exists and leads to multiple equilibria, as shown in regions 2 and 4 of Figure 1. If θ is too high or too low, however, there is an overwhelming tendency to end up in equilibrium a or c; this leads to uniqueness, as shown in regions 1 and 3.[14]

IV. Private Information: Monetary Equilibria

In this section, we demonstrate that with private information there can exist active monetary equilibria under circumstances in which the only nonmonetary equilibrium is degenerate, and that even if an active nonmonetary equilibrium exists there may also exist a monetary equilibrium that entails higher welfare. We also demonstrate that several different types of monetary equilibria are possible, although we give conditions under which a unique outcome exists. We also interpret the mechanism by which the use of money mitigates the private-information problem, and discuss some welfare implications.

We restrict attention to equilibria with $0 < p < 1$. One can show that there exists a monetary equilibrium with $p = 1$ if and only if $\theta \geq \theta_4$, where $\theta_4 = (1+r)\gamma/[(1-M)u + M\gamma]$.[15] Since $\theta_4 > \theta_1$, whenever this equilibrium exists there also exists an active nonmonetary equilibrium with $p = 1$, and the

[13]The existence of equilibrium c depends on the willingness of agents with good commodities sometimes to forgo a trade for something that they cannot recognize in favor of waiting to trade later for something that they can recognize. This willingness to wait depends on the discount rate being sufficiently low, which explains the role of r in the discussion. Similar results obtain in other private-information models. For example, in Townsend's (1982) environment, optimal allocations exhibit intertemporal links which are absent in the corresponding optimal allocations under full information, due to the fact that under private information agents may have incentives to make apparently inferior trades in the present in order to gain something for the future.

[14]Welfare does not depend on θ in equilibrium a, is decreasing in θ in equilibrium b, and is increasing in θ in equilibrium c. In this sense equilibrium b seems perverse: welfare is higher the greater the private-information problem. This results from the fact that the fraction of high-quality goods, p_b, depends negatively on θ in this equilibrium, as can be seen from (15). Of course, in the presence of multiplicity it is typical for some equilibria to have counterintuitive comparative-static properties.

[15]Equivalently, a monetary equilibrium with $p = 1$ exists if and only if $M < 1 - (1-\theta+r)\gamma/(u-\gamma)$. This generalizes the necessary and sufficient condition for the existence of a monetary equilibrium when $\theta = 1$, given in (7).

latter generates greater welfare. Thus, if money is to have a welfare-enhancing role in this economy, it cannot *completely* alleviate the private-information problem by driving out *all* bad commodities. Since $0 < p < 1$, in what follows, $V_g - \gamma = V_b$. We also restrict attention to equilibria where money is universally accepted, which implies that $V_m \geq V_g$. In contrast to the situation with $\theta = 1$, when $\theta < 1$ there may well exist mixed monetary equilibria where money is sometimes but not universally accepted. However, it can be shown that if such an equilibrium exists then there also exists a nonmonetary equilibrium that yields higher welfare (details are available from the authors upon request). Since we are interested in equilibria where the use of money potentially leads to an increase in welfare, we do not pursue mixed monetary equilibria here.

Denote the probabilities that good commodity traders and money traders accept commodities that they cannot recognize by Σ and Ω, respectively. Then the best-response problem is described by (again, see Appendix A):

$$(21)\quad rV_g = (1-M)\theta p[\theta+(1-\theta)\Sigma](u-\gamma) + M[\theta+(1-\theta)\Omega](V_m - V_g) + (1-M)(1-\theta) \times \max_{\sigma} \sigma\{p[\theta+(1-\theta)\Sigma] \times (u-\gamma) - (1-p)\gamma\}$$

$$(22)\quad rV_b = (1-M)p(1-\theta)\Sigma u + M(1-\theta)\Omega(V_m - V_b)$$

$$(23)\quad rV_m = (1-M)\theta p(u-\gamma+V_g-V_m) + (1-M)(1-\theta) \times \max_{\omega} \omega\{p(u-\gamma+V_g-V_m) + (1-p)(-\gamma+V_g-V_m)\}.$$

Equation (21) sets the return to holding a good commodity equal to the sum of three terms. The first term is the probability that the agent meets a commodity trader with a good commodity he recognizes, $(1-M)\theta p$, multipled by the probability that the other agent is willing to trade, $\theta+(1-\theta)\Sigma$, multiplied by $u-\gamma$. The second term is the probability that the agent meets a money trader who is willing to trade, $M[\theta+(1-\theta)\Omega]$, multiplied by $V_m - V_g$. The final term is the probability that the agent meets a commodity trader with something he cannot recognize, $(1-M)(1-\theta)$, multiplied by the gain from choosing σ. Equations (22) and (23) have similar interpretations.

TABLE 1—CANDIDATE MONETARY EQUILIBRIA

Σ	$\Omega = 0$	$\Omega = \Phi$	$\Omega = 1$
0	N	P	P
Φ	P	P	P
1	D	D	D

Key: N, monetary equilibrium does not exist; D, dominated by nonmonetary equilibrium; P, money potentially improves welfare.

Several different types of equilibria are possible, depending on whether Ω and Σ are elements of $\{0\}$, $\{1\}$, or Φ, where Φ denotes the open interval $(0,1)$. The set of possibilities is shown in Table 1. We give each case a label in terms of (Σ,Ω): for instance, equilibrium $(0,0)$ has $\Sigma = \Omega = 0$, equilibrium $(0,\Phi)$ has $\Sigma = 0$ and $0 < \Omega < 1$, and so on. There could never exist a $(0,0)$ equilibrium with $p < 1$, since someone has to accept commodities of unrecognized quality in order for bad commodities to be produced. Furthermore, it may be shown that whenever there exists an equilibrium with $\Sigma = 1$ there also exists a nonmonetary equilibrium which implies a higher level of welfare (details are available from the authors upon request). This leaves us with exactly five candidate equilibria in Table 1 that have the potential to Pareto-dominate active nonmonetary equilibria: $(0,\Phi)$, $(0,1)$, $(\Phi,0)$, (Φ,Φ), and $(\Phi,1)$.

Because there are several qualitatively different types of equilibria and some of them are not amenable to closed-form solutions, we do not attempt a complete analytical characterization, as we provided for nonmonetary equilibria in the previous section. Below we will describe the set of equi-

libria numerically for certain parameter values. However, we begin with a case in which analytical results are fairly tractable: the equilibrium with $(\Sigma, \Omega) = (0, \Phi)$. This case is important because there is a region of parameter space, characterized by low values of θ, in which this equilibrium exists, none of the other candidate monetary equilibria exist, and the only nonmonetary equilibrium is degenerate.[16]

When $(\Sigma, \Omega) = (0, \Phi)$, $0 < \Omega < 1$ implies that the final term in (23) vanishes. Then, since $\Sigma = 0$, (21)–(23) simplify to

$$(24) \quad rV_g = (1-M)\theta^2 p(u-\gamma) + M[\theta + (1-\theta)\Omega](V_m - V_g)$$

$$(25) \quad rV_b = M(1-\theta)\Omega(V_m - V_b)$$

$$(26) \quad rV_m = (1-M)\theta p(u - \gamma + V_g - V_m).$$

To verify that $(0, \Phi)$ is an equilibrium, we need to find values for p and Ω in $(0,1)$ with the following properties:[17]

$$(27) \quad V_m - V_g \geq 0$$

(so that accepting money is a best response);

$$(28) \quad V_g - \gamma = V_b$$

(so that $0 < p < 1$ is a best response);

$$(29) \quad p\theta(u-\gamma) - (1-p)\gamma \leq 0$$

(so that $\sigma = 0$ is a best response); and

$$(30) \quad pu - \gamma + V_g - V_m = 0$$

(so that $0 < \omega < 1$ is a best response).

In Appendix B we show that, as long as r is not too large given u, γ, and θ, these conditions will be satisfied, and hence the $(0, \Phi)$ equilibrium will exist, for all M in an interval $(\underline{M}, \overline{M})$ where $0 < \underline{M} < \overline{M} < 1$. As stated above, in a region of parameter space with low values of θ, this equilibrium exists, none of the other monetary equilibria in Table 1 exist, and the only nonmonetary equilibrium is the degenerate one that yields $Z = 0$. The $(0, \Phi)$ monetary equilibrium yields $Z > 0$, and therefore it Pareto-dominates the only (inactive) nonmonetary equilibrium. This illustrates how the introduction of fiat currency can improve welfare by allowing the existence of an active monetary equilibrium when the private-information problem is so severe that economic activity would otherwise shut down.

How is it that the use of fiat money reduces the information problem in this case? First, an agent with a good commodity who is confronted with an offer of an unrecognized commodity has the luxury of rejecting the offer and demanding either cash or a good that can be recognized. If the probability p is low, it is advantageous to incur the waiting cost and hold out for either money or something that can be recognized, rather than taking a chance. Moreover, the $(\Sigma, \Omega) = (0, \Phi)$ strategy imposes the greatest amount of discipline on the producers of bad commodities. Since $\Sigma = 0$, bad-commodity holders can never trade directly for a good commodity. They must trade first for money, which is possible since $\Omega > 0$ (although not automatic since $\Omega < 1$), and then use this money to make a purchase. This effectively subjects

[16] Since the only nonmonetary equilibrium is degenerate, this region is contained in region 5 in Figure 1. In particular, at the point where $u = 2\gamma$ and $r = 0.01$ the situation is as depicted in the first panel of Figure 2, with $\theta_3 = 0.11$, $\theta_1 = 0.505$, and $\theta_2 = 0.673$. For $\theta = 0.10$, no active nonmonetary equilibrium exists, and one can show numerically that equilibrium $(0, \Phi)$ exists at least for some M, while no other candidate monetary equilibrium exists for any value of M. An appropriate appeal to continuity guarantees that the situation will be qualitatively similar in an open neighborhood of this point.

[17] Notice how we exploit the unimprovability criterion here. Since at each instant in time the agent only needs to make a single decision (accept or reject money, produce a good or a bad commodity, etc.), we can demonstrate that a given strategy is a best response by showing that the agent's payoff cannot be increased by deviating from this strategy at any single decision point. We do not also have to show that the agent's payoff cannot be improved by combinations of deviations (e.g., stop accepting money and simultaneously start producing only bad commodities, etc.).

bad-commodity traders to a cash-in-advance constraint, while good-commodity holders can also trade for cash but additionally can barter directly whenever they meet another good-commodity trader and both recognize the other's inventories.[18]

This suggests that it is *socially* desirable to subject bad-commodity traders to cash-in-advance—but how is it also *individually* optimal? There seems to be a paradox here. In order for money to be accepted we require $V_m \geq V_g$; yet at the same time money traders are more willing than commodity traders to trade for something that they cannot recognize. The resolution of this apparent paradox is that a money trader has a greater probability than a commodity trader of receiving high quality in an exchange opportunity. In general, the probability that trading money for an unrecognized commodity yields a good commodity is p, while the probability that trading a commodity for an unrecognized commodity yields a good commodity is only $p[\theta+(1-\theta)\Sigma]$. Since money is universally recognizable, you have a greater chance of getting high quality when you offer money rather than another commodity. Therefore you are more likely to take a chance on something you cannot recognize when you have money, even though $V_m \geq V_g$.

Again, the $(0, \Phi)$ equilibrium exists uniquely when θ is small. If θ becomes larger and the private-information problem becomes less severe, then other equilibria may appear. For $u = 2\gamma$ and $r = 0.01$, when $\theta = 0.10$, for example, $(0, \Phi)$ is the only active equilibrium; but when we increase θ to 0.20 all five of our candidate monetary equilibria exist for some values of M, as does a unique active nonmonetary equilibrium which is of type c. Figure 3 shows equilibrium welfare for a range of values of M for each of the equilibria when they exist. All of the monetary equilibria exist and dominate the nonmonetary equilibrium for some values of M.

[18]There is always *some* barter in any active equilibrium. If an agent has a good commodity, the probability of a direct exchange for another good commodity is bounded below by $(1-M)p\theta^2$.

At the risk of taking liberty with conventional usage, we call the value of M that maximizes welfare across all equilibria the *optimal quantity of money*. Then the *optimal monetary equilibrium* is the one that yields the highest welfare at the optimal quantity of money. As seen in Figure 3, in this example the optimal monetary equilibrium is $(\Sigma, \Omega) = (0, \Phi)$, the one we previously analyzed analytically. For a range of parameter values that we examined, the optimal monetary equilibrium was either of the type $(0, \Phi)$ or $(0, 1)$ (the latter being a limiting case of the former). Both of these equilibria impose discipline on the producers of bad commodities by effectively subjecting them to a cash-in-advance constraint.[19]

We computed the optimal quantity of money for a range of values for θ. One perhaps surprising finding is that the optimal quantity of money is not monotonic in θ. For low values of θ, the optimal monetary equilibrium is of type $(0, \Phi)$, and the optimal quantity of money falls with θ. For higher values of θ, the optimal monetary equilibrium is of type $(0, 1)$, and the optimal quantity of money rises with θ. For sufficiently high values of θ the optimal quantity of money is zero, since there exists a type-a nonmonetary equilibrium. Our calculations also reveal that p, the fraction of commodities that are good-quality, can be lower in the optimal monetary equilibrium than in the nonmonetary equilibrium, and that $(1-M)p$, the fraction of all traders holding

[19]It might be thought that a more natural equilibrium would involve $\Omega = 0$ (money traders never accept commodities they do not recognize), which imposes discipline on the producers of bad commodities by forcing them to barter directly and by not allowing them to sell their goods for cash. As seen in Figure 3, an equilibrium with $(\Sigma, \Omega) = (\Phi, 0)$ exists and dominates the nonmonetary equilibrium for some values of M, but it can be dominated by other monetary equilibria. Direct barter is not particularly difficult in this environment, and therefore, forcing producers of bad commodities to barter directly does not impose a very effective discipline on them. It is more effective to force them to use money, since this requires making two trades instead of one and increases the expected time required to acquire a consumption good.

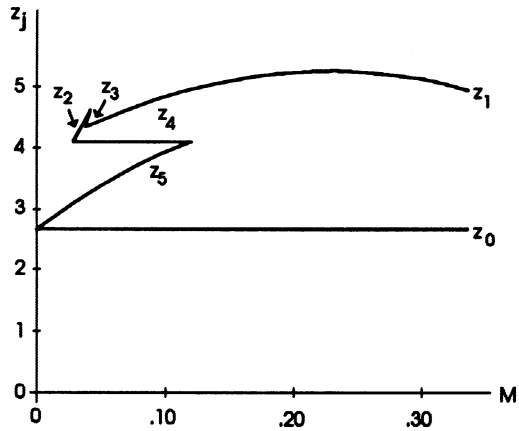

FIGURE 3. EQUILIBRIUM WELFARE FOR A RANGE OF VALUES OF M

Notes: z_0, welfare in nonmonetary equilibrium; z_1, welfare in $(0,\Phi)$ equilibrium; z_2, welfare in $(\Phi,1)$ equilibrium; z_3, welfare in $(0,1)$ equilibrium; z_4, welfare in (Φ,Φ) equilibrium; z_5, welfare in $(\Phi,0)$ equilibrium.

good commodities, can be higher or lower. However, the *probability of acquiring a good commodity* each period was always highest in the optimal monetary equilibrium. Money works by promoting useful exchange, in the sense that it not only can lead to more transactions, as in existing search models, but it also leads to the exchange of more high-quality output.

We close this section with a caveat. In the interest of tractability, it has been assumed throughout that agents carry either one unit of money or one real commodity, but not both simultaneously. This means that we have had to restrict attention to a very particular type of monetary intervention: a change in the initial endowment of M, such that when we increase the number of agents endowed with money we simultaneously reduce the number with real commodities. Not only is this a somewhat special experiment, it means that increasing M always tends to reduce welfare to a certain extent. What is interesting is that, despite this restriction and despite the assumption that there is no double-coincidence problem, the introduction of some fiat money can be welfare-improving when there is private information. The point is that it is harder, not easier, for money to have a welfare-improving role under our assumptions, and therefore it is all the more striking that such a role has been shown to exist.

At the same time, the results have an interpretation in terms of "real-world" policy issues. Since fiat money is universally recognizable and acceptable, buying with cash conveys an obvious *liquidity* advantage over attempting to barter in the equilibria with private information. The variable M measures the fraction of the population with liquid assets, and an increase in M affects average liquidity by increasing the fraction of traders with the universally acceptable asset. To the extent that policymakers have some control over the amount of liquidity in the system, in the sense of the ratio of traders with more liquid assets to traders with less liquid assets, this is a relevant issue. What has been shown is that increasing liquidity can increase welfare in the presence of private information, even under extreme assumptions that are not otherwise conducive to an efficiency role for money or liquidity.

V. Conclusion

This paper has analyzed a model of production and exchange under private information, abstracting from the double-coincidence problem in order to isolate the impact of informational frictions. Our framework is related to Akerlof's (1970) model of qualitative uncertainty, but it is also very different, as we abstract from the impact of differentially informed agents on price and study the impact on the time it takes to buy and sell. With no private information, there is a nonmonetary equilibrium in which all agents produce good-quality commodities, trade, and consume every period. There is no role for money, in the sense that a monetary equilibrium may exist but it is Pareto dominated by the nonmonetary equilibrium. With a little private information, the complete-information outcome

can still be supported as an equilibrium, but as the private-information problem becomes severe other equilibria emerge with production of bad commodities. For some parameter values there exist multiple Pareto-ranked nonmonetary equilibria.

The strategic complementarity behind the multiplicity seems different from those that have been discussed in related models in the literature. For instance, in Diamond (1982, 1984), the existence of multiple Pareto-ranked equilibria results from increasing returns to scale in the meeting technology and suggests that policy interventions which promote search and production might be beneficial. In Kiyotaki and Wright (1991, 1993), the existence of multiple Pareto-ranked equilibria results from a coordination problem in the social choice of media of exchange and suggests that the government may have some role in helping to influence this choice. Here the results suggest that a welfare-improving direct intervention might take the form of monitoring production or trade to ensure quality. However, a central point of the paper is that welfare can potentially be improved through a more indirect and presumably less costly means: the introduction and maintenance of a generally recognizable currency.

When the private-information problem is severe, the introduction of fiat currency can lead to active equilibria when the only nonmonetary equilibrium is degenerate. In this case, there can be no economic activity without money. Even when active nonmonetary equilibria exist, the introduction of fiat money can improve welfare. However, money never completely alleviates the private-information problem, since welfare is always lower in the optimal monetary equilibrium than in the first-best outcome. Any monetary equilibrium that dominates an active nonmonetary equilibrium has the property that some bad-quality commodities are produced and traded. Money does not drive out all bad commodities. Rather, it allows agents to adopt trading strategies that ultimately increase the probability of acquiring high quality. Attempting to buy with money rather than barter increases the probability that what is obtained will be high quality, precisely because the seller recognizes money but might not recognize another commodity.

The model also illustrates another fundamentally important property of fiat money. Notice that bad commodities are very similar to money in the sense that they are perfectly durable objects with zero consumption value and can be produced at zero cost; hence, one might conjecture that bad commodities could serve as a medium of exchange. There is a critical difference between fiat money and bad commodities, however: the former cannot be produced privately. If bad commodities are to be accepted in exchange by holders of good commodities, we require $V_b \geq V_g$; but if any good commodities are to be produced at all, we require $V_g - \gamma \geq V_b$. Hence, if bad commodities are accepted as a medium of exchange no one will ever produce good commodities. Either bad commodities will have to cease being media of exchange or the economy will be stuck in an inactive equilibrium.

The important characteristics of money clearly include durability and recognizability; but it is also critical that money cannot be produced privately at too low a cost. There can exist equilibria with privately produced money if the cost of producing it is sufficiently high. Suppose there is some intrinsically worthless and perfectly recognizable and durable object—a precious metal, say—that can be produced privately at a cost per unit that exceeds the cost of producing a good commodity. There will be equilibria where some agents produce the precious metal and others produce commodities, with the amount of privately produced money endogenous. As Milton Friedman (1960) has pointed out, however, it is socially preferable to adopt a fiat currency which can be produced costlessly as long as private production or counterfeiting can be controlled, since this avoids the initial cost of mining the precious metal.

Several extensions come to mind. An ambitious project would be to incorporate bi-

lateral bargaining theory into the framework in order to endogenize the price level, as is done in Shi (1992) or Trejos and Wright (1992) in search models of money without the (possibly severe) complication of private information.[20] In terms of privately produced money, one could imagine introducing a variety of objects with different degrees of recognizability and asking whether more recognizable ones necessarily arise endogenously as media of exchange; our guess is that the answer may be no. One could allow agents to differ in their ability to recognize quality, or allow agents to invest in information, or in some other way make θ endogenous. Potentially, a role could be derived for specialized traders or other types of intermediaries.

Concerning intermediaries, although the environment as currently specified rules out credit, the potential superiority of fiat money can be interpreted as suggesting that a monetary system with publicly provided currency dominates one with private currency created by a laissez-faire banking system. There is a body of research supporting the notion that movement toward laissez-faire banking would be socially desirable (see the discussion in Williamson [1992]), although there are also arguments to the effect that legal restrictions on the issue of private money might be beneficial (see Bernhardt and Engineer, 1991; Williamson, 1992). Our model provides support for the latter, more traditional, position: that private note provision under laissez-faire banking is undesirable. Of course, this discussion is only meant to be suggestive, and there are many details that need to be considered. We relegate further exploration of these topics to future work.

[20] It is possible that using bargaining theory to determine prices may not affect the results qualitatively. Suppose you are reluctant to buy something because you think it might be a lemon. Then any offer by the seller to reduce the price may only increase your belief that it is a lemon. Hence, take-it-or-leave-it offers at a fixed price could be a feature of some equilibria. On the other hand, it may be worth introducing bargaining to see how average price is affected by private information and the introduction of money.

Appendix A

Here we derive equations (21)–(23) describing the best-response problem for an agent in a monetary equilibrium with private information. The best-response problem in a nonmonetary equilibrium or without private information is a special case, derived by setting $M = 0$ or $\theta = 1$. To reduce notation, let

$$\lambda_1 = M[\theta + (1-\theta)\Omega]$$

be the probability of meeting a money trader who is willing to trade; let

$$\lambda_2 = (1-M)\theta p[\theta + (1-\theta)\Sigma]$$

be the probability of meeting a commodity trader with a good commodity that can be recognized who is willing to trade; let

$$\lambda_3 = (1-M)(1-\theta)$$

be the probability of meeting a commodity trader with a commodity that cannot be recognized; and let

$$\Delta = [\theta + (1-\theta)\Sigma]$$

be the probability that an agent with a good commodity is willing to trade.

Consider an agent with a good commodity. Bellman's equation of dynamic programming says that V_g satisfies

$$\text{(A1)}\quad V_g = \frac{1}{1+r}\max_{\sigma}\{\lambda_1 V_m + \lambda_2(u+W) + \lambda_3\sigma p\Delta(u+W) + \lambda_3\sigma(1-p)W + [\lambda_3\sigma p(1-\Delta) + \lambda_3(1-\sigma) + (1-\lambda_1-\lambda_2-\lambda_3)]V_g\}.$$

Simplification yields

$$(A2)\quad rV_g = \lambda_1(V_m - V_g) + \lambda_2(u + W - V_g) + \lambda_3 \max_{\sigma} \sigma\{p\Delta(u + W - V_g) + (1-p)(W - V_g)\}.$$

Upon substitution of $W = V_g - \gamma$, λ_j, and Δ, (A2) reduces to (21). The derivations of (22) and (23) are very similar and are therefore omitted.

APPENDIX B

Here we show that $(0, \Phi)$ is a monetary equilibrium for all M in a nondegenerate interval $(\underline{M}, \overline{M})$, as long as r is not too big given u, γ, and θ. First, note that (30) implies $V_m - V_g = pu - \gamma$. Using this and subtracting (24) from (26) we find

$$(B1)\quad M(pu-\gamma)(1-\theta)\Omega = (1-M)\theta(1-\theta)p(u-\gamma) - [r + (1-M)\theta p + \theta M](pu-\gamma).$$

Now using (28) and subtracting (25) from (24), we find

$$(B2)\quad M\gamma(1-\theta)\Omega = M\theta(pu-\gamma) + (1-M)\theta^2 p(u-\gamma) + r\gamma.$$

Solving (B1) and (B2), we find

$$(B3)\quad p = \frac{\gamma}{\gamma + (u-\gamma)[\theta + (1-\theta)M]}$$

$$(B4)\quad \Omega = \frac{(1-M)\theta(u-\gamma)[\theta + (1-\theta)M] - rK}{(1-\theta)MK}$$

where $K = \gamma + (u-\gamma)[\theta + (1-\theta)M]$.

Equilibrium conditions (28) and (30) are satisfied by construction (we used them to solve for p and Ω). Simple algebra implies that (27) and (29) hold for all parameter values. Clearly, (B3) implies that $0 < p < 1$. Therefore, all that remains to check in order to verify that $(0, \Phi)$ is an equilibrium is the condition $0 < \Omega < 1$. Equation (B4) implies that $\Omega > 0$ if and only if $\varphi(M) > 0$ and that $\Omega < 1$ if and only if $\psi(M) > 0$, where

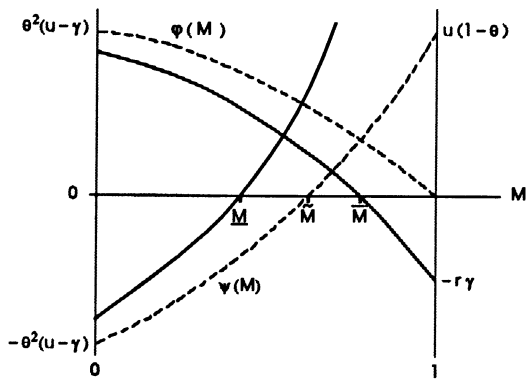

FIGURE B1. THE FUNCTIONS φ AND ψ

Note: The dashed curves depict the functions for $r = 0$; as r increases, the functions shift, as indicated by the solid curves.

$$(B5)\quad \varphi(M) = -\theta(1-\theta)(u-\gamma)M^2 + [(1-\theta)(\theta - r) - \theta^2](u-\gamma)M + C$$

$$(B6)\quad \psi(M) = (1-\theta)(u-\gamma)M^2 + [(1-\theta)\gamma + (r - \theta r + \theta^2)(u-\gamma)]M - C$$

and $C = \theta(u-\gamma)(\theta - r) - r\gamma$. The functions φ and ψ are shown for the case $r = 0$ as the dashed curves in Figure B1, and both are positive if and only if M is in the nondegenerate interval $(\tilde{M}, 1)$. As r increases, the functions shift as indicated by the solid

curves. We conclude that, as long as r is not too large given u, γ, and θ, there will exist a nondegenerate interval $(\underline{M}, \overline{M})$, where $0 < \underline{M} < \overline{M} < 1$, such that the equilibrium $(\Sigma, \Omega) = (0, \Phi)$ exists if and only if $M \in (\underline{M}, \overline{M})$.

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