The Real-Bills Doctrine versus the Quantity Theory: A Reconsideration

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Two competing monetary policy prescriptions are analyzed within the context of overlapping generations models. The real-bills prescription is for unfettered private intermediation or central bank operations designed to produce the effects of such intermediation. The quantity-theory prescription, in contrast, is for restrictions on private intermediation designed to separate "money" from credit. Although our models are consistent with quantity-theory predictions about money supply and price-level behavior under these two policy prescriptions, the models imply that the quantity-theory prescription is not Pareto optimal and the real-bills prescription is.

This paper studies aspects of three interrelated issues: (a) the appropriate government regulation of financial intermediaries; (b) the proper conduct of central bank open-market and discount-window policy; and (c) the definition of money. As they have been for over two centuries, these remain among the most important issues in monetary economics. A useful way to organize the discussion of these issues is in terms of two long-standing and opposing doctrines: the quantity theory of money and the free-banking or real-bills doctrine.

The real-bills doctrine asserts that unrestricted intermediation either by private banks or by a central bank has beneficial economic effects and should be promoted by public policy. The doctrine proposes that there be unrestricted discounting of real bills—evidences of

indebtedness, which, in accordance with Adam Smith's definition in *The Wealth of Nations* (1776/1937), are safe or free of default risk.[1] As we interpret the doctrine, it asserts that one function of banks is to issue bank notes or similar liabilities that, because they are issued in small and standard denominations, are more easily held as assets by ultimate lenders than the bills of the ultimate borrowers that the banks are discounting. The key prescription of the doctrine is that no government regulations ought to restrict the scope of such intermediation. The doctrine relies on market forces to prevent excessive "credit creation" by private banks.

The doctrine also has an interpretation in terms of a proposed central bank open-market strategy. If regulations happen to exist that inhibit private intermediation—for example, regulations that prohibit private banks from issuing bearer notes and that make the central bank a monopoly issuer of currency-like assets—then the real-bills doctrine instructs the central bank to conduct open-market operations in private securities or operate a discount window in a way that vitiates the restrictions against private intermediation. The central bank can accomplish this by freely rediscounting paper for private banks or by lending directly at an appropriate nominal interest rate. By doing this, it brings together borrowers and lenders who otherwise might not be matched because of the restriction against private intermediation.

The real-bills doctrine has long been disputed by advocates of classical macroeconomic theories, which usually embody some version of the quantity theory of money. One seemingly telling criticism is that, under a real-bills regime, the economy is exposed to the Wicksellian situation in which both the price level and the money supply are indeterminate. Presentations of this criticism of the real-bills doctrine are contained in Sargent and Wallace (1975, sec. 5) and Sargent (1979, chaps. 5, 15). Those presentations are extreme versions of the common criticism that a real-bills regime permits excessive fluctuations in the supply of money and, hence, in the price level.

Motivated partly by a desire to avoid such price-level fluctuations and possible Wicksellian price-level indeterminacy, quantity theorists have advocated legal restrictions on private intermediation. The legal restrictions are intended to separate "money creation" from credit creation, that is, from the process of intermediation. Thus, for

[1] Smith says that "a real bill of exchange [is] drawn by a real creditor upon a real debtor, and . . . , as soon as it becomes due, is really paid by that debtor" (p. 288). According to Mints (1945, p. 9), a critic of the real-bills doctrine, Smith provided the most elegant statement of the doctrine. However, even Smith qualified his advocacy of free banking by proposing two restrictions on private banks: They should not be allowed to issue notes in small denominations, and all notes should be payable upon demand. (See Smith's discussion in bk. 2, chap. 2.)

example, Friedman (1959, p. 21) hailed the feature of the National Banking System that taxed state bank notes out of existence and has advocated 100 percent reserves against bank liabilities called demand deposits. The virtue claimed for those measures is that isolation of the money market from the credit market makes it easier for the authorities to control the money supply.

The purpose of this paper is to represent and compare the real-bills doctrine and the quantity theory in a simple model that is compatible with the principle of finance theory that assets are valued according to the streams of returns that back them. The analysis is intended as an alternative to the one represented in Friedman and Schwartz (1963, pp. 169, 191) and Sargent and Wallace (1975, sec. 5).

We describe an economy in which, under a free-banking or real-bills regime, fluctuations in the demand for credit cause fluctuations in the price level and can even produce an indeterminate price level. The economy is such that, in the absence of government restrictions upon intermediation, private credit instruments and government-issued currency are perfect substitutes from the point of view of asset holders. Thus, from the quantity-theory point of view, the economy is one in which credit creation is simultaneously money creation when there are no government restrictions on intermediation. It is also one in which a "quantity-theory" restriction, which can be interpreted either as granting the government a monopoly of note issues or as imposing 100 percent reserves on intermediaries' small-denomination notes, stabilizes and lowers both the price level and the concept of the money supply that is relevant to the quantity theory and causes a fluctuating credit demand to show up in a fluctuating nominal interest rate.

Despite all of this, the model does not support the quantity theory's prescriptions. In terms of the Pareto criterion, we find that the real-bills doctrine is consistent with the existence of an optimal equilibrium and that the sort of restrictions suggested by the quantity theorists are not. Thus, our analysis calls for something of a rehabilitation of the real-bills doctrine.

Section I of the paper contains analyses using unbacked or fiat currency. A discussion in terms of fiat currency seems relevant to contemporary monetary institutions. However, in much of the original debate between quantity theorists and real-bills advocates, it was assumed that a commodity standard was in effect, in particular, a gold standard. For example, Adam Smith's defense of the real-bills doctrine assumed a gold standard. For this reason, in Section II we analyze the controversy in the context of a model with commodity money. There we outline a model of a small country operating under a gold standard and show that the model reproduces Smith's main conclusions.

Our analysis is designed, in part, to mimic features of the U.S. financial system in the late nineteenth century. We interpret the monetary system under the National Banking Act as one with a quantity-theory-like restriction in effect—namely, an effective government monopoly on note issue along with other rules that kept this note issue largely fixed in the face of fluctuating credit demands. Real-bills advocates decried this aspect of the system and ascribed to it the alternating periods of ease and stringency in the credit market. We build a fluctuating demand for credit into our model. Under a restriction that is meant to resemble the note-issue restriction in effect under the National Banking Act, this produces a fluctuating nominal interest rate. We interpret the real-bills advocates of the period as favoring some mechanism for weakening that restriction.

The paper is also motivated by a desire to provide a theoretical framework for understanding some empirical observations that seem anomalous from the viewpoint of classical macroeconomic theory. Typical of these are the observations described by Sargent (1980) of the behavior of the stocks of high-powered money after the ends of hyperinflations in four European countries after World War I. In each country, the stock of high-powered money increased sharply for some time, even after the price level had stabilized abruptly. To a first approximation, these observations can be explained as reflecting the operation of the Modigliani-Miller-like result on indeterminacy of portfolios reported below. As explained by Sargent (1980), those postinflation increases in high-powered money were backed 100 percent by real bills, gold, and foreign exchange and were very different from the largely unbacked increases in high-powered money that occurred during the inflations. The finance theory outlook of this paper naturally directs our attention to the backing behind government liabilities and to interpreting the central bank as an intermediary.

I.   A Fiat Currency Model

Our purposes are served by using a simple version of Samuelson's (1958) consumption loans model. One advantage of that model is that for some specifications it permits unbacked government currency to have value in a perfect-foresight equilibrium, even if private credit instruments are perfect substitutes for such currency for asset holders. Another advantage is that it permits us to easily specify a fluctuating private demand for credit.

The model we study is peopled by two-period-lived, overlapping generations of size N. At each date t, $t \geqslant 1$, there is a single nonproduced, nonstorable consumption good. Each member h of generation t (young at t, old at $t + 1$) has a symmetric Cobb-Douglas utility

function, the arguments of which are consumption of time t and time t + 1 good.

Although individuals have identical preferences, they differ as regards endowments. Each generation is composed of three groups. Letting $w_s^h(t)$ be the endowment of time t good of member h of generation s, we assume

$$[w_t^h(t), w_t^h(t + 1)] =$$

$$\begin{cases} (\alpha, 0) \text{ for } h = 1, 2, \ldots, N_1 \text{ and } t \geq 1, \\ (\beta, 0) \text{ for } h = N_1 + 1, N_1 + 2, \ldots, N_1 + N_2 \text{ and } t \geq 1, \\ (0, \gamma_1) \text{ for } h = N_1 + N_2 + 1, N_1 + N_2 + 2, \ldots, N \\ \qquad \text{and } t = 1, 3, 5, \ldots, \\ (0, \gamma_2) \text{ for } h = N_1 + N_2 + 1, N_1 + N_2 + 2, \ldots, N \\ \qquad \text{and } t = 2, 4, 6, \ldots, \end{cases} \qquad (1)$$

and $\gamma_2/2 > \gamma_1/2 \geq \beta/2 > \alpha > 0$ and $N - N_1 - N_2 = N_2$. Thus, there are N_1 "poor savers" with endowments $(\alpha, 0)$ and N_2 "rich savers" with endowments $(\beta, 0)$. The difference, $(\beta/2) - \alpha$, serves a definite purpose. Under an appropriate legal restriction on the minimum size of privately issued securities, poor savers, because of their small endowment, are unable to purchase such securities, while rich savers can. Private securities are issued by the N_2 "borrowers" with endowments $(0, \gamma_i)$, γ_1 in odd periods, γ_2 in even periods. This periodicity produces a periodic or fluctuating demand for private credit. We impose equality between the number of rich savers and the number of borrowers in order to avoid an integer constraint that could otherwise arise under the kind of legal restriction we study. Moreover, given that equality, the inequality $\gamma_i \geq \beta$ makes it easy to devise a legal restriction that separates money from credit.

Below, we will be describing how this economy evolves from $t = 1$ on in a perfect-foresight competitive equilibrium under different policy regimes. To do that, we must note that at $t = 1$, in addition to the N members of generation 1, there are N members of generation 0, the initial "old." We assume that they are endowed in the aggregate with H units of unbacked or fiat government currency and that each attempts to maximize consumption of time 1 good.

We study three policy regimes, two real-bills regimes and a quantity-theory regime. In the first real-bills regime, the government does nothing. We dub this laissez-faire. In the quantity-theory regime, the government does nothing except impose a prohibition, costlessly enforced, against issuing securities in denominations smaller than some magnitude. The third policy regime is the other version of real bills. Under it, the quantity-theory denomination restriction is in

effect, but the government or central bank operates a discount window: It stands ready to grant safe loans at a zero nominal interest rate.

Given the way we specify policy, it is sufficient for most of what we do to describe how each member of generation t chooses a lifetime consumption bundle when he or she faces a given terms of trade between time t and time $t + 1$ good. This involves choosing nonnegative $[c_t^h(t), c_t^h(t + 1)]$ to maximize $c_t^h(t)c_t^h(t + 1)$ subject to

$$c_t^h(t) + [c_t^h(t + 1)/R^h(t)] \leq w_t^h(t) + [w_t^h(t + 1)/R^h(t)], \qquad (2)$$

where $c_s^h(t)$ is consumption of time t good by member h of generation s and $R^h(t)$ is the terms of trade, unity plus the real interest rate, faced by agent h at time t. The solution to this maximization problem can be described by the following saving function:

$$w_t^h(t) - c_t^h(t) = \{w_t^h(t) - [w_t^h(t + 1)/R^h(t)]\}/2. \qquad (3)$$

In addition to this description of the behavior of the young, we need a description of the competitive behavior of the old; they supply all their government currency inelastically.

A.   Equilibrium under a Laissez-Faire (LF) Regime

Let $p(t)$ be the time t price of a unit of government currency in terms of time t good, the inverse of the time t price level. The terms of trade implied by holding government currency from t to $t + 1$ is $p(t + 1)/p(t)$ if $p(t) > 0$. Under LF, we assume that nothing prevents borrowers from issuing securities that are perfect substitutes for government currency in the portfolios of savers. (Readers are free to suppose that this is accomplished through the costless operation of intermediaries, mutual funds, or banks.)[2] It follows that, if government currency has value in an equilibrium, any private securities held in equilibrium must bear the same real return as currency. Therefore, we have the following.

DEFINITION: A monetary equilibrium under LF is a positive sequence $\{p(t)\}$, $t \geq 1$, such that for all $t \geq 1$,

$$R^h(t) = p(t + 1)/p(t) \text{ for all } h, \qquad (4)$$

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[2] We should, perhaps, emphasize that assuming costless and unfettered intermediation is not the same as assuming costless and unfettered counterfeiting. The fact that borrowers issue IOUs that end up being perfect substitutes for fiat currency in the portfolios of savers does not imply that they issue pieces of paper that are indistinguishable from fiat currency. (Suppose that a mortgage on Sargent's residence is a perfect substitute in a bank's portfolio for a mortgage on Wallace's residence. Does it follow that one cannot be distinguished from the other?)

and

$$\sum_{h=1}^{N} \{w_t^h(t) - [w_t^h(t + 1)/R^h(t)]\}/2 = p(t)H. \qquad (5)$$

Note that (5) equates aggregate saving of all the young at t as given by (3) to aggregate dissaving of the old at t.

Using (1), the above definition takes the particular form: A monetary equilibrium under LF is a positive sequence $\{p(t)\}$ that for all $t \geq 1$ satisfies

$$(N_1\alpha/2) + (N_2\beta/2) - [N_2(\gamma_t/2)p(t)/p(t + 1)] = p(t)H, \qquad (6)$$

where $\gamma_t = \gamma_1$ if t is odd and $\gamma_t = \gamma_2$ if t is even.

To solve for a monetary equilibrium, we notice that equation (6) can be expressed as

$$[(N_1\alpha + N_2\beta)/p(t)] - [N_2\gamma_t/p(t + 1)] = 2H, \quad t \geq 1.$$

This is a linear difference equation in the price level, $1/p(t)$, with time-varying coefficients. By repeating substitution, the general solution to this difference equation is found to be

$$1/p(t) = [2H(N_1\alpha + N_2\beta + N_2\gamma_1)/A] + k\theta^{(t-1)/2}, \quad t = 1, 3, 5, \ldots,$$

$$1/p(t) = [2H(N_1\alpha + N_2\beta + N_2\gamma_2)/A]$$

$$+ k[N_2\gamma_2/(N_1\alpha + N_2\beta)]\theta^{(t/2)}, \quad t = 2, 4, \ldots,$$

where $\theta = (N_1\alpha + N_2\beta)^2/(N_2^2\gamma_1\gamma_2)$, $A = (N_1\alpha + N_2\beta)^2 - N_2^2\gamma_1\gamma_2$, and k is *any* constant. Notice that $A > 0$ if and only if $\theta > 1$. Therefore, the form of the above general solution for $1/p(t)$ implies that $A > 0$ is a necessary and sufficient condition for the existence of a monetary equilibrium (i.e., a positive price sequence). Supposing that $A > 0$, any $k \geq 0$ leads to a positive equilibrium sequence of prices, so that for $A > 0$ there exists a continuum of equilibrium price paths. Since $A > 0$ implies that $\theta > 1$, all of the solutions with $k > 0$ imply a limiting value of money of zero.[3]

From this point on, we choose to focus on the stationary or periodic equilibrium that results when $k = 0$. We summarize this equilibrium in:

PROPOSITION 1: If $A \equiv (N_1\alpha + N_2\beta)^2 - N_2^2\gamma_1\gamma_2 > 0$, then the periodic sequence $\{\hat{p}_1, \hat{p}_2, \hat{p}_1, \ldots\}$ with

$$\hat{p}_i = A/2H(N_1\alpha + N_2\beta + N_2\gamma_i) \qquad (7)$$

is a monetary equilibrium under LF.

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[3] The equilibrium sequences with $k > 0$ have the property that they approach in the limit the nonmonetary equilibrium sequence described in proposition 2.

This proposition can be proved either by setting $k = 0$ in the above general solution for $1/p(t)$ or else by directly verifying that the price sequence given by (7) satisfies the equilibrium condition (6).

As promised, this is an equilibrium in which the nominal interest rate is always zero (see eq. [4]) and in which the price level fluctuates; it is high when the demand for credit as measured by γ_i is high and vice versa. Also, as quantity theorists define money, there is no basis for distinguishing among assets. No saver cares about the *composition* of his or her portfolio as between holdings of currency issued by the government and holdings of securities issued by borrowers. Thus, according to the quantity-theory view described in the introduction, the nominal money supply at time t is the nominal value of all assets, $(N_1\alpha + N_2\beta)/2p(t)$. Obviously, with $\{p(t)\}$ given by the proposition 1 equilibrium, this total fluctuates: It is high when the demand for private credit is high and vice versa.[4] One dividend for quantity theorists from defining money this way is constant "real balances"— that is, perfect proportionality between money and the price level.[5] Finally, it seems plausible that a quantity theorist observing nothing more than the government policy and the time series of the money stock, the price level, and the nominal interest rate for a proposition 1 equilibrium would urge intervention.

Before we appraise this interventionist view, we describe the non-monetary equilibrium of the model. This equilibrium always exists, even when the parameters of the model imply $A \leqslant 0$. We employ the following.

DEFINITION: A nonmonetary equilibrium under LF is an identically zero $p(t)$ sequence and a positive sequence $\{R(t)\}$ that for all $t \geqslant 1$ satisfies (5) and $R(t) = R^h(t)$ for all h.

The following proposition can be verified directly from (5).

PROPOSITION 2:   There is one and only one nonmonetary equilibrium. It is given by the periodic sequence $\{R_1^*, R_2^*, R_1^*, \ldots\}$ with

$$R_i^* = N_2\gamma_i/(N_1\alpha + N_2\beta). \tag{8}$$

We can now assert:

PROPOSITION 3:   Under LF, there exists a Pareto-optimal equilibrium.

If $A > 0$, then we claim optimality for the proposition 1 equilibrium, while if $A \leqslant 0$, then we claim optimality for the proposition 2 equilib-

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[4] Of course, it is the "inside money" component that fluctuates. By (6), $(N_1\alpha + N_2\beta)/2p(t) = H + [N_2(\gamma_t/2)/p(t + 1)]$. The second term on the right-hand side is the nominal value of private securities issued at t. Using (7), one sees that this fluctuates as asserted.

[5] However, if income is taken to be the total time t endowment, $N_1\alpha + N_2\beta + N_2\gamma_i$, then constant income velocity is not observed.

rium. Since each displays a common marginal rate of substitution between time t good and time $t + 1$ good for all members of generation t, the only matter of concern is whether the implied real rates of interest are high enough. By a simple extension of results in Kareken and Wallace (1977) or Balasko and Shell (1980), it is sufficient for optimality that the product $\Pi_1^t R(i)$ be bounded away from zero.[6] If $A > 0$, then, since we then claim optimality for the proposition 1 equilibrium, the relevant product is either unity or $\hat{p}_2/\hat{p}_1$. If $A \leq 0$, then, since we then claim optimality for the proposition 2 equilibrium, the relevant product is either $(R_1^* R_2^*)^t$ or $(R_1^* R_2^*)^{t-1} R_1^*$. Since $A \leq 0$ implies $R_1^* R_2^* \geq 1$ (see eq. [8]), we have boundedness away from zero.[7]

Proposition 3 offers grounds for questioning the quantity-theory call for intervention. Not only is there an optimal equilibrium under LF, but the proposition 1 equilibrium, which displays the features that quantity theorists have used as a basis for advocating intervention, is itself an optimal equilibrium. This establishes, by counterexample, that those features alone—price-level fluctuations matched by money stock fluctuations—do not justify intervention designed to separate credit from money. We next pursue this matter further by explicitly analyzing what we interpret as a quantity-theory-like restriction, one that separates credit from money.

B. Equilibrium under a Quantity-Theory (QT) Regime

Here we impose a legal restriction, a minimum size on privately issued securities. We state this minimum in terms of the time t goods value of private securities issued at t. Thus, if we denote by $v(t)$ the time t goods value of a security issued at t, then the restriction is $v(t) \geq v$. We assume $\alpha < v < \beta/2$.

The first inequality limits poor savers to holding government currency. No matter how high is the yield on private securities, no poor saver can afford to buy one. It follows that if $p(t)$ and $p(t + 1)$ are positive, then poor savers face constraint (2) with $R^h(t) = p(t + 1)/p(t)$. Their saving, then, satisfies (3) and is entirely in the form of holdings of government currency. If $p(t) = p(t + 1) = 0$, then poor savers consume their endowment.

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[6] Kareken and Wallace (1977) formally consider only setups with endowment patterns that are constant over time, whereas Balasko and Shell (1980) formally consider only setups with identical agents in a given generation. The result we need is given by Millan (1981) as lemma 3 in chap. 3.

[7] It can easily be shown that the nonmonetary equilibrium is not optimal if $A > 0$, because $A > 0$ implies $R_1^* R_2^* < 1$. This is a variant of the intertemporal inefficiency that arises in growth models when equilibrium interest rates are "too low."
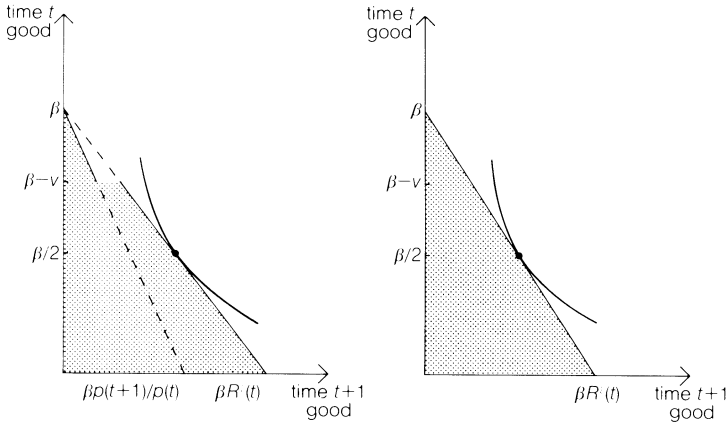
FIG. 1.—Rich lender budget sets under the QT regime

The inequality, $v < \beta/2$, insures an internal solution for rich savers. Letting $R^L(t)$ be the gross real return on private securities, we have two situations of interest: $R^L(t) > p(t + 1)/p(t)$ and $R^L(t) = p(t + 1)/p(t)$.[8] The implied respective budget sets facing a rich saver are depicted in figure 1.

In the first situation, desired saving is $\beta/2$ and is composed entirely of a demand for private securities. This is also the case if $p(t) = p(t + 1) = 0$. In the second situation, desired saving is also $\beta/2$, but there is indifference about its composition.

As for borrowers, we regard each as choosing the value of a single security subject to $v(t) \geq v$. Figure 2 depicts the implied budget set.

It follows that the security supplied by a borrower is given by the following function of $R^L(t)$,

$$g_i[R^L(t)] = \begin{cases} (\gamma_i/2)/R^L(t), & \text{if } R^L(t) \leq (\gamma_i/2)/v, \\ v, & \text{if } (\gamma_i/2)/v \leq R^L(t) \leq \gamma_i/v, \\ 0, & \text{if } R^L(t) \geq \gamma_i/v, \end{cases} \qquad (9)$$

where $i = 1$ if t is odd and $i = 2$ if t is even.[9]

With these descriptions of asset demands and supplies, we define a monetary equilibrium under the QT regime as follows.

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[8] Since rich savers are always free to hold government currency, $R^L(t) < p(t + 1)/p(t)$ cannot be an equilibrium.

[9] As described, g_i is a function except at $R^L(t) = \gamma_i/v$. There it is a correspondence with values v and 0, both of which imply zero utility.
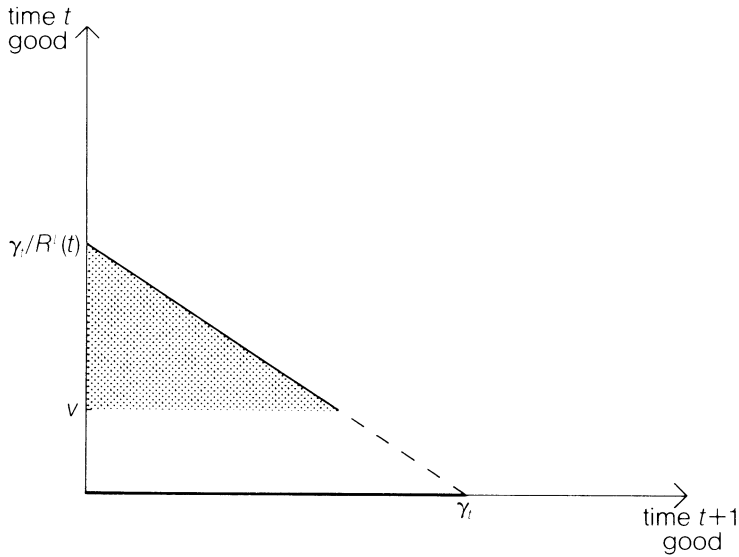
FIG. 2.—Borrower budget set under the QT regime

DEFINITION: A monetary equilibrium under the QT regime is a positive sequence $\{p(t), R^L(t)\}$ that for all $t \geq 1$ satisfies

$$(N_1\alpha/2) + (N_2\beta/2) - N_2 g_i[R^L(t)] = p(t)H, \qquad (10)$$

$$R^L(t) \geq p(t + 1)/p(t) \text{ and } \beta/2 \geq g_i[R^L(t)], \qquad (11)$$

and with at least one of the inequalities of (11) at equality.
This definition implies:
PROPOSITION 4: Under the QT regime, there exists a unique monetary equilibrium. It is given by the periodic sequence $\{(\tilde{p}, \tilde{R}_1^L), (\tilde{p}, \tilde{R}_2^L), (\tilde{p}, \tilde{R}_1^L), \ldots\}$ with

$$\tilde{p} = N_1\alpha/2H \text{ and } \tilde{R}_i^L = \gamma_i/\beta. \qquad (12)$$

The private securities market under the proposed equilibrium is depicted in figure 3, which displays the supply of private securities by a borrower (see eq. [9]) and the demand for private securities of a rich lender. Since there are N_2 identical borrowers and N_2 identical rich lenders, the intersection of these individual supply and demand schedules determines the equilibrium interest rate and size of each private security.

There are two parts of the proof of proposition 4. Existence is proved by showing that the proposed solution satisfies (10) and (11). This follows from noting that (9) implies $g_i(\gamma_i/\beta) = \beta/2$. This and $\gamma_i \geq$
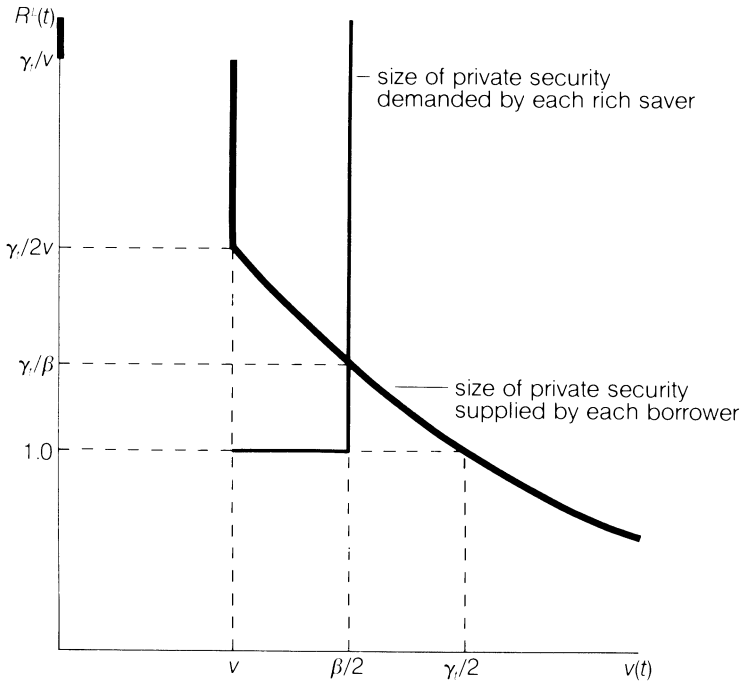
FIG. 3.—Equilibrium in the credit market under the QT regime

β imply that the proposed solution satisfies (10) and (11) with at least the second inequality of (11) at equality.

We prove uniqueness in the Appendix, in part because not much should be made of it. Uniqueness is not robust. There would be a continuum of monetary equilibria if the endowment of poor savers was assumed to be $(\alpha, \lambda\alpha)$ for some $0 < \lambda < 1$ instead of $(\alpha, 0)$.

Next, note that the unique nonmonetary equilibrium under the QT regime is $p(t) = 0$ for all t and $R^L(t)$ as given by the proposition 4 equilibrium. In such an equilibrium, poor savers consume their endowment, while rich savers and borrowers have the consumption bundle they have in the proposition 4 equilibrium.

This brings us to:

PROPOSITION 5: There does not exist a Pareto-optimal equilibrium under the QT regime.

The proof is simple. A necessary condition for optimality is identical marginal rates of substitution between time t and time $t + 1$ good for all members of generation t. At least at even dates, poor savers and

rich savers have different marginal rates of substitution in any equilibrium under the QT regime.[10]

Nonoptimality obtains under the QT regime despite the fact that the quantity-theory goals are achieved in the proposition 4 equilibrium. First, because government currency and private securities are not substitutes in that equilibrium, a quantity theorist would probably identify as money only government currency, the stock of which is fixed at H. Second, and most important, the price level is constant at $1/\tilde{p}$.

We now compare the proposition 1 and proposition 4 equilibria and establish their noncomparability. By (7) and (12), $\tilde{p} > \hat{p}_i$ for $i = 1$, 2. Thus, government currency is always more valuable under the QT monetary equilibrium. This implies that at any given time, the old would prefer to be living under that equilibrium. As for poor savers, those born in even periods prefer the proposition 1 equilibrium while those born in odd periods prefer the proposition 4 equilibrium. To judge the well-being of rich savers and of borrowers, we need to compare real returns on private securities. Using (7) and (12) we see that the return in the proposition 4 equilibrium, γ_i/β, is higher than that in the corresponding period of the proposition 1 equilibrium, where it is $\hat{p}_2/\hat{p}_1$ in odd periods and $\hat{p}_1/\hat{p}_2$ in even periods. Thus, borrowers prefer the proposition 1 equilibrium and rich savers the proposition 4 equilibrium. (In this example, it is, indeed, the monied interests that benefit from an inelastic currency.)

Next, we wish to comment on determinacy of equilibrium under the different regimes.

One possible interpretation identifies determinacy with existence and indeterminacy with nonexistence of a monetary equilibrium. As noted above, monetary equilibria do not exist under the LF regime if $A \leq 0$. In contrast, as proposition 4 shows, a monetary equilibrium always exists under the QT regime. However, existence of monetary equilibrium per se seems a weak basis for choice of a policy regime, especially given propositions 3 and 5.

Another interpretation identifies determinacy with uniqueness of equilibrium and indeterminacy with nonuniqueness. Our model does not suggest any robust differences between the two regimes with regard to multiplicity of equilibria.

We conclude this section with the observation that the equilibria that we have calculated can be reinterpreted as referring to economies in which there are banks. To this point, our discussion has assigned no role to banks or intermediaries. In effect, our model assumes that

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[10] This result *is* robust. It does not depend on poor savers having a boundary endowment.

there is a costless technology for issuing arbitrarily small debt instruments and that this technology is available to private agents and government alike. This means that, strictly speaking, there is no need for intermediaries since large borrowers can costlessly tailor their issues of debt for numerous small lenders. However, one is free to reinterpret the equilibria that we have calculated as describing a situation in which zero-cost, competitive intermediaries are necessary in order to bring together borrowers and small lenders. On the one hand, the LF regime can be interpreted as one in which there is free banking with intermediaries subject to no reserve requirements. On the other hand, the QT regime can be interpreted as one in which intermediaries are permitted to operate but are required to hold 100 percent reserves of government currency as backing for their liabilities.

C. Equilibrium under a Real-Bills Discount-Window (DW) Regime

The real-bills doctrine often took the form of a proposed central bank policy. As we interpret the doctrine, it took this form when legal restrictions on private intermediation were taken as a given. In this section, we demonstrate that a central bank or government policy like that proposed by real-bills advocates can offset the consequences of restrictions on private intermediation.

In order to make that point, we suppose that a version of the legal restriction analyzed in the last section is in effect but that the government stands ready at every date t to grant safe one-period loans in the form of (newly printed) government currency at a *zero* nominal rate of interest; if someone borrows h units of government currency at t, he or she is required to pay back h units at $t + 1$. We assume that the denomination restriction analyzed in Section IB is in effect on all loans except those granted by the government. We call this policy regime the DW regime.

To show that this discount window policy can offset the legal restriction, we prove that the set of monetary equilibria under the DW regime coincides with the set of monetary equilibria under the LF regime.

First, if $p(t) > 0$ in an equilibrium under the DW regime, then $p(t + 1)/p(t)$ is the single terms of trade facing all members of generation t in that equilibrium. Since asset holders are free to hold government currency, the return on private securities cannot be less than $p(t + 1)/p(t)$. And it cannot exceed the real return on money because anyone can borrow from the government at $p(t + 1)/p(t)$. Thus, in any monetary equilibrium under the DW regime, all members of generation t face constraint (2) with $R^h(t)$ given by (4).

Now we must show that (5) holds in any monetary equilibrium under the DW regime. By the argument of the last paragraph, the left-hand side of (5) is the excess supply of time t good on the part of all members of generation t in a monetary equilibrium under the DW regime. Thus it remains to show that the right-hand side of (5) is the excess demand for time t good on the part of members of generation $t - 1$. This follows from the fact that loans from the government are at a zero nominal interest rate. While members of generation $t - 1$ may have contracted for loans from the government at $t - 1$ and, therefore, may have held from $t - 1$ to t an amount of government currency in excess of what the old at $t - 1$ offered, all such loans must be repaid at t. It follows—formally, by induction—that if H is the stock of government currency that the old at $t = 1$ offer for sale, then H is the stock offered by the old at every date.

This shows that any monetary equilibrium under the DW regime is a monetary equilibrium under the LF regime. To prove the converse, one takes a $p(t)$ sequence that satisfies (4) and (5) and shows that there is a sequence of loans from the government and private transactions consistent with the legal restriction that support all the real trades implied by (4) and (5). One sequence that works has borrowers getting all their loans from the government. Then savers, both rich and poor, hold only government currency. But other sequences in which borrowers get some loans directly from rich savers and some from the government may also work. The implied indeterminacy of gross portfolios is an example of the operation of the Modigliani-Miller theorem.[11]

It follows from equivalence between the set of monetary equilibria under the LF and DW regimes that, if $A \leq 0$, then a monetary equilibrium does not exist under the DW regime. As noted above, some may wish to interpret this nonexistence as indeterminacy of equilibrium under a real-bills regime. In any case, if $A \leq 0$, then the

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[11] See Wallace (1981) for a detailed discussion of the applicability of the theorem to the government's portfolio. This result on indeterminacy of gross portfolios provides the analytics that can be used to explain the observations of the movements of base money and inflation that occurred after the hyperinflations, and which we cited in the introduction (see Sargent 1980). The basic idea is that once the government has purchased sufficient private debt to offset completely the government restriction on minimal denomination, further government purchases of private debt with government currency are superfluous. Such open-market operations leave the equilibrium price path and all real variables unaffected. Of course, open-market operations could be used to peg the nominal rate of interest on private securities at values between the positive values given by (12) and the zero nominal rates attained under our version of the real-bills regime. Government open-market purchases would have to be in amounts between zero (which implies nominal rates given by [12]) and the minimal amount needed to support our real-bills regime. In analyzing such schemes, one must remember to dispose of the government's interest earnings.

only equilibrium under the DW regime is a nonmonetary equilibrium. And, as we have described that regime, the nonmonetary equilibrium under the DW regime is the nonmonetary equilibrium of the QT regime, not the nonmonetary equilibrium of the LF regime. Obviously, if $p(t) = 0$ for all t, a willingness to lend government currency means nothing. Thus, if $p(t) = 0$ for all t, then the DW regime is identical to the QT regime.

This completes what we have to say about interpreting the real-bills doctrine and the quantity-theory proposal for separating money and credit within a model of fiat currency.[12] In one sense, it is misleading to discuss these doctrines in such a model because most discussions took place against the background of a commodity-money system. Yet, as we will see, much of what we have demonstrated also applies in versions of commodity-money models.

II.   A Smithian, Small-Country, Commodity-Money Model

In this section, we show how elements of the model of Section I can be reinterpreted to provide a model of a small country that is part of a world economy operating under a commodity-money standard. This exercise is of interest because it permits a formal representation of arguments that Adam Smith advanced in propounding a version of the real-bills doctrine. In *The Wealth of Nations* Smith wrote:

> It is not by augmenting the capital of the country, but by rendering a greater part of that capital active and productive than would otherwise be so, that the most judicious operations of banking can increase the industry of the country. That part of his capital which a dealer is obliged to keep by him unemployed, and in ready money for answering occasional demands, is so much dead stock, which, so long as it remains in this situation, produces nothing either to him or to his country. The judicious operations of banking enable him to convert this dead stock into active and productive stock; into materials to work upon, into tools to work with, and into provisions and subsistence to work for; into stock

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[12] The resemblance of the fluctuations in our model to seasonal fluctuations brings to mind the following considerations. Many writers, including some in the quantity-theory tradition, have advocated that the central bank accommodate seasonal fluctuations in the demand for credit by rediscounting real bills, but that it not accommodate nonseasonal fluctuations. Simply by selecting an appropriate γ_t sequence, we can readily extend the model of this paper to generate nonseasonal and irregular fluctuations in prices and interest rates. Therefore, our model provides no justification for adopting a different rediscounting policy for seasonal as opposed to nonseasonal fluctuations in credit demand (see also Dewald 1972).

which produces something both to himself and to his coun-
try. The gold and silver money which circulates in any
country, and by means of which the produce of its land and
labour is annually circulated and distributed to the proper
consumers, is, in the same manner as the ready money of the
dealer, all dead stock. It is a very valuable part of the capital
of the country, which produces nothing to the country. The
judicious operations of banking, by substituting paper in
the room of a great part of this gold and silver, enables the
country to convert a great part of this dead stock into active
and productive stock; into stock which produces something
to the country. [Pp. 304–5]

Smith pointed out that the substitution of titles to productive capital
for gold in lenders' portfolios would drive gold out of the country:

Where paper money, it is to be observed, is pretty much
confined to the circulation between dealers and dealers, as at
London, there is always plenty of gold and silver. Where it
extends itself to a considerable part of the circulation be-
tween dealers and consumers, as in Scotland, and still more
in North America, it banishes gold and silver almost entirely
from the country; almost all the ordinary transactions of its
interior commerce being thus carried on by paper. The
suppression of ten and five shilling bank notes, somewhat
relieved the scarcity of gold and silver in Scotland; and the
suppression of twenty shilling notes, would probably relieve
it still more. Those metals are said to have become more
abundant in America, since the suppression of some of their
paper currencies. They are said, likewise, to have been more
abundant before the institution of those currencies. [P. 307]

Thus, Smith argued that, through substitution of paper liabilities
for gold in lenders' portfolios, the spread of banking would provide
more favorable terms for borrowers. That is how it makes possible
"rendering a greater part of [a country's] capital active and productive
than would otherwise be so" (p. 304). However, for a small country
operating under an international gold standard, the spread of bank-
ing would not raise the domestic price level:

A paper money consisting in bank notes, issued by people
of undoubted credit, payable upon demand without any
condition, and in fact always readily paid as soon as pre-
sented, is, in every respect, equal in value to gold and silver
money; since gold and silver money can at any time be had
for it. Whatever is either bought or sold for such paper, must

necessarily be bought or sold as cheap as it could have been
for gold and silver.

   The increase of paper money, it has been said, by aug-
menting the quantity, and consequently diminishing the
value of the whole currency, necessarily augments the money
price of commodities. But as the quantity of gold and silver,
which is taken from the currency is always equal to the
quantity of paper which is added to it, paper money does not
necessarily increase the quantity of the whole currency.
From the beginning of the last century to the present time,
provisions never were cheaper in Scotland than in 1759,
though, from the circulation of ten and five shilling bank
notes, there was then more paper money in the country than
at present. The proportion between the price of provisions
in Scotland and that in England, is the same now as before
the great multiplication of banking companies in Scotland.
Corn is, upon most occasions, fully as cheap in England as in
France; though there is a great deal of paper money in
England, and scarce any in France. [Pp. 308–9]

We interpret Smith as assuming that the price of goods in terms of
gold is exogenous to the country under study. We also interpret him
as assuming that the market for loans is local, so that government
rules about the kind of banking or intermediation allowed within a
country can affect the terms on which residents can obtain loans. We
regard Smith as arguing that interest rates on loans would be lower
and less gold would be held if banks were left largely unregulated.

   We now show that elements of the model of Section I can be
reinterpreted as a model of commodity money that implies most of
Smith's conclusions.

   As part of our reinterpretation of the Section I model, we identify
the government currency in Section I with gold, so that $p(t)$ is the time
t price of gold in terms of the single consumption good. The small-
country assumption we make is that the $p(t)$ sequence is exogenous to
the country under consideration. We justify zero consumption of gold
and its nonappearance as an argument of utility functions by assum-
ing that $p(t)$ for all t is so high that it produces a corner solution: zero
consumption of gold.[13] We also assume that home residents—except,
perhaps, for the old at $t = 1$—have no endowment of gold. This is to
say that home residents never "find" or "discover" gold. These as-

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[13] E.g., if the common underlying utility function is $[c(1) + \delta g(1)][c(2) + \delta g(2)]$,
where $c(i)$ is consumption of the nonstorable good at age i, $g(i)$ is consumption of gold
at age i, and $\delta > 0$ is a parameter, then $p(t) > \delta$ for all t implies a corner solution with
zero gold consumption for all t.

sumptions permit us to maintain the endowment and preferences assumptions of the Section I model. As regards regimes, we treat the LF regime as an approximation to the regime Smith advocated and treat the QT regime as one he would have viewed as involving excessive restriction of banking. (So far as we can tell, central banking was not considered in Smith's discussion.)

Consider, first, the situation under LF. As was true in the Section I model, under LF a single terms of trade given by $p(t + 1)/p(t)$ prevails for all members of generation t. The amount of gold held from t to $t + 1$ is determined by the behavior of generation t; in units of the consumption good, it is given by the left-hand side of (5). Net gold imports at t are, of course, given by the first difference of this quantity.

Now consider the situation under the QT regime. We depict the determination of equilibrium in figure 4, which is a version of figure 3 modified by setting the horizontal portion of a rich saver's demand for private securities at the level $p(t + 1)/p(t)$ given by the world market rather than at unity. If $p(t + 1)/p(t) < \gamma_t/\beta$, we obtain the Smithian result that less gold is held from t to $t + 1$ and the interest
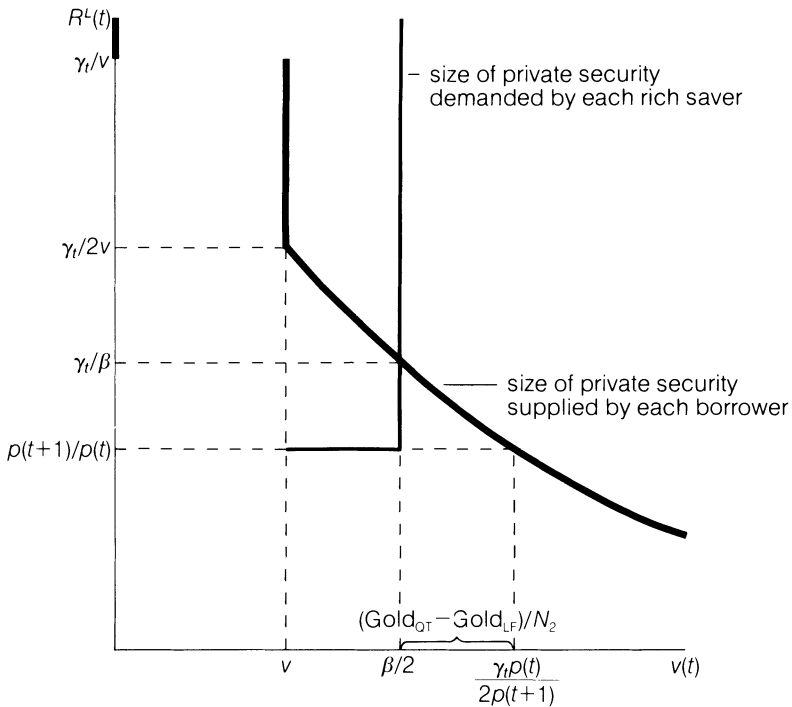

FIG. 4.—Equilibrium in the credit market under the QT regime in a Smithian model

rate is lower under the LF regime than under the QT regime. This is because under the QT regime the interest rate on private securities is γ_t/β, whereas under the LF regime it is $p(t + 1)/p(t) < \gamma_t/\beta$. Under the LF regime each borrower issues private securities worth $\gamma_t p(t)/2p(t + 1)$, whereas under the QT regime each borrower issues only $\beta/2$ private securities. Therefore, under the LF regime the country holds $N_2\{[\gamma_t p(t)/2p(t + 1)] - (\beta/2)\}$ fewer units of gold than under the QT regime. The difference represents the total additional securities issued by borrowers, all of which can be thought of as being held by poor lenders.

Thus, on their own small-country, local-credit-market terms, Smith's conclusions seem plausible. They are certainly in the right direction. But, like all partial equilibrium analyses, this one leaves many questions unanswered. What if the same banking developments are going on in every country? What happens if every country either encourages or discourages such intermediation? Such policies can affect $p(t)$ and, hence, can have consequences for how much gold gets consumed and, perhaps, produced. A small-country analysis can at best provide only hints about such consequences and their implications for efficiency.

One way of addressing the above questions is to turn the model of this section into a general equilibrium model. This can be done by (1) taking explicit account of how utility depends on consumption of gold, (2) being explicit about the time path of endowments of gold ("new finds"), (3) considering in a more explicit way the storage technology for gold—does gold physically appreciate with storage, depreciate, or remain unaffected as was assumed above—and (4) treating the world as a single economy. If all of that is done, then the model that results is a growth model with two consumption goods and one capital good (gold). Without going into the details, we can say quite a bit about such a model based on what is known about related models.

If gold does not appreciate physically, then it can easily happen that any LF equilibrium for such a model is nonoptimal. Any such equilibrium can display the kind of capital-theoretic nonoptimality that the Section I LF nonmonetary equilibrium, the proposition 2 equilibrium, displays if $A > 0$. In fact, it can easily happen that any LF equilibrium is inefficient in the usual growth theory sense.[14]

In other respects, though, any LF equilibrium for such a model resembles those of the model described above. Differences among members of a given generation in the time profile of endowments

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[14] When the LF equilibrium of this model is inefficient, the model satisfies the widely held notion that the existence of fiat currency can help by freeing resources, in this case gold, from their role as money for other uses, namely, consumption.

generate within-generation borrowing and lending. And, if gold is held as an asset—which happens under conditions analogous to $A > 0$—then the yield on loans measured, say, in terms of the nonstorable consumption good must be the same as the yield on gold measured in the same way. Thus, in such an equilibrium, the nominal interest rate is zero and savers are indifferent about the composition of their portfolios as between gold and private securities, the IOUs of borrowers. Once again, then, a quantity theorist would identify as money the total of assets held, gold plus the IOUs of borrowers. As for fluctuations, although the construction of examples is more difficult than for the Section I model, it is obvious that a fluctuating demand for credit is produced in the same way and that it gets reflected in a fluctuating price level, that is, a fluctuating price of the nonstorable consumption good in terms of gold. In other words, nothing precludes the construction of examples in which an LF equilibrium displays those features that lead a quantity theorist to call for restrictions designed to separate money from credit.

This brings us to our main concern. Are such restrictions more easily justified in this sort of model than they are in the Section I model? This would be the case if such restrictions tended to correct the possible nonoptimality of any LF equilibrium. It seems clear, however, that they would not work that way. Restrictions that tend to separate money from credit—for example, a restriction like that of the QT regime—work in the wrong direction. Not only can such restrictions give rise to within-generation discrepancies among intertemporal marginal rates of substitution, but, as we have seen, they can increase the demand for gold as an asset. In the kind of model under discussion, such increases can come at the expense of gold consumption. If they do, then restrictions produce inefficiency when LF is efficient and exacerbate inefficiency when LF is inefficient. In summary, then, quantity-theory-like restrictions that inhibit substitution between gold and private securities do not look any better in a general equilibrium version of the model of this section than they do in the fiat currency model of Section I.

III. Concluding Remarks

This paper has produced an example of an economy in which many of the quantity theorists' assertions about the effects of instituting different monetary regimes are valid. In particular, under a real-bills regime both the price level and the money supply fluctuate more than they do under restrictions that isolate the money market from the market for private credit. Even so, for the sample economy that we have analyzed, it is not possible to argue that the real-bills regime is

worse than the regime with restrictions. Some agents are better off under one regime, whereas others are better off under the other. Indeed, the real-bills regime is consistent with the existence of a Pareto-optimal equilibrium, whereas the regime with restrictions is not.

Some of what we have said is not new. Uneasiness about restricting one particular class of financial intermediaries motivates Becker's (1956) "A Proposal for Free Banking."[15] Also, the essential similarity between banks and other intermediaries is the main message of Tobin (1963) and Fama (1980). We pursue matters further by questioning the basing of policy conclusions on assertions about desirable price-level paths. Any defense of versions of the real-bills doctrine vis-à-vis the quantity-theory view must do this: If one accepts the quantity-theory view about desirable price-level paths, then, depending on the environment, all sorts of restrictions are called for.

In order to confront claims about desirable price-level paths, one must use more explicit models than either adherents of the real-bills doctrine or their critics have produced. Of course, having been explicit in laying out a model that represents something of a counterexample to the quantity-theory view, we are subject to the criticism that we do not have the right model. Some may argue that our model is rigged against the quantity-theory view because it abstracts from uncertainty and from business-cycle phenomena. We doubt that merely complicating the model to deal with additional phenomena or generalizing it would change its basic message. Thus, for example, the mere presence of uncertainty does not destroy the underlying logic of the free-banking position.[16]

We are ready to concede, however, that our conclusions may not survive radical departures from the tenets that underlie the asset demands of our model.[17] In particular, our model is true to Keynes's dictum (1924, p. 83) that "money as such has no utility except what is derived from its exchange value, that is to say, from the utility of the things it can buy." Many models are not. Some violate it in a direct way by putting "real balances" into utility functions (e.g., Sidrauski 1967; Brock 1974) or into production functions (e.g., Levhari and Patinkin 1968). Other theories violate it in a more subtle way by imposing transaction technologies—special cases of which are various Clower-

[15] For a brief description of Becker's proposal, see Friedman (1959, p. 108).

[16] Uncertainty does, though, change the form of a central bank discount window version of the doctrine. With uncertainty, no simple scheme that amounts to granting all loans of a certain kind at some fixed nominal interest rate necessarily duplicates unfettered private intermediation.

[17] For a discussion of the tenets that underlie our model, see Bryant and Wallace (1980).

type, cash-in-advance constraints—not implied by the physical characteristics (depreciation rates) of the objects traded or any physical characteristics of the environment (e.g., Heller and Starr 1976; Bryant and Wallace 1979; Lucas 1980; Martins 1980). We suspect that the implications of those models for the questions addressed in this paper depend, in part, on how money or cash is defined. It is precisely because we have rigidly adhered to Keynes's dictum that our model provides new insights and a different perspective on the issues at stake between real-bills advocates and their critics than do standard macroeconomic models (e.g., Sargent and Wallace 1975).

Finally, we emphasize that our goal has been exactly that—to provide new insights into the issues at stake between real-bills advocates and their critics. We are not making a plea for a laissez-faire monetary system. First, as happens in the examples presented above, equilibria with and without restrictions are almost certainly noncomparable. Second, if the only taxation options are distorting taxes, then it may be beneficial to impose financial sector restrictions that enhance the demand for government liabilities (see Bryant and Wallace [1980] for examples). In this regard, note that the intermediation restrictions in force in the United States in the late nineteenth century were initiated by the North during the Civil War, at least in part as a way to enhance the demand for its liabilities at a time when it was experiencing a very large deficit. By the last quarter of the nineteenth century, however, the problem was not how to finance a deficit but, more often, how to dispose of a surplus (see Timberlake 1978, chap. 10). Since the financial regulations in force were of no help in dealing with a surplus, it is not surprising that the real-bills doctrine was influential at that time, influential enough so that features of it got reflected in the legislation that created the Federal Reserve System. Indeed, it is tempting to regard the creation of the Federal Reserve System as an attempt to implement a commodity-money version of our real-bills discount-window regime.

Appendix

Proof of Uniqueness of the Proposition 4 Equilibrium

Suppose to the contrary that $\{\bar{p}(t), \bar{R}(t)\}$ denotes another monetary equilibrium. If $\bar{p}(t) = \bar{p}$ for all t, then $\bar{R}^L(t) = \bar{R}^L(t)$ for all t. Therefore, for some t, say T, $\bar{p}(T) = \bar{p}$. Since $\bar{p}$ is the minimum price of currency in a monetary equilibrium under the QT regime (this is the nonrobust fact), it follows that $p(T) > \bar{p}$, which, in turn, implies that rich savers hold some currency at $t = T$.

Two implications follow: $\bar{p}(T + 1)/\bar{p}(T) = R^L(T)$ and $R^L(T)$ is high enough to make borrowers willing to borrow less than $\beta/2$. The second implies $\bar{R}^L(t) > (\gamma_1/\beta) \geq 1$ or $\bar{p}(T + 1) > \bar{p}(T)$. This, in turn, implies $\bar{p}(T + 1) > \bar{p}$ so that, by induction, it follows that $\{p(t)\}$ is strictly increasing for $t \geq T$.

But $\{\bar{p}(t)\}$ is bounded, say, by $(N_1\alpha + N_2\beta)/2$. Therefore $\bar{p}(t)$ approaches a limit and $\bar{p}(t+1)/\bar{p}(t)$ approaches unity. But, then, there exists some J such that for all $j \geqslant J$, $\bar{p}(T+j+1)/\bar{p}(T+j) < \gamma_2/\beta$. This condition and $\bar{p}(T+j) > \bar{p}$ contradict equilibrium in the market for private securities.

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