Short-Run and Long-Run Effects of Changes in Money in a Random-Matching Model

Neil Wallace

Pennsylvania State University

A random-matching model of money is used to deduce the effects of a once-for-all change in the quantity of money. It is shown that the change has short-run effects that are predominantly real and long-run effects that are in the direction of being predominantly nominal provided that the change is random and people learn its realization only with a lag. The change in the quantity of money comes about through a random process of discovery that does not permit anyone to deduce the aggregate amount discovered when the change actually occurs.

I show that a random-matching model of money implies the kind of qualitative short-run and long-run effects of changes in the quantity of money that have often been observed, namely, short-run effects that are predominantly real and long-run effects that are in the direction of being predominantly nominal. Those effects occur in the particular random-matching model studied here, the model in Aiyagari, Wallace, and Wright (1996), provided that two conditions are met: the quantity of money is random, and people learn about what happened to it only with a lag.

Those conditions on changes in the quantity of money are, of course, not new; they are important ingredients in several models consistent with the observed short-run and long-run effects of

An earlier version of this paper was presented at the Federal Reserve Bank of Minneapolis. I am indebted to Beth Allen, Ed Green, Tom Holmes, Narayana Kocherlakota, Alexander Taber, and a referee for helpful comments.

[*Journal of Political Economy,* 1997, vol. 105, no. 6]

changes in the quantity of money (see Lucas 1996). Therefore, one may wonder why it is worthwhile showing that those conditions give rise to similar effects in a random-matching model. Doing so demonstrates that the ingredients of the matching model, ingredients that give outside money a role in overcoming double-coincidence problems, are sufficient to account for those effects. In addition, those conditions and the ingredients of the matching model closely resemble long-held views about what accounts for the observed short-run and long-run effects of changes in the quantity of money:

> Accordingly we find, that, in every kingdom, into which money begins to flow in greater abundance than formerly, every thing takes a new face: labour and industry gain life; the merchant becomes more enterprising. . . .
>
> To account, then, for this phenomenon, we must consider, that though the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon that encrease; but some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people. At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another; till the whole at last reaches a just proportion with the new quantity of specie in the kingdom. In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favorable to industry. When any quantity of money is imported into a nation, it is not at first dispersed into many hands but is confined to the coffers of a few persons, who immediately seek to employ it to advantage. Here are a set of manufacturers or merchants, we shall suppose, who have received returns of gold and silver for goods which they sent to Cadiz. They are thereby enabled to employ more workmen than formerly, who never dream of demanding higher wages, but are glad of employment from such good paymasters. . . . [The artisan] carries his money to market, where he finds every thing at the same price as formerly, but returns with greater quantity and of better kinds for the use of his family. The farmer and gardener, finding that all their commodities are taken off, apply themselves with alacrity to raising more. . . . It is easy to trace the money in its progress through the whole commonwealth, where we shall find that it must first quicken

> the diligence of every individual before it encrease the price of labour. [Hume 1970, pp. 37, 38]

There seem to be two explanatory ingredients in Hume's discussion: decentralized trade and incomplete information about the quantity of money. Those are the two main ingredients in the model described below.

The rest of the paper proceeds as follows. In Section I, I set out the model. In Section II, I describe equilibrium short-run and long-run effects of once-for-all changes in the quantity of money that come about in a way that implies satisfaction of the conditions discussed above. In Section III, I discuss extensions of the model and suspicions about the robustness of the results to extensions. I offer concluding remarks in Section IV.

I. The Model

Because the model is identical to that in Aiyagari et al. (1996), I shall be brief. Time is discrete and the horizon is infinite. There are N divisible and perishable types of goods at each date, and there is a [0, 1] continuum of each of N types of people. Each type is specialized in consumption and production in the following way: a type i person consumes only good i and produces only good $i + 1$ (modulo N), for $i = 1, 2, \ldots, N$, where $N \geq 3$. Each type i person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. Utility in a period is given by $u(x) - y$, where x is the amount of the good consumed and y is the amount of the good produced.[1] The function u is defined on $[0, \infty)$, is increasing and twice differentiable, and satisfies $u(0) = 0$, $u'' < 0$, and $u'(0) = \infty$.

People meet pairwise at random, people cannot commit to future actions, and each person's trading history is private information to the person. Together, these assumptions rule out all but quid pro quo trade for optimizing people. In particular, they rule out private credit. The only storable objects are indivisible units of (fiat) money, and each person has a storage capacity of one unit. In a meeting, each person sees the trading partner's type and amount of money held.

The sequence of actions within a period is as follows. Each person begins a period holding either one unit of money or nothing. Then people meet pairwise at random. Because of the upper bound on

[1] The assumption that the disutility of production is equal to the amount produced is made without loss of generality. For details, see Aiyagari et al. (1996).

individual holdings of money and the indivisibility, there is a potential for trade only when a type i person meets a type $i + 1$ person and the type $i + 1$ person, the potential consumer, has money and the type i person, the potential producer, does not. I call such meetings *trade* meetings. People in trade meetings bargain. If the outcome of bargaining implies exchange, then production and consumption occur. Then people begin the next period. Throughout the paper the following simple bargaining rule is assumed: the potential consumer makes a take-it-or-leave-it offer and the potential producer accepts if made no worse off by accepting. The offer, a scalar, consists of a demand for an amount of production, which, if accepted by the producer, gives rise to the exchange of the consumer's unit of money for that amount of production.

All of this is the same as in Aiyagari et al. (1996). Their model, in turn, follows closely the models in Shi (1995) and Trejos and Wright (1995). The only addition made here is the following specification of how changes in the quantity of money come about. Let the initial date be date 0 and let $m_0 > 0$ be the initial amount of money per type. At the end of date 0, there is a once-for-all increase in the amount of money. This increase per type, denoted Δ, is a drawing from the following distribution, which is common knowledge at the beginning of date 0: $\Delta = \Delta_k$ with probability p_k, $k = 1, 2, \ldots, K$, where $p_k > 0$, $K \geq 2$, $\Delta_{k+1} > \Delta_k$, $\Delta_1 \geq 0$, and $m_0 + \Delta K \leq 1/2$; the range of Δ, $\Delta_K - \Delta_1$, is sufficiently small in a way to be described later. Conditional on Δ, each person who exits a meeting without money at date 0 discovers a unit of money with probability $\Delta/(1 - m_0)$. (This possibility of discovery, which is present only at date 0, was not included in the sequence of actions given above.) At date 1, no one observes Δ, although people use their experience to update the prior given by the p_k; at date 2, prior to meetings, the realization of Δ is revealed to everyone.

The model above is structured so that there can be equilibria that are symmetric across person types. To permit there to be such equilibria, I assume that the initial money distribution is symmetric across types. Notice that if the money distribution at the beginning of a date is symmetric and trades and discoveries are symmetric, then the money distribution remains symmetric. Given the unit upper bound on holdings of money, at any date there is only one symmetric distribution consistent with all money being held: if m is the amount of money per type, then a fraction m of each type has a unit of money and a fraction $1 - m$ has nothing. In what follows, I limit attention to symmetric equilibria. In such equilibria, it follows that the sequence of money distributions is very simple: the date 0 distribution is the unique symmetric one with $m = m_0$, and the distribution at all other dates is the unique symmetric one with $m = m_0 + \Delta$.

Although most of the special assumptions will be discussed in Section III, the specification of changes in the amount of money deserves some comment now. First, I study a once-for-all change in the quantity of money because it is simple. Second, only those who exit trade without money are eligible to discover a unit of money, because those with money would have to discard a unit if they discovered money.[2] Third, the assumption that $m_0 + \Delta \leq 1/2$ restricts the quantity of money to a range in which the probability of a trade meeting is nondecreasing in the quantity of money. If there were no upper bound on individual holdings, then increases in the quantity of money would never reduce the probability of a trade meeting. Since the upper bound is adopted only for tractability, it seems sensible to restrict the quantity of money to a range in which it does not crowd out trade meetings. That range is $[0, 1/2]$ because the fraction of all meetings that are trade meetings is $(1 - m)m(2/N)$, where m is the fraction of each type with a unit of money. Finally, the assumption that Δ is revealed to everyone at the beginning of date 2 is also made for simplicity. It allows me to easily describe what happens at date 2 and then, by working backward, describe what happens at dates 1 and 0.

II. A Symmetric Monetary Equilibrium

An equilibrium is a description of what happens in all meetings—essentially a description of what is produced (and consumed) in trade meetings. The equilibrium concept is the take-it-or-leave-it bargaining described above along with rational expectations. I shall construct the simplest kind of monetary equilibrium, one that is constant from date 2 onward. By long-run effects of changes in the quantity of money, I mean the dependence on Δ of what happens in that equilibrium at date 2 and thereafter; by short-run effects, I mean the dependence on Δ of what happens in that equilibrium at date 1. In other words, I shall be describing equilibrium cross-section observations at date 2 and thereafter (the long-run) and equilibrium cross-section observations at date 1 (the short-run): cross sections in that they come from economies that are identical except for the

[2] A version in which everyone could discover money would differ only in insignificant details. Alternatively, a version in which, after date 0 trade, people choose whether to expend some small amount of effort in order to be eligible to discover money would not differ at all.

realization of Δ. I begin with a summary of those short-run and long-run effects.

Each producer in a trade meeting at date 1 has the same experience: each exited a meeting at date 0 without money, did not discover a unit of money, and met someone with a unit of money (and does not know the source of the consumer's money). Therefore, each has the same posterior. Since the posterior of the producer is known to the producer's trading partner, because the partner knows what happened to the producer, the maximum amount produced in every trade meeting is the same and can be denoted c_1. (An explicit expression for the producer's updated prior and c_1 is given below.) Suppose, as is demonstrated below, that trade occurs in each trade meeting and, therefore, that c_1 is produced in each such meeting. Because all trade at date 1 consists of the exchange of c_1 for one unit of money, the price level at date 1 is $1/c_1$. Therefore, it does not depend on the realization of Δ. Total output can be expressed in terms of c_1 and the realization of Δ. Total output per type is $c_1(m_0 + \Delta)(1 - m_0 - \Delta)(2/N)$. Therefore, total output, denoted $Y_1(\Delta)$, arrived at by summing over types, is given by

$$Y_1(\Delta) = 2c_1(m_0 + \Delta)(1 - m_0 - \Delta). \qquad (1)$$

It follows, from the assumption that $m_0 + \Delta_K \leq 1/2$, that $Y_1(\Delta)$ is increasing in Δ.

The date 2 effects are quite different. At the beginning of date 2, everyone knows Δ. Thus, beginning at date 2, the economy has a constant and known amount of money per type. If $c_2(\Delta)$ denotes the amount produced in exchange for a unit of money when the constant quantity of money is $m_0 + \Delta$, then, as shown below, $c_2(\Delta)$ is decreasing in Δ. Since the price level is $1/c_2(\Delta)$, the price level is increasing in Δ. Total output at date 2, denoted $Y_2(\Delta)$, is given by

$$Y_2(\Delta) = 2c_2(\Delta)(m_0 + \Delta)(1 - m_0 - \Delta). \qquad (2)$$

The assumptions do not imply that $Y_2(\Delta)$ is monotone in Δ or, if monotone, the direction of the monotonicity. Thus there is no obvious association at date 2 between total output and the realization of Δ.

Notice that the form of the total output function is the same for dates 1 and 2; it is the product of two functions. One function is the probability of a trade meeting. That part, given by $2(m_0 + \Delta)(1 - m_0 - \Delta)$, is identical at dates 1 and 2 and, under my assumption about the range of Δ, is increasing in Δ. The other function is the amount produced in a trade meeting. At date 1, that part is a constant, whereas at date 2, it is a decreasing function of Δ. That difference between the total output functions captures the sense in which total output varies more strongly with Δ, the realization of the quantity of money, at date 1, the short run, than at date 2, the long run.[3]

I now show how to construct the equilibrium just described. As noted above, the idea is to work backward from the date 2 constant equilibrium monetary equilibrium that depends on the realization of Δ.

Date 2 and thereafter.—For an economy with a constant and known quantity of money per type, let $v(j)$ denote the constant expected discounted value of starting a period with j units of money ($j = 0, 1$) and let c denote the amount produced in each trade meeting. The bargaining rule implies that $v(0) = 0$ because all the trading gains go to the consumer and $u(0) = 0$. Therefore, $v(1)$ and c must be a solution to

$$v(1) = \alpha \max[\max_c u(c), \beta v(1)] + (1 - \alpha)\beta v(1), \tag{3}$$

where the maximum over c is subject to

$$c \leq \beta v(1) \tag{4}$$

and $\alpha = (1 - m)/N$, the probability of meeting a potential producer who has no money. Equation (3) is Bellman's equation, and (4) says that the disutility to the producer cannot exceed the producer's gain. (The result, $v(0) = 0$, has been substituted into [3] and [4].)

Because (4) holds at equality at a solution, if the outer maximum in (3) is $u(c)$, then (3) and (4) imply, by substitution,

$$\left(\alpha + \frac{1 - \beta}{\beta}\right)c = \alpha u(c). \tag{5}$$

Equation (5) has two solutions for c: zero and a positive solution, which I denote by $f(m)$. Because a positive solution to (5) satisfies $u(c) > c$, it follows that $c = f(m)$ and $v(1) = f(m)/\beta$ are such that the outer maximum in (3) is $u(c)$. Therefore, they are a solution to the problem above. Moreover, differentiation of (5), which gives $dc/d\alpha > 0$ at $c = f(m)$, implies that f is decreasing.

The first step in constructing an equilibrium satisfying the claims made above is to let

$$\begin{aligned} c_t(\Delta) &= f(m_0 + \Delta), \quad v_t(0; \Delta) = 0, \\ v_t(1; \Delta) &= \frac{f(m_0 + \Delta)}{\beta}; \quad t \geq 2, \end{aligned} \tag{6}$$

[3] If the support of Δ is an interval, then the derivative of $Y_2(\Delta)$, evaluated at the magnitude of Δ at which $c_2(\Delta) = c_1$, is less than the derivative of $Y_1(\Delta)$.

where $c_t(\Delta)$ denotes production in a trade meeting at t and $v_t(j; \Delta)$ denotes expected discounted utility at the beginning of date t from beginning with j units of money. Equation (6) gives us the long-run effects asserted above. In particular, since f is decreasing, the price level is increasing in the realization of Δ.

Date 1.—I now describe c_1. That is done by finding the maximum amount each producer in a trade meeting would be willing to produce in exchange for a unit of money and then showing that such a trade actually occurs. I begin by computing the posterior of a producer in a trade meeting.

Let I denote information and let I_p denote the specific information of a producer in a trade meeting: I_p consists of not discovering a unit of money and, subsequently, meeting someone with money. Conditional on the realization of Δ, those are independent events. Therefore,

$$P(I = I_p|\Delta = \Delta_k) = \left(1 - \frac{\Delta_k}{1 - m_0}\right)(m_0 + \Delta_k) = \frac{(1 - m_0 - \Delta_k)(m_0 + \Delta_k)}{1 - m_0}. \tag{7}$$

Then Bayes's rule gives

$$P(\Delta = \Delta_k|I = I_p) = \frac{p_k(1 - m_0 - \Delta_k)(m_0 + \Delta_k)}{\sum_j [p_j(1 - m_0 - \Delta_j)(m_0 + \Delta_j)]}. \tag{8}$$

It follows that the maximum amount a producer is willing to produce in exchange for a unit of money at date 1 is

$$c_1 = \beta \sum_k P(\Delta = \Delta_k|I = I_p)\, v_2(1; \Delta_k) = \sum_k P(\Delta = \Delta_k|I = I_p) f(m_0 + \Delta_k), \tag{9}$$

where the first equality follows from noting that the producer's gain is the expected utility of beginning date 2 with a unit of money and the second equality follows from (6), which gives the realized utility at date 2 of beginning with money for each possible Δ.

The next step is to assure that each potential consumer in a trade meeting wants to surrender a unit of money for c_1 as given by (9). That happens if $u(c_1)$ is not less than the discounted expected utility for the consumer of beginning date 2 with a unit of money. If p'_k denotes the posterior of a consumer in a trade meeting, then the condition for trade is[4]

$$u\left[\sum_k P(\Delta = \Delta_k | I = I_p) f(m_0 + \Delta_k)\right] \geq \sum_k p'_k f(m_0 + \Delta_k). \quad (10)$$

If the posteriors of the producer and the consumer were the same, then (10) would be an implication of $u[f(m_0 + \Delta_k)] > f(m_0 + \Delta_k)$ for each k, which follows from (5). However, the posteriors are not the same.[5] Therefore, as I now explain, I obtain (10) from the assumption that the range of Δ is sufficiently small.

Because f is decreasing, a sufficient condition for (10) is $u[f(m_0 + \Delta_K)] \geq f(m_0 + \Delta_1)$. Let $\Delta_K - \Delta_1 \equiv r$ (for range) and let $g(r) \equiv u[f(m_0 + \Delta_1 + r)]/f(m_0 + \Delta_1)$. The function g is continuous and decreasing and satisfies $g(0) > 1$ and $g(1 - m_0 - \Delta_1) = 0$. Therefore, there exists a unique and positive r, say r^*, such that $g(r^*) = 1$. Thus if $\Delta_K - \Delta_1 \leq r^*$, then (10) holds.

Date 0.—Although a description of what happens at date 0 is not needed for my claims about short-run and long-run effects, it is needed to complete the description of the equilibrium. The first step is to compute (beginning of) date 1 expected discounted utilities. As at date 2, the expected discounted utility of beginning date 1 without money is zero. There are two distinct expected discounted utilities of beginning date 1 with a unit of money: one is for those who exited trade at date 0 with a unit of money and, therefore, were not in a position to discover a unit of money; the other is for those who exited trade without a unit of money and discovered a unit of money. They are distinct because such people have different information. Once again, with I standing for information, both expected

[4] There are two types of consumers: one type exited trade at date 0 with a unit of money; the other did not and discovered a unit of money. They have distinct posteriors, despite my use of a single symbol, p'_k.

[5] One way to see why is to consider the consumer who exited trade with a unit of money. Such a consumer updates his or her prior through the experience of having met someone without money. That information leads such a consumer to revise the prior p_i by putting more weight on lower realizations of Δ, which is not the same as what the producer does. If this seems paradoxical, it may help to consider the following. In terms of meetings, the producer draws from a sample space with the following two elements: (i) neither person has money or (ii) one does and one does not. In contrast, the consumer draws from a sample space with the following two elements: (i′) both people have money or (ii) one does and one does not. Since points i and i′ differ, observing point ii is interpreted differently by the producer and the consumer.

discounted utilities can be expressed as

$$v_1(1; I) = \sum_k P(\Delta = \Delta_k | I)\left[\left(\frac{1 - m_0 - \Delta_k}{N}\right)u(c_1) + \left(1 - \frac{1 - m_0 - \Delta_k}{N}\right)\beta v_2(1; \Delta_k)\right], \tag{11}$$

where $v_1(1; I)$ denotes expected discounted utility at date 1 (the subscript) of holding one unit of money in information state I, and $P(\Delta = \Delta_k | I)$ denotes the posterior conditional on information I.

The person who exited trade at date 0 with money has no information because the person was not in a position to discover a unit of money. I denote this absence of information by $I = \varnothing$. Obviously, $P(\Delta = \Delta_k | I = \varnothing) = p_k$. I let $I = D$ denote the information at the beginning of date 1 of the person who discovered a unit of money. Because $P(I = D | \Delta = \Delta_k) = \Delta_k / (1 - m_0)$, Bayes's rule implies

$$P(\Delta = \Delta_k | I = D) = \frac{p_k \Delta_k}{\theta}, \tag{12}$$

where θ denotes the unconditional expected value of Δ, $\sum_k p_k \Delta_k$. That completes the description of date 1 discounted expected utilities.

Now I can describe what happens at date 0. Because there is no information about Δ at the beginning of date 0, I let $v_0(j)$ denote the discounted expected utility at the beginning of date 0 of someone with j units of money. At date 0, someone who starts with no money has a chance of discovering a unit. It follows that

$$v_0(0) = \beta\left(\frac{\theta}{1 - m_0}\right)v_1(1; D), \tag{13}$$

where $v_1(1; D)$ is implied by (11) with $I = D$, and $\theta/(1 - m_0)$ is the unconditional probability of discovering a unit of money. As regards someone who starts with a unit of money,

$$v_0(1) = \left(\frac{1 - m_0}{N}\right) \times \max\left\{\max_c u(c) + \beta\left(\frac{\theta}{1 - m_0}\right)v_1(1; D), \beta v_1(1; \varnothing)\right\} + \left(1 - \frac{1 - m_0}{N}\right)\beta v_1(1; \varnothing), \tag{14}$$

where the maximization over c is subject to

$$c \leq \beta\left[v_1(1; \varnothing) - \left(\frac{\theta}{1 - m_0}\right)v_1(1; D)\right] \equiv c_0. \tag{15}$$

I now show that production in each date 0 trade meeting is equal to c_0. First, by (11) and (12),

$$\begin{aligned} v_1(1; \varnothing) - \left(\frac{\theta}{1 - m_0}\right)v_1(1; D) = \sum_k p_k\left(1 - \frac{\Delta_k}{1 - m_0}\right) \\ \times \left[\left(\frac{1 - m_0 - \Delta_k}{N}\right)u(c_1)\right. \\ \left. + \left(1 - \frac{1 - m_0 - \Delta_k}{N}\right)\beta v_2(1; \Delta_k)\right], \end{aligned} \tag{16}$$

which is positive because $\Delta_k < 1 - m_0$. Therefore, $c_0 > 0$. Second, because the maximum of $u(c)$ over c is $u(c_0)$, the condition that the outer maximum in (14) involves trade is $u(c_0) \geq c_0$. From (11), we have $v_1(1; \varnothing) < v_2(1; \Delta_1)$. Therefore, from (15), $c_0 < \beta v_1(1; \varnothing) < \beta v_2(1; \Delta_1) = f(m_0 + \Delta_1)$. Since $u[f(m_0 + \Delta_1)] > f(m_0 + \Delta_1)$ and $0 < c_0 < f(m_0 + \Delta_1)$, it follows that $u(c_0) > c_0$. Therefore,

$$\begin{aligned} v_0(1) = \left(\frac{1 - m_0}{N}\right) \\ \times \left[u(c_0) + \beta\left(\frac{\theta}{1 - m_0}\right)v_1(1; D)\right] \\ + \left(1 - \frac{1 - m_0}{N}\right)\beta v_1(1; \varnothing). \end{aligned} \tag{17}$$

That completes the construction of an equilibrium.

III. The Assumptions and Robustness

Although the model contains many extreme assumptions, three deserve special attention: the inability of producers at date 1 to distinguish the source of the consumer's money, the public knowledge at the beginning of date 2 about the realized change in the quantity of money, and the indivisibility of money and the upper bound on individual holdings.

As I have specified the form of offers, consumers at date 1 are unable to signal the source of their money holdings.[6] Were they able to, either those who have newly discovered ("new") money or those who have "old" money would want to signal the source. Their information is different and if known by the producer would give different producer posteriors, one of which would be consistent with higher production than is implied by pooling. One way to think about the possible consequences of such signaling is to examine an alternative in which new money looks different from old money for one period. Then there is no relevant asymmetric information, but there are different posteriors for producers depending on whether they meet new or old money. That being so, different amounts are produced in the two meetings and we no longer get the implication that the price level at date 1 is independent of Δ. Because different amounts are produced in the different kinds of date 1 single-coincidence meetings, the price level must be computed using an implicit deflator. Also, total output is a weighted sum of the amounts produced in the two meetings. Nevertheless, the result that the date 1 effects are predominantly real and expansionary holds provided that the range of Δ is sufficiently small. To see this, let c_{old} denote the amount produced in each old-money meeting at date 1 and let c_{new} denote the amount produced in each new-money meeting. Neither depends on the realization of Δ, but both depend on all the parameters, including the range of Δ. Total output at date 1 is

$$Y_1(\Delta) = 2c_{\text{new}}(1 - m_0 - \Delta)\Delta + 2c_{\text{old}}(1 - m_0 - \Delta)m_0, \quad (18)$$

and therefore

$$\frac{\partial Y_1(\Delta)}{\partial \Delta} = 2\{c_{\text{new}}[1 - 2(m_0 + \Delta)] - (c_{\text{old}} - c_{\text{new}})m_0\}. \quad (19)$$

Now consider what happens as the range of Δ, r, gets small. As $r \to 0$, $c_{\text{old}} \to c_1$ and $c_{\text{new}} \to c_1$. Therefore, the price level becomes independent of Δ. As regards total output, from (19), as $r \to 0$, $\partial Y_1(\Delta)/\partial\Delta \to 2c_1[1 - 2(m_0 + \Delta)]$, which is positive and identical to what is implied by the version examined above. Thus such a symmetric information version gives qualitative implications similar to those of the version studied above provided that the range of Δ is small enough.

In contrast to the assumption that the realized change in the quantity of money is revealed to everyone with a one-period lag, the natural assumption is that it is never revealed. I see two difficulties in

[6] I am indebted to Tom Holmes for discussions that greatly influenced the content of this paragraph. However, he is not responsible for any errors that I may have made.

working with that specification or even one that lengthens the lag beyond one period. First, priors get revised in accord with experience (at least experience regarding what the trading partner has). Since experience is diverse, one would have to keep track of groups that are diverse in terms of their posteriors over the realized change in the amount of money. Second, the bargaining would then occur between two people who do not know each other's posteriors. Despite these possible difficulties, it is plausible that the qualitative features found for the one-period information lag formulation would continue to hold, but not in the same way. Under the natural specification, because people would learn the realization in the limit, there ought to be an equilibrium that converges to what happens at date 2 under the one-period information lag formulation. Moreover, although the implied "short run" would then merge smoothly into the "long run," rather than ending abruptly after one period as under my specification, the effects at date 1 would again be entirely real.

The assumption that money is indivisible and that there is a unit upper bound on individual holdings plays an important role. To consider that role, suppose instead that money is divisible and that there is no bound on individual holdings. Under that alternative, the first issue that arises is how to have the change in the quantity of money come about. In order that people might not be able to infer the aggregate change from their own discoveries of money, each person's discovery should not be proportional to the person's initial holdings with a proportionality factor equal to the proportional change in the aggregate quantity of money. In the absence of such proportionality, even if the initial money distribution is a steady state, the money distribution after the change occurs is not a steady-state distribution. That will make it difficult to deduce the properties of the equilibrium path.

More interesting, in my formulation, those who discover money are not producers at date 1; they either are consumers or do not trade, a consequence of the indivisibility and the upper bound. If there is no upper bound, then the process of discovery could be random among everyone. Given such randomness, total output at date 1 may not be increasing in the aggregate discovery of money because producers who have discovered money will tend to produce less.[7] One way to amend the model to restore such dependence is

[7] The assumption that new money goes to consumers appears in many other models; see, e.g., Lucas (1972), Eden (1994), or Lucas and Woodford (1994). Barro and King (1984) emphasize the important role of the assumption and question the rationale for it.

to allow some choice about whether to produce or consume. If there is such a choice, then those who discover money would tend to be consumers. Although such a choice appears in some closely related models, they also include indivisible money and a unit upper bound on individual holdings (see Diamond 1984; Kiyotaki and Wright 1991).

Although the remarks above are necessarily speculative, there are grounds for supposing that the main qualitative finding regarding the effects of once-for-all changes in the quantity of money will survive generalizations of the model in several directions. In addition, it seems clear that the features that produce the distinct short-run and long-run effects in cross sections in the model of this paper would also produce similar effects in time series, if we were able to analyze a version with a stationary process for changes in the quantity of money.

IV. Concluding Remarks

Although I have emphasized the positive implications of the model, the model can also be used to judge the welfare consequences of different distributions for the one-time change in the quantity of money, different distributions for Δ. Such comparisons among different distributions for Δ can be regarded as a policy analysis if we suppose that there are multiple fiat objects and that a policy choice determines which is used as money. If there are multiple fiat objects, then there is an equilibrium in which all but one are valueless. We can regard the equilibrium of Section II as that equilibrium. As regards a welfare criterion, the model lends itself to a representative agent welfare criterion, namely, $(1 - m_0) v_0(0) + m_0 v_0$. This can be interpreted as the expected discounted utility of each person at date 0 prior to learning whether or not the person starts out with a unit of money: $(1 - m_0)$ being the probability of starting without money and m_0 being the probability of starting with money. Since the v's are given in Section II, we have all the ingredients for evaluating this welfare measure for different probability distributions for Δ.

The ability to conduct a welfare analysis is one main advantage of the model over Hume's discussion. Hume, in fact, seemed to advocate increases in the quantity of money, no matter how they were brought about. He said, "it is of no manner of consequence, with regard to the domestic happiness of a state, whether money be in a greater or less quantity. The good policy of the magistrate consists only in keeping it, if possible, still encreasing" (Hume 1970, pp. 39–40). In advocating this policy, Hume can be accused of forgetting the prominent role played by incomplete information in his explanation of different short-run and long-run effects of changes in the quantity of money.

References

Aiyagari, S. Rao; Wallace, Neil; and Wright, Randall. "Coexistence of Money and Interest-Bearing Securities." *J. Monetary Econ.* 37 (June 1996): 397–419.

Barro, Robert J., and King, Robert G. "Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles." *Q.J.E.* 99 (November 1984): 817–39.

Diamond, Peter A. "Money in Search Equilibrium." *Econometrica* 52 (January 1984): 1–20.

Eden, Benjamin. "The Adjustment of Prices to Monetary Shocks When Trade Is Uncertain and Sequential." *J.P.E.* 102 (June 1994): 493–509.

Hume, David. "Of Money." In *Writings on Economics,* edited by Eugene Rotwein. Madison: Univ. Wisconsin Press, 1970.

Kiyotaki, Nobuhiro, and Wright, Randall. "A Contribution to the Pure Theory of Money." *J. Econ. Theory* 53 (April 1991): 215–35.

Lucas, Robert E., Jr. "Expectations and the Neutrality of Money." *J. Econ. Theory* 4 (April 1972): 103–24.

———. "Nobel Lecture: Monetary Neutrality." *J.P.E.* 104 (August 1996): 661–82.

Lucas, Robert E., Jr., and Woodford, Michael. "Real Effects of Monetary Shocks in an Economy with Sequential Purchases." Manuscript. Chicago: Univ. Chicago, Dept. Econ., 1994.

Shi, Shouyong. "Money and Prices: A Model of Search and Bargaining." *J. Econ. Theory* 67 (December 1995): 467–96.

Trejos, Alberto, and Wright, Randall. "Search, Bargaining, Money, and Prices." *J.P.E.* 103 (February 1995): 118–41.