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SEARCH, BARGAINING, MONEY, AND PRICES UNDER PRIVATE INFORMATION*

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This paper presents a model of money and search where bargaining determines prices and the quality of goods is private information. It studies how a lemons problem affects the purchasing power of money. There are multiple, Pareto-ranked equilibria. The superior equilibrium, where no lemons are produced, exists even if information about quality is relatively scarce. In other equilibria, there is price dispersion, and uninformed buyers pay higher prices than informed buyers for all goods. Taxing money balances (a proxy for inflation) makes buyers less selective, thus reducing the average quality of supply and the premium paid for known quality.

1. INTRODUCTION

As first suggested by Menger (1892) and articulated by Alchian (1977), one of the roles of fiat money is to help alleviate *lemons problems* (problems that emerge when the quality of goods is private information). When the quality and authenticity of money are easy to ascertain but the same is not true about consumption goods, monetary transactions are easier to arrange than direct barter transactions. The first reason for this is that in monetary transactions the private information problem is, at most, one-sided. The second reason is that money holders, unlike goods holders, are not concerned that their willingness to trade may act as a negative signal about the quality of what they offer. Williamson and Wright (1994) use a search model to formalize these ideas and show that qualitative uncertainty can by itself make media of exchange necessary, even if there is no double-coincidence-of-wants problem. This has prompted other analyses of monetary-search economies with private-information problems; for instance, Kim (1996) studies the endogenous acquisition of information, Trejos (1997) studies the effects of monetary matters on the incentives to produce high-quality output, and Cuadras-Morató (1994) and Li (1995) look at economies where lemons may be used as commodity money.[2]

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[2] In related work, Li (1996) shows how money and middlemen can play complementary roles when a lemons problem is not the only friction affecting barter. For other work on monetary theory with private information, with a very different model, see Banerjee and Maskin (1996). There is also an extensive literature on lemons problems in general; this is not the place to survey it, since the intended contribution here is primarily to monetary theory.

In all the work cited, attention is focused on the exchange process (when does trade take place?) while ignoring the terms of exchange. In contrast, in this paper I study a search model of money and private information in which bilateral bargaining determines the purchasing power of money. The motivation for this is twofold. On the one hand, a series of new issues can be addressed with endogenous prices, including the pattern of price dispersion, the relationship between information and the value of money, and the effects of inflation on quality, bargaining, and exchange. On the other hand, one also can verify the robustness of previous results when one takes away the assumptions that make prices exogenous and force all qualities to trade at the same price.[3]

In the economy considered here, individuals meet bilaterally, and all exchange is quid pro quo. A double-coincidence problem and a private-information problem jointly generate a role for fiat money as a medium of exchange. Each agent can invest in the ability to produce high-quality output or can opt to save costs and produce low-quality lemons. In a particular bilateral meeting, a buyer may be informed or uninformed about the quality of the seller's output. Because searching for another producer is time-consuming, a buyer may still be willing to purchase something whose quality he or she cannot recognize, depending on various considerations, including general market conditions that are determined as part of the equilibrium. Whether or not the buyer is informed, the terms of trade are determined through bilateral bargaining.

I first describe the various combinations of the value of money, the pattern of price dispersion, and the average quality of output that constitute equilibria. It results that there can be multiple, Pareto-ranked equilibria for given combinations of the exogenous parameters. An equilibrium where only high quality is produced and qualitative uncertainty fails to affect prices exists even when information is relatively scarce. In fact, the strategic complementarity is such that whenever monetary equilibria with lemons exist, so does a superior equilibrium without lemons. Among the relevant exogenous parameters in the model is the amount of fiat money in the system (which in models of this kind affects the matching rate and the relative ease of selling versus buying). One can show that the optimal quantity of money is reduced by the presence of private information relative to the one in a model with a search friction only; the reason is that, on the margin, one wants to make selling difficult, since this lowers the incentives to produce hard-to-sell lemons. Regarding the pattern of price dispersion, the main finding is that contrary to what one might expect, buyers tend to pay higher prices when they are uninformed about the quality they are purchasing than when they know they are getting high quality. Finally, I introduce a proxy for inflation into the model. It follows that since it makes buyers eager to spend quickly and less discriminating about their purchases, inflation reduces the average quality of supply, reduces the premium paid for high quality, and reduces welfare.

[3] For instance, the existence of equilibria where low quality does not drive high quality out of the market despite the lemons problem may depend on the fact that both qualities are forced to trade at the same price. As an example of this, in Akerlof (1970) the price mechanism plays a key role in driving the high-quality goods out of the market.

2. THE ECONOMY

Time is continuous and unbounded. There is a continuum of infinitely lived agents. There are many varieties of consumption goods, all of which are perfectly divisible but not storable. Each variety of these goods comes in high quality or low quality, with low-quality goods also being referred to as *lemons*. Production is specialized, in the sense that each agent can only produce one variety (in either quality). Every time an agent consumes, he or she decides the quality of his or her next output. If his or her choice is to produce high quality, he or she incurs *at that time* a disutility cost $\gamma > 0$; no such cost is incurred if the choice is to produce low quality. Besides this fixed cost faced before production, there is a variable cost *at the time of production*: Making q units of output of either quality generates a disutility $c(q)$, where $c(0) = c'(0) = 0$ and $c'(q) > 0, c''(q) > 0\ \forall q > 0$. Consumer's tastes are specialized as well, in the sense that at a given moment an agent desires to consume one variety but not the others. If an agent consumes q units of high-quality output of the variety he or she desires, he or she derives instantaneous utility $u(q) = q$; he or she gets no utility from consuming any amount of low quality or of other varieties. Agents discount the future, and the rate of time preference is denoted $r > 0$.

Each individual knows which agents can produce the variety that he or she wants to consume. Furthermore, he or she has access to a Poisson matching technology, with arrival rate normalized to unity, by which he or she can generate bilateral meetings with those producers. He or she does not know, however, what varieties are desired by each of those other agents (perhaps because tastes are not advertised like production specialties or because tastes change with one's consumption history). This creates a double-coincidence-of-wants problem: Although one can always search for a producer of what one wants, one can only assign probability $y < 1$ that, once found, he or she also will want to consume the variety one can produce. The assumption made here is that this problem completely prevents direct barter, since I focus on the limit case as the number of varieties grows large, which means $y \to 0$.

While the *variety* of someone's output is public information, the *quality* of that output is private information. To be precise, consider a match between two agents, where the first is able to produce the variety that the second desires. Independent of the identity of these agents, with probability θ, the second agent will enter the match being informed about the quality of the output of the first; with probability $1 - \theta$, the second agent will be uninformed. Either way, the first agent always knows if the second agent is informed.

There is no Walrasian auctioneer in this economy, nor any other institution to arrange multilateral transactions or to enforce long-term agreements. Hence, if there is to be trade, agents must use the matching technology described earlier to find one another, and all transactions must be bilateral and quid pro quo. Also, since goods cannot be stored, their production, exchange, and consumption have to be simultaneous. Furthermore, as discussed earlier, the problem of double coincidence of wants makes direct barter of goods for goods impossible. Finally, agents act anonymously in the sense that their previous actions cannot be observed by others. All this means that credit, barter, reputation, commodity monies, and centralized markets are not viable mechanisms to generate trade, and the use of a fiat medium of exchange is the only alternative to autarchy.

To play the role of fiat medium of exchange, I assume that in addition to the consumption goods, there is another object called *money*. In contrast to the consumption goods, there is only one kind of money (it comes in only one quality and one variety), it cannot be consumed or privately produced, and it is storable but indivisible. At the beginning of time, some agents are endowed with one unit of money each. Assuming that an agent who already has one unit of money cannot acquire a second unit, and given that the money is indivisible, the stationary distribution of money holdings is very simple: At any point in time, a given fraction of the population, denoted M, holds exactly one unit of money each, and the remaining $1 - M$ holds no money.[4] I refer to agents holding money as *buyers* and to those with no money as *sellers*. Obviously, as money changes hands, the identities of the buyers and sellers change, but their numbers are not affected.

Once a buyer finds a seller of the variety he or she wants, each of them decides whether to negotiate a trade with that partner or to wait for a different trading partner. Following Trejos-Wright (1995) and Shi (1995), the object of negotiation is the amount of output that the seller produces in exchange for the buyer's unit of money. In the bargaining game, modeled after Rubinstein (1982), a random selection is made to determine who makes the first offer. The recipient of the offer can either accept it, reject it and permanently walk away, or reject it and let an interval of length Δ go by before another random selection is made and the process is repeated. No exogenous events can terminate the bargaining during the wait; in particular, neither buyer nor seller can meet another potential trading partner in that interval, unless they walk away definitively from their current match. We will mainly be interested in the limit case as $\Delta \longrightarrow 0$, in which the outcome of the game approaches the axiomatic bargaining solution of Nash (1950).[5]

To finish the description of the environment, I provide some notation. Let π denote the fraction of sellers that have the ability to produce high quality. Let V_M, V_H, and V_L denote the expected discounted lifetime net utility (the value function) for a buyer, a high-quality seller, and a low-quality seller, respectively. Define $V = (V_M, V_H, V_L)$. In trades taking place between a high-quality seller and an informed buyer, I call Q_i the amount of output that the seller produces in exchange for the buyer's unit of money. Similarly, in transactions between an uninformed buyer and (any kind of) seller, Q_u denotes the amount of output traded.[6] Define $Q = (Q_i, Q_u)$. Finally, I will assume

[4] The literature shows several ways to motivate the assumption that one does not get a second unit of money before spending the first. For instance, one can assume that agents cannot produce twice without consuming; one also can assume that production needs to take place in an idiosyncratic location or outright limit the ability to store more than one object. Either way, these assumptions are just ways of making sure that the distribution of money holdings is tractable. The more general problem, where money is divisible or gets to be accummulated in arbitrarily large amounts, has been a focus of study recently (see Green and Zhou, 1995; Camera and Corbae, 1996; Lang, Li, and Wang, 1996; Molico, 1996; and Shi, 1996). However, for a first attempt to endogenize prices in the model with private information, it seems appropriate to start with this better known and more tractable case.

[5] Private information sometimes prevents the outcome to approach the Nash solution in the limit. As is explained below, that is not the case here.

[6] This means that the price paid in a transaction by an informed buyer is $P_i \equiv 1/Q_i$ and by an uninformed buyer is $P_u = 1/Q_u$, since all exchanges involve exactly one unit of money. This notation is implicitly assuming that all equilibria yield pooling solutions to the problems concerning uninformed

complete symmetry across varieties; no variety is produced by more agents or desired with more frequency than any other variety.

3. EQUILIBRIUM

There is always a degenerate *nonmonetary* equilibrium, in which people believe money has no value and therefore $Q_i = Q_u = 0$. In this case, trade never takes place, and $\pi = 0$. I shall concentrate instead on *monetary* equilibria, where some high quality is produced, money is exchanged for output with positive frequency, and all agents are strictly better off than in autarchy. I also shall concentrate on steady states.

Given that buyers never buy lemons knowingly and that sellers accept money with probability one,[7] the value functions in a stationary equilibrium satisfy the flow equations

$$rV_M = (1-M)\left[\theta\pi\sigma_i(\Phi - V_M + Q_i) + (1-\theta)\sigma_u(\Phi - V_M + \pi Q_u)\right] \tag{1}$$

for the buyers,

$$rV_H = M\left\{\theta\sigma_i[V_M - V_H - c(Q_i)] + (1-\theta)\sigma_u[V_M - V_H - c(Q_u)]\right\} \tag{2}$$

for the high-quality sellers, and

$$rV_L = M(1-\theta)\sigma_u[V_M - V_L - c(Q_u)] \tag{3}$$

for the sellers of lemons. In equation (1), Φ denotes the continuation value for a buyer who has just spent his or her money and consists of the better option between becoming a high-quality producer or a low-quality producer: $\Phi = \max\{V_H - \gamma, V_L\}$. Also, in equations (1) through (3) σ_i denotes the probability that if a high-quality seller meets a buyer and the buyer is informed, they will agree to trade, and σ_u denotes the analogous probability in matches involving an uninformed buyer.

Equation (1) describes the flow value of being a buyer (rV_M) as equal to the rate at which a buyer meets producers of his or her desired variety (which has been normalized to unity) times the probability that those producers happen to be selling rather than buying at the moment $(1 - M)$ times the expected gain from such a meeting. This gain is determined as follows: With probability $\pi\theta\sigma_i$ the seller carries high quality, the buyer recognizes it, and they agree to trade; then the buyer obtains the utility from consuming Q_i plus the net gain of switching from buyer to high-quality seller $\Phi - V_M$; with probability $(1-\theta)\sigma_u$, the buyer cannot identify the seller's quality, but they trade anyway, and so the buyer obtains the expected utility πQ_u from

buyers; i.e., uninformed buyers get the same quantity of goods from both high-quality and low-quality sellers. This will be obtained below as a result. Trivially, I need no notation for the price of lemons that are recognized by the buyer as such, because in such circumstances no trade would occur.

[7] In Kiyotaki and Wright (1991) there are mixed-strategy equilibria, where sellers are indifferent about accepting or rejecting money and thus randomize. As shown in Shi (1995), this is a consequence of indivisibility; with divisible output buyers are always able to provide marginal incentives during bargaining to guarantee trade.

consumption, plus $\Phi - V_M$ from the change of status. Equations (2) and (3) can be interpreted analogously.

The strategic decisions that determine π lead to the condition[8]

$$\pi = \begin{cases} 0 & \text{if } V_H - \gamma < V_L \\ \in [0,1] & \text{if } V_H - \gamma = V_L \\ 1 & \text{if } V_H - \gamma > V_L \end{cases} \tag{4}$$

Equation (4) shows that the strategic decision of investing in the ability to produce high quality is determined by whether the benefits of such investment compensate for the fixed cost γ. As long as we only focus on nondegenerate, monetary equilibria, where $\pi > 0$ [and hence, by equation (4), where $V_H - \gamma \geq V_L$], we can substitute $\Phi = V_H - \gamma$ in equation (1) in what follows.

In the Appendix it is shown that in any nondegenerate equilibrium it has to be the case that $\sigma_i = \sigma_u = 1$; the corresponding incentive compatibility conditions are

$$\begin{aligned} Q_i - \gamma \geq V_M - V_H \geq c(Q_i) \\ \pi Q_u - \gamma \geq V_M - V_H \geq c(Q_u) \end{aligned} \tag{5}$$

The first line of equation (5) indicates that when a high-quality seller and an informed buyer meet, it is rational for them to agree to trade. The second line indicates the same for an uninformed buyer and a high-quality seller. We do not need a third equation for uninformed buyers and low-quality sellers because it is made redundant by the condition $V_H - \gamma \geq V_L$.

The next step is to determine prices. Consider the amount q_i that is traded between an informed buyer and a high-quality seller, who take as given the amounts $Q = \{Q_i, Q_u\}$ that are being traded in the rest of the market. Define q_u analogously. Clearly, in equilibrium, $q_i = Q_i$ and $q_u = Q_u$, but to get there we first derive q_i and q_u, taking Q_i and Q_u as given. Using standard methods, one can derive that for small Δ, the outcome of the bargaining game has the following characteristics: (1) in equilibrium, the first offer is always accepted, regardless of who poses it, (2) out of equilibrium, if an offer was ever rejected, nobody would walk away from the bargaining table until the next offer, and (3) as the waiting time between rounds vanishes ($\Delta \longrightarrow 0$), the offers made by either buyer or seller converge to the same quantity, which is the maximand of an appropriately defined Nash product. Hence

$$\begin{aligned} q_i &= \arg\max_q [V_M - c(q)][V_H - \gamma + q] \\ &\quad \text{subject to} \quad V_M - c(q) \geq V_H \\ &\quad \text{and} \qquad V_H - \gamma + q \geq V_M \\ q_u &= \arg\max_q [V_M - c(q)][V_H - \gamma + \pi q] \\ &\quad \text{subject to} \quad V_M - c(q) \geq V_H \\ &\quad \text{and} \qquad V_H - \gamma + \pi q \geq V_M \end{aligned} \tag{6}$$

[8] There is an underlying decision variable, say, Π, that determines the probability that an agent chooses to produce high- instead of low-quality output. But there is a one-to-one mapping between Π and π, so one needs only consider the latter.

where the value functions in expression (6) are obtained taking the quantities Q as given and thus are not varying with q.

In deriving expression (6) I use the fact that at the bargaining table the relevant payoff function for both parties is common knowledge. In other words, although there is private information in the model, at bargaining time, the payoffs of both parties are not private information, so the determination of q_u takes the familiar form displayed earlier. This is important because without complete information, the bargaining game can display multiple equilibria, delay, and other complications [see Kennan and Wilson (1993) for a discussion]. In my model, however, such complications do not arise. There is no signaling on the part of sellers; lemons sellers would never signal, since that would mean they would not trade, and $V_H > V_L$ implies that high quality sellers cannot credibly signal, since any offer or delay that a high-quality seller could make can be profitably imitated by a lemons seller, given that the latter has a slacker participation constraint and a lower value of outside search. In other words, a pooling solution follows because total cost curves do not intersect.

Even if the buyer ignores the quality of what he or she is being offered, he or she does know the seller's continuation value $V_M(Q)$ and cost function $c(q)$ because those do not depend on whether the seller has high or low quality. Similarly, the seller knows the buyer's continuation value $V_H(Q) - \gamma$ and expected utility (since he or she knows whether the seller is informed or not). This is the sense in which there is complete information in the bargaining game, even though there is private information in the economy as a whole. This tractability follows from two critical assumptions: that the variable cost function is the same for both qualities and that there is no search during bargaining (by which I mean that in the event an offer was rejected and the trading parties decided to wait for another round, they would make no new matches with other partners during the wait). The same variable cost for quality and lemons implies that the seller's payoff function $V_M(Q) - c(q)$ is always common knowledge. The lack of search between offers implies that the expressions $V_H(Q)$ and $V_L(Q)$ do not appear in the Nash product, since all that matters is the options that the buyer and seller have on the sale (known to everybody) and not the status that the seller is leaving behind. [See Shi (1995) and Trejos and Wright (1995) for the derivation of the Nash product that applies for the bargaining games in this type of model, with and without outside search.] If negotiations could break up exogenously, the payoff for the seller from rejecting an offer would include the value of renewing search without making a sale, which is affected by the quality he or she can produce.

The Nash products in expression (6) are strictly concave in q, so there is a unique solution to the bargaining game. It will be convenient below to refer to the solution of the maximization by using the functions

(7)
$$\begin{aligned}\Gamma_i &= [V_M(Q_i, Q_u) - c(Q_i)] - c'(Q_i)[V_H(Q_i, Q_u) - \gamma + Q_i]\\ \Gamma_u &= \pi[V_M(Q_i, Q_u) - c(Q_u)] - c'(Q_u)[V_H(Q_i, Q_u) - \gamma + \pi Q_u]\end{aligned}$$

where one first derives the first order conditions for equation (6), taking V_M and V_H as given, and then substitutes $q_i = Q_i$ and $q_u = Q_u$, having solved equations (1) through (3) for the value functions.

If the solution to expression (6) is interior, both trading partners derive a positive surplus from exchange, and we say that the bargaining game has an *unconstrained* solution (because the constraints in the maximization are not binding). If expression (6) has a corner solution, then either buyer or seller is trading at his or her reservation q, and we say that the bargaining game has a *constrained* solution. For instance, for the game between an informed buyer and a high-quality seller, an unconstrained solution would be a value Q_i that satisfies $\Gamma_i = 0$, $V_M - c(Q_i) - V_H \geq 0$ and $V_H - \gamma - V_M + Q_i \geq 0$. In a constrained solution, either $\Gamma_i < 0$ and $V_H - \gamma - V_M + Q_i = 0$ (the buyer gets no surplus, while the Nash product is decreasing in Q_i) or $\Gamma_i > 0$ and $V_M - c(Q_i) - V_H = 0$ (the seller gets no surplus, while the Nash product is increasing in Q_i). Analogous statements apply to a constrained or unconstrained solution for Q_u.

Before proceeding, let me define an equilibrium formally. A stationary, monetary equilibrium for this economy is a combination (π, Q, V) such that, given prices Q and production strategy π, the value functions V satisfy the Bellman equations (1) through (3); given V and Q, the production strategy π is consistent with equation (4); given V and π, the market prices are consistent with bargaining, so the maximization problems in expression (6) are solved by $q_i = Q_i$, $q_u = Q_u$, and finally, the incentive-compatibility conditions (5) are satisfied.

I start now by looking for an equilibrium where there are no lemons, $\pi = 1$, which means that sellers always choose to invest in the ability to produce high-quality output. To clarify the exposition, I will later refer to this as a *class 1* equilibrium. Presumably, this type of arrangement can be an equilibrium when investment is cheap enough or when buyers are informed often enough about quality that nobody would find it profitable to try to sell lemons. This equilibrium, as will be shown, closely resembles the equilibrium of similar models with only one quality. The only difference is that here agents *could* produce low-quality goods but *choose* not to do it (so we have to specify the conditions that make this choice rational).

If $\pi = 1$, it follows that $Q_i = Q_u = Q$, and one can rewrite equations (1) through (3) as

$$\begin{aligned} rV_M &= (1-M)(V_H - \gamma - V_M + Q) \\ rV_H &= M[V_M - V_H - c(Q)] \\ rV_L &= M(1-\theta)[V_M - V_L - c(Q)]. \end{aligned} \tag{8}$$

The third equation in expression (8) shows that a seller of lemons not only would get away with selling them when he or she meets an uninformed buyer but also will do so at the price that informed buyers pay for high quality. This being a stationary problem, the equilibrium exists only if the strategy to always produce quality is unimprovable by a one-time deviation, i.e., $V_H - \gamma \geq V_L$. This requires information to be abundant enough as to make selling lemons too time-consuming. Simple algebra leads us to solutions for the value functions in terms of Q, and from there it follows that $V_H - \gamma \geq V_L$ if and only if

$$\theta \geq \frac{(1+r)(M+r)\gamma}{M[(1-M)Q + (M+r)\gamma - (1-M+r)c(Q)]} \equiv \theta_1(Q) \tag{9}$$

We are left with the determination of Q. Substitute the solution for expression (8) into expression (7) to derive

$$\Gamma_i = (M+r)\rho(Q) - (1-M+r)c'(Q)\Psi(Q) \tag{10}$$

where $\Psi(Q) \equiv (M+r)Q - Mc(Q)$ and $\rho(Q) \equiv (1-M)[Q-\gamma] - (1-M+r)c(Q)$. One can show that the constraints in expression (6) are satisfied if and only if $\Psi(Q) \geq 0$ and $\rho(Q) \geq 0$; furthermore, only the latter constraint binds. The procedure to find an unconstrained solution is to solve equation (10) for a root to $\Gamma_i = 0$ and verify that it satisfies equation (9) and $\rho(Q) \geq 0$. Constrained equilibria with $\pi = 1$ do not exist because, as can be seen from equation (10), $\rho(Q) = 0$ implies $\Gamma_i < 0$, and $\Psi(Q) = 0$ implies $\rho(Q) < 0$. Since under very high values of r it is the case that $\Gamma_i < 0\ \forall Q$, it follows that there is a maximum value r_1 such that this equilibrium can exist only if $r < r_1$. The critical value θ_1 is unambiguously increasing in γ and in r, while the sign of $\partial\theta_1/\partial M$ is ambiguous. In summary, a *class 1* equilibrium exists when agents are patient enough and trade is easy enough that investing on future trade is worthwhile (r is low and M is neither too high nor too low), information is abundant enough that selling lemons is hard (θ is high), and quality is not too expensive to produce when compared with lemons (γ is low).

The problem of private information becomes more interesting when $\pi \in (0,1)$ so that supply is heterogeneous in quality. Since all agents are in effect identical in this model, to observe output of both qualities, we need agents to be indifferent about whether or not to make the investment γ. There are only two possible kinds of equilibria that satisfy $0 < \pi < 1$, and for both, a condition for existence is that r is low and that θ is neither too high nor too low.[9] On the one hand, there is the case that I will call a *class 2* equilibrium that has the characteristic that Q_i and Q_u are both unconstrained solutions to the bargaining games in expression (6). Then the solution is a (Q, V, π) combination that satisfies equations (1) through (3) and (5), $\Gamma_i = \Gamma_u = 0$, and $V_H - \gamma = V_L$. On the other hand, there is the case that I will call a *class 3* equilibrium, characterized by the fact that Q_u is a constrained solution, since uninformed buyers purchase at their reservation price, while Q_i is unconstrained. Then the solution is a (Q, V, π) combination that satisfies equation (1) through (3) and (5), and $\Gamma_i = 0$, $V_H - \gamma + \pi Q_u = V_M$, $V_H - \gamma = V_L$, and $\Gamma_u \leq 0$. Other equilibria with $0 < \pi < 1$ cannot exist, as is demonstrated in the Appendix.

Consider first the situation where $0 < \pi < 1$ that has been labeled a *class 2* equilibrium. It is straightforward to derive solutions for V as functions of Q, and also, from $V_H - \gamma = V_L$,

$$\pi = \frac{(M+r)(1-\theta+r)\gamma + \theta M(1-\theta+r)c(Q_i) - \theta M^2(1-\theta)c(Q_u)}{\theta(1-M)[\theta M u(Q_i) + M(1-\theta)u(Q_u) - M\theta c(Q_i) - (M+r)\gamma]} \tag{11}$$

[9] As in the *class 1* equilibrium, a low r is necessary to guarantee that some agents will be willing to make an investment in producing high quality. If θ is too high, selling lemons is too hard, so nobody produces them; if θ is too low, the investment in quality seldom pays off.

A *class 2* equilibrium is then a combination (V, Q, π) that satisfies equations (1) through (3) and (11), the first-order conditions

$$\begin{aligned} V_M - c(Q_i) &= c'(Q_i)(V_H - \gamma + Q_i) \\ \pi[V_M - c(Q_u)] &= c'(Q_u)(V_H - \gamma + \pi Q_u) \end{aligned} \tag{12}$$

and satisfies $0 < \pi < 1$ and expression (5). In an equilibrium of this type, some agents choose to produce lemons because in that way they can save the upfront investment cost γ and because they can get away with selling to uninformed buyers at the same price that they would pay for quality. Some other agents choose to produce quality because it has a larger market and therefore can be sold faster, since all buyers, informed or not, would spend their money on it when given the opportunity.

One interesting feature of this equilibrium is that one can show that in it $Q_i > Q_u$. In other words, when a buyer is informed, not only can he or she be sure that he or she is not being sold a lemon, he or she also can extract a higher amount of output for his or her money than if he or she was uninformed. To see why, substitute first $Q_i = Q_u$ into the second equation of expression (12), and use $\Gamma_i = 0$ from the first equation to derive

$$\begin{aligned} \Gamma_u &= \pi c'(Q_i)(V_H - \gamma + Q_i) - c'(Q_i)(V_H - \gamma + \pi Q_i) \\ &= -(1-\pi)c'(Q_i)(V_H - \gamma) < 0 \end{aligned} \tag{13}$$

Then the Nash product for the uninformed match is decreasing in Q_u at $Q_u = Q_i$, and thus $Q_u < Q_i$. This result means that information enables a buyer to get quality at better prices. This is somewhat surprising; when $\pi < 1$, an informed buyer is effectively more eager to consume than an uninformed buyer, and presumably this should hurt his or her bargaining position, yet he or she gets away with paying a lower price. One way of finding this less counterintuitive is noting that the continuation payoff is a high fraction of the total payoff of the uninformed buyer when compared with an informed buyer, and this makes him or her in a way less patient, thus giving him or her a weaker bargaining position.

The difference between a *class 2* and a *class 3* equilibrium does not lie in who agrees to trade with whom or in how quickly the different goods are sold, since the same matches lead to a trade in both equilibria. The differences lie, simply, in prices and average quality and hence in the liquidity and value of money. A *class 3* equilibrium is a combination (V, π, Q) that solves equations (1) through (3), as well as $V_H - \gamma = V_L$, $V_M - V_H = \pi Q_u - \gamma$ and $\Gamma_i = 0$ while satisfying expressions (14) and (5) and $\Gamma_u \leq 0$. Algebraic manipulation allows one to derive the simpler expressions:

$$\begin{aligned} rV_M &= \theta\pi(1-M)(Q_i - \pi Q_u) \\ rV_H &= M\theta[\pi Q_u - \gamma - c(Q_i)] + M(1-\theta)[\pi Q_u - \gamma - c(Q_u)] \\ rV_L &= M(1-\theta)[\pi Q_u - c(Q_u)] \\ \pi &= \frac{M\theta c(Q_i) + (M+r)\gamma}{M\theta Q_u} \end{aligned} \tag{14}$$

In the case of a *class 3* equilibrium, it is not necessarily true anymore that $Q_i > Q_u$; it can be shown, however, that $Q_i > \pi Q_u$. In the next section I use a specific functional form for the cost function $c(q)$ that allows me to derive more properties about this and the other equilibria.

3.1. *Functional Forms*. This section presents some additional results that can be obtained by studying numerically the model under the functional form $c(q) = q^2$, which satisfies the properties we have assumed. Under this functional form, for any given set of parameter values, it is possible to compute numerically the endogenous variables and verify the existence conditions for all three classes of equilibria. This allows, for instance, characterization of the sensitivity of the different equilibria to parameter variations and description the portions of the parameter space at which each equilibrium exists. The rest of this section describes the outcome of this numerical analysis. It only presents those results which were shown to be robust across an extensive search through parameter space. Details of the computation for this numerical analysis appear in the Appendix.

A first robust finding from all the numerical solutions studied is that for a given set of parameter values, at most one combination of (Q, V, π) constitutes an equilibrium for each class of equilibria discussed before. That is, there are not multiple equilibria of the same class. However, one may find multiple classes of equilibria coexisting. For instance, an equilibrium of *class 1* can coexist with an equilibrium of *class 3*. To illustrate this, Fig. 1 describes the combinations of the parameters M and θ for which the different nondegenerate equilibria exist, drawn for given values of γ and r (the figure is drawn for $\gamma = 0.02$ and $r = 0.001$; several other values of γ and r were tried and yielded identical qualitative results).

When information is abundant (θ high), lemons take too long to sell because uninformed agents are too hard to find, and thus equilibria with lemons do not exist; in such circumstances, the unique monetary equilibrium involves only high quality being sold. When information is scarce (θ low), nobody would invest in quality because it

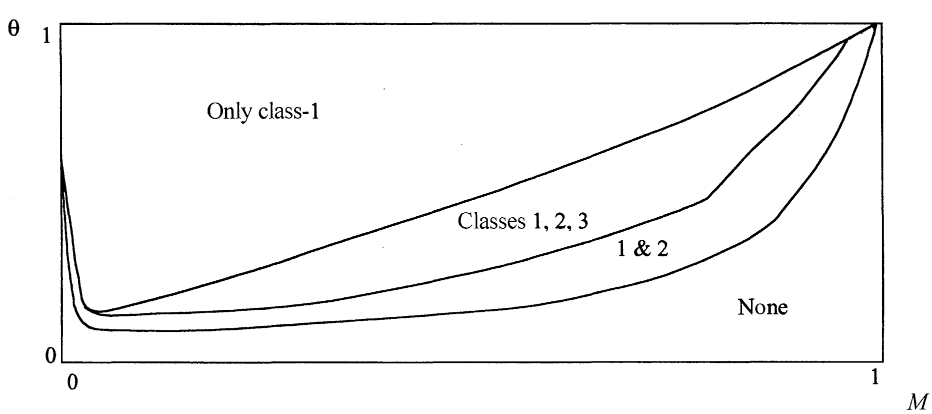

FIGURE 1

EXISTENCE OF THE DIFFERENT MONETARY EQUILIBRIA would seldom be recognized; thus money has no value and no purchasing power, and there is no monetary equilibrium. Intermediate values of θ allow all three equilibria to exist. In such a case, the expectation that only quality will be produced would raise the value of money and make it worthwhile to invest in quality. Similarly, information is scarce enough that the expectation that money sometimes buys lemons reduces its value in a way that makes it worthwhile to produce lemons. This type of strategic complementarity is what leads to multiple equilibria: higher average quality of output means higher value of exchange, and higher value of exchange is an incentive to produce output of higher quality; analogously, higher value of money means lower monetary prices, and lower prices mean that money is more valuable.

Another robust finding, which can be seen from Fig. 1, is that whenever an equilibrium with lemons exists, an equilibrium without lemons also exists. In particular, one finds that if a *class 3* equilibrium exists, then a *class 2* equilibrium also exists, and if a *class 2* equilibrium exists, then a *class 1* equilibrium also exists (although the converse statements are not true). This contrasts with the results in some of the previous work, which assumed fixed prices. In Williamson and Wright (1994) and in Trejos (1997), there are parameter values for which the *only* monetary equilibrium involves lemons in circulation.[10] More precisely, in those models, for θ low enough, there is no *class 1* equilibrium (i.e., one characterized by $\pi = 1$), but there may be a *class 3* equilibrium (where $0 < \pi < 1$). The usual interpretation is that one may have to settle for a second-best outcome with some lemons produced when information is scarce enough that the first-best allocation where $\pi = 1$ cannot be supported. In contrast, in the endogenous-price model, the production of lemons is always the consequence of coordination failure.

This contrast is an artifact of the fixed-price assumption in the previous models. To illustrate this, consider the lowest value of θ, call it $\underline{\theta}$, for which a *class 3* equilibrium exists, given other parameter values, in the current model. Inspection of the numerical solutions reveals that at $\underline{\theta}$ high-quality sellers get exactly zero surplus from selling to uninformed buyers, since Q_u is much higher than Q_i. This means that for $\theta < \underline{\theta}$, high-quality sellers prefer to wait for an informed buyer, and this prevents the existence of the equilibrium, because if an uninformed buyer saw a seller willing to trade with him or her, he or she would learn from that willingness that the seller has lemons. If one exogenously forced both Q_i and Q_u to be equal (in fact, what is done in the fixed-price model), the high-quality seller would not discriminate among buyers, and all the other conditions for the *class 3* equilibrium would be satisfied for $\theta < \underline{\theta}$. In fact, if one forces $Q_i = Q_u$, one obtains that there are values of θ so low that a *class 1* equilibrium does not exist, and yet a *class 3* equilibrium does exist.

Studying the effect of the parameters on prices and quality, one finds the following. First, increases in the money supply M decrease Q_i—that is, increase the price of known quality—except for extremely low values of M. Also, by making selling easy and buying hard, a high M means that average quality is low and that the premium for

[10] A deeper contrast is observed with respect to Kim (1996) and Kim and Yao (1996), where it is shown that with endogenous acquisition of information, a *class 1* equilibrium can *never* exist, since when $\pi = 1$ there is no incentive to acquire information, and with no information there is no incentive to produce quality.

known quality $Q_i - Q_u$ is low. Similarly, in the *class 3* equilibrium, more information (high θ) means more goods of high quality.[11] At given parameter values, if both equilibria with lemons exist, then the *class 2* equilibrium displays a higher π than the *class 3* equilibrium. This also means more transactions taking place in a unit time and so a higher velocity of money.

To make welfare comparisons across the different classes of equilibria, one can use the criterion $W = MV_M + (1-M)\pi V_H + (1-M)(1-\pi)V_L$. The welfare function W can be interpreted as the objective function for a planner that weighs all members of this economy equally, and it is also the expected utility of every agent before the initial distribution of money. Robust across all the attempted parameterizations was the finding that whenever a *class 2* equilibrium existed, it had lower welfare (as described by W) than the also existing *class 1* equilibrium; similarly, whenever a *class 3* equilibrium existed, *it* had lower welfare than the also existing *class 2* equilibrium. The intuition from this finding can be obtained from the way in which the different endogenous variables were found to rank across equilibria. We observe that whenever the different outcomes coexist, Q_i and π are higher in the *class 1* than in the *class 2* equilibrium and also higher in the *class 2* than in the *class 3* equilibrium. Since the maximand of W involves $Q_i = Q^*$ and $\pi = 1$ and we have found that in each equilibrium $Q_i < Q^*$, the welfare ranking becomes intuitive.[12]

We also can use W to study the optimal money supply M^* (the optimal fraction of agents who are buyers rather than liquidity-constrained sellers). The frequency of trade is maximized at $M = \frac{1}{2}$. However, in the *class 1* equilibrium $M^* < \frac{1}{2}$, because on the margin making it harder for sellers to find buyers increases the already lower than optimal equilibrium value of Q_i. In the *class 3* equilibrium, M^* is even lower, because on the margin reducing M increases π, and W is increasing in the amount of high quality. These results are not surprising considering related models. With fixed prices and fixed quality, as in Kiyotaki and Wright (1993), the optimal money supply is $M^* = \frac{1}{2}$. With endogenous prices and fixed quality, as in Trejos and Wright (1995), $M^* < \frac{1}{2}$. With endogenous quality and fixed prices, as in Trejos (1997), $M^* = \frac{1}{2}$ in the equilibrium with no lemons, and $M^* < \frac{1}{2}$ in the equilibrium with lemons.

A last consideration for the numerical analysis is to study the effect of inflation on both the quality of supply and the pattern of price dispersion.[13] Since introducing a true inflationary process in this model is complicated, I instead use the proxy proposed in Li (1994), which is of common use in this literature. This proxy consists of assuming that with arrival rate μ, agents holding money have it confiscated by the

[11] Of course, for the *class 1* equilibrium, $\pi = 1$. For the *class 2* equilibrium, some of these relationships are of the opposite sign. It is often the case that there exist unstable "intermediate" equilibria that display counterintuitive comparative statics. While stability is not studied here, it has been shown that in the model with fixed prices, the *class 2* equilibrium is unstable, while the others are stable (see Trejos, 1997).

[12] Of course, the comparison between the *class 2* and *class 3* equilibria is also affected by Q_u. In fact, the ranking of Q_u across both classes of equilibria was not robust, nor was the sign of $\partial W/\partial Q_u$. Nevertheless, the ranking among the variables Q_i and π seemed to be the strongest factor, since no exceptions to the dominance of the former equilibrium over the latter could be found.

[13] To motivate this, one may wonder whether this model is consistent with the common casual observation that lemons problems seem to worsen during hyperinflations. On related issues, see Tomassi (1994).

government, who put it back in circulation by spending it on goods. This resembles inflation because it lowers the value of money and because the expected "tax" collected from a particular agent is proportional to the length of time that agent holds onto his or her money. To be specific about the exercise here, I modify the model by having that, by a Poisson process with arrival rate μ, money holders have their money confiscated [this adds the term $\mu(V_H - V_M - \gamma)$ to the right-hand side of equation (1)]. This means that the flow of money confiscated from buyers is of size μM, so with arrival rate $\mu M/(1 - M)$ sellers get an opportunity to sell to the government. To keep matters simple, I assume that the government puts the confiscated money back in circulation by making take-it-or-leave-it offers to high-quality sellers, which means that equations (2) and (3) are unaffected.

For the *class 1* and *class 3* allocations, we look for sets of parameter values under which the corresponding equilibrium exists for $\mu = 0$ and then study the effects on existence and on the endogenous variables as μ increases.[14] First, and not surprisingly, we find that in both classes, a high enough rate of inflation μ means that the equilibrium no longer exists, since intuitively the premium associated with being a buyer $(V_M - \Phi)$ falls with μ. This also means that increases in μ reduce Q_i. Also, in the *class 3* equilibrium, the tax on money holdings, proxy for inflation, reduces the average quality of supply π, intuitively because buyers are eager to spend before money loses its value and thus are less selective about quality. Finally, the tax also reduces the premium paid for known quality $Q_i - Q_u$ (in the two equilibria where lemons are produced), for similar reasons. As μ reduces Q_i and π, welfare is decreasing in the inflation rate.

4. CONCLUDING REMARKS

This paper has presented a search theoretic monetary model where the quality of consumption goods is private information, and prices are determined through bilateral bargaining. The model is used to study how the properties of monetary exchange, and in particular, the purchasing power of fiat money, depend on a lemons problem. Some of the previous results from models with fixed prices are shown to be robust, and some generalized results are also found.

There are three types of equilibria in this model. For some functional forms, I find that under certain parameter values one equilibrium of each type exists, and it is possible to make welfare comparisons across them. What distinguishes each equilibrium is the average quality of supply and the amount of goods that are produced and exchanged in each trading opportunity.

In one of these equilibria, lemons are not produced at all, nor does private information affect prices or output. Unlike Akerlof (1970), this equilibrium is sustainable even when information about quality is relatively scarce. Furthermore, whenever other nondegenerate outcomes are sustainable in the economy, so is this particular equilibrium; in this sense, in this monetary economy the presence of lemons is always a coordination failure. As in Chan and Leland (1982) and Cooper and Ross

[14] I also found parameter combinations under which the *class 3* equilibrium did not exist for $\mu = 0$ but did exist for higher inflation rates. Those cases were not studied for comparative statics.

(1984), the equilibria where different qualities of consumption goods are in supply display price dispersion. However, unlike those papers, observing different prices for the same good does not require heterogeneity among agents here. The pattern of the price dispersion is somewhat surprising: All goods (not only lemons) are more expensive to those buyers who cannot recognize their quality, even when their ignorance makes them less "eager" to purchase. Finally, by introducing a proxy for inflation in the model, one finds that lemons problems are aggravated when inflation is high. For instance, increases in the inflation rate lead to reductions in the average quality of supply. Also, when inflation is high, the price of goods of recognized quality goes up, yet it goes down relative to the price of other goods (including lemons).

APPENDIX

I now prove that in any nondegenerate equilibrium where $\pi > 0$ it must hold that $\sigma_i = \sigma_u = 1$. This can be shown by contradiction. Assume that it is not case that $\sigma_i = \sigma_u = 1$, which means (since I do not consider mixed strategies) that either $\sigma_i = \sigma_u = 0$, or $\sigma_i = 0$, $\sigma_u = 1$, or $\sigma_i = 1$, $\sigma_u = 0$. Then $\pi > 0$ is contradicted by $\sigma_i = \sigma_u = 0$ (since it implies $\gamma > V_H = 0$ and thus $\pi = 0$). Neither can it be that $\sigma_i = 0$ and $\sigma_u = 1$, since this implies that $V_H = V_L$ and thus $\pi = 0$ as well. Finally, if it was the case that $\sigma_i = 1$ and $\sigma_u = 0$, then lemons would never be sold; this would imply in a nondegenerate equilibrium that $\pi = 1$, which in turn contradicts $\sigma_i \neq \sigma_u$, since uninformed buyers would assume they are being offered high quality and act like informed buyers. Consequently, it must be that in a nondegenerate equilibrium $\sigma_i = \sigma_u = 1$.

Now I show why other equilibria with $0 < \pi < 1$, besides those I have labeled *class 2* and *class 3* equilibria, cannot exist. To see why, study in turn other types of *nondegenerate* equilibria. First, consider allocations where high-quality sellers get zero surplus from trading with informed buyers. From equations (2) and (3), it would follow that $V_H = V_L$, and thus $\pi > 0$ would be inconsistent with individual rationality. Second, consider allocations where informed buyers get zero surplus from high-quality sellers. Then, when informed buyers derived zero surplus, they would not purchase at all when uninformed [by the same argument that lead to expression (13)], which contradicts $\pi < 1$. Both things together imply that in trades between a high-quality seller and an informed buyer, both parties derive strictly positive surplus, and so any equilibrium must satisfy $\Gamma_i = 0$. Go now to transactions involving uninformed buyers. First, notice that if lemons sellers derived zero surplus from selling to uninformed buyers, then $V_L = 0$, which under $\pi < 1$ is contradictory with a nondegenerate equilibrium. Second, consider an allocation where high-quality sellers derive zero surplus from selling to uninformed buyers, which would require as equilibrium conditions that $V_M - V_H = c(Q_u)$ and $\Gamma_u > 0$. But the former implies that $rV_H = M\theta[c(Q_u) - c(Q_i)]$; this implies $Q_u > Q_i$, which in turn implies $\Gamma_u - \Gamma_i < 0$, a contradiction. In conclusion, in any equilibrium it must be the case that all sellers get a strictly positive surplus from trading with uninformed buyers. Thus, in equilibrium, it must be that $\Gamma_u \leq 0$. This leaves only two possibilities for nondegenerate equilibria with $0 < \pi < 1$: Either $\Gamma_u = 0$ and $V_H - \gamma + \pi Q_i \geq V_M$ (which I labeled *class 2* equilibria above) or $\Gamma_u < 0$ and $V_H - \gamma + \pi Q_i = V_M$ (which I labeled *class 3*).

Now proceed to describe the numerical procedure used in Section 3.1. For the *class 1* equilibrium, first solve in closed form equations (8) for the variables V_i; then substitute the solution into equation (10), which results in a cubic equation in Q_i. Finally, for each set of parameter values (γ, M, r, θ), this cubic equation is solved (using *Mathematica*), and its three (positive) roots are tested for constraints (5) and (9).

For the *class 2* equilibrium, the subsystem (1) through (3) and $V_H - \gamma = V_L$ was solved closed form for the variables V_M, V_H, V_L, and π. The unique solution was substituted into expression (12), resulting in a 2×2 nonlinear system in Q_i and Q_u. For each set of parameter values (γ, M, r, θ) this system was solved (*Mathematica* can yield an exhaustive set of solutions for this type of system, so no roots are known to be missing), and then for each (real, positive) solution the constraints (5) were checked.

For the *class 3* equilibrium, equations (1) and (2) and $\Gamma_i = 0$ are solved for V_M, V_H, and π, with the solution substituted into $V_M - V_H = \pi Q_u - \gamma$. This can be solved in closed form for Q_u. Substituting this solution into equation (3) and applying $V_H - \gamma = V_L$ results in a degree 11 polynomial equation in Q_i. Again, the set of roots was solved using *Mathematica*, and each (real, positive) solution was tested for the constraints (5).

Because these procedures included a numerical stage, the results in that section (with the exception of a handful that were proven analytically) are not guaranteed globally. They were robust to the very large number of sets of parameters that was tried. For instance, to derive Fig. 1, extensive search (thousands of points) was made over (M, θ) space for several combinations of (r, γ).

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