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The Efficiency of Monetary Exchange in Search Equilibrium

Search theoretic models of money emphasize monetary exchange as the outcome of economic environments characterized by bilateral trading frictions. This paper analyzes the efficiency of monetary exchange in a search model of fiat money where individuals invest costly effort in the exchange process. Because the optimal effort each individual trader invests in exchange is based upon the private rather than social gains from trade, decentralized monetary equilibria are shown to be inefficient relative to the social optimum. However, for an appropriate division of the gains from trade given to buyers and sellers, it is possible to attain social efficiency. The nature of these search externalities for monetary exchange and their implications for policy are evaluated and discussed.

THE INTRODUCTION OF MONEY serves to alleviate the informational barriers associated with the lack of a centralized commodity market and riskless Arrow-Debreu futures contracts. However, the need for reopening markets over time and the lack of trade coordination leads to the inefficiency of monetary equilibria. In cash-in-advance models (Grandmont and Younes 1972 and Lucas 1980, 1982), the restriction that current sales cannot be used to finance current consumption purchases binds the allocation of resources away from the frictionless ideal. Similarly, shopping-time models (McCallum and Goodfriend 1988 and Marshall 1990) alter household resource constraints so that transactions costs rule out otherwise Pareto improving barter exchanges. However, by starting with a competitive Arrow-Debreu framework and approximating trade frictions with the a priori use of money, these models are unable to address important questions regarding welfare and the efficiency of monetary exchange. Given a decentralized trading environment, what determines how much effort individuals in a monetary economy invest in exchange activity? How do trading frictions impose inefficiencies in the

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exchange process and what role does the quantity of money play in resolving such inefficiencies?

This paper will explore the answers to these questions in the context of an economic environment where fiat money has an explicit role in facilitating transactions. In particular, decentralized exchange is characterized in a search-theoretic context where spatially separated agents must engage in search activity to locate potential trading partners. Search theory provides a natural characterization of an exchange process that is both time consuming and costly. By doing so, it directly captures the fundamental friction that gives rise to money as a medium of exchange. Given that individuals have heterogenous preferences over a large number of types of goods, money serves to minimize the search costs associated with exchange and overcome the classic "double coincidence of wants" problem with barter. Formal demonstrations of this search aspect of monetary exchange are given by Jones (1976), Iwai (1988), Kiyotaki and Wright (1989), and Oh (1989). Most recently, Kiyotaki and Wright (1991, 1993) demonstrate the existence and robustness of monetary equilibria by extending the dynamic "search equilibrium" framework of Diamond (1982, 1984). In these models, individuals can store either goods or money and exchanges are one-for-one swaps of an agent's inventory. Trade is coordinated via a stochastic transactions technology by which the meeting rate between buyers and sellers depends on the number of buyers and sellers in the market. Kiyotaki and Wright show that valued fiat money can result as a Nash equilibrium in these models given a belief that all other agents in the economy find it acceptable in trade.

This paper explores the welfare properties of search models of money by generalizing the transactions technology and considering the issue of efficiency in the monetary exchange process. Instead of fixed arrival rates of potential trading partners, we allow individual traders to choose how much effort to invest in search. Endogenizing search intensity leads to the more realistic assumption that agents can influence their expected rate of transactions and consumption. Since the search effort of each individual trader will influence the aggregate rate of transactions, a natural trading externality arises. First, more search by an individual agent increases the meeting probability of agents on the other side of the market. Secondly, by increasing the competition for contacting those on the other side of the market, higher search effort also reduces the meeting probability of other agents on the same side of the market. Thus, it is shown that monetary equilibria will generally be inefficient relative to the social optimum. Agents take only their private rather than the social gains from exchange into account when choosing optimal search efforts. These external effects arising from search activity in general equilibrium are similar to those uncovered in the labor market literature and other dynamic economic contexts by Mortensen (1982a, b), Pissarides (1984), and Hosios (1990).

Section 1 of the paper presents the basic search model where both commodity and money traders search and barter co-exists with monetary exchange. The optimal search decisions of individual buyers and sellers are derived and compared with the socially efficient outcome. It is shown that the nature of these inefficiencies depend critically on how the gains from trade are divided between commodity and money traders. Since this share of the trading surplus also depends on individual choices of search effort in the decentralized equilibrium, there is no internal mechanism that causes these market choices to be socially efficient. Section 1 then provides a couple of examples that highlight some implications of this efficiency result. The first offers some interesting parallels with previous studies which emphasize other sources of external effects in search economies. The second example analyzes how changes in the quantity of money in the search economy can affect the efficiency of decentralized exchange. Intuitively, the real quantity of money is able to influence the incentive to search by directly affecting the gains from trade for buyers and sellers. This leads to the interesting question of whether policies that adjust the quantity of money can lead to more efficient trading and improve welfare. Section 2 concludes with a summary.

1. THE MODEL

A. The Transactions Technology

The economy is populated by a continuum of infinite-lived agents with a population mass of unity. The storage technology of each agent is restricted to holding either one unit of a real commodity or one unit of real money balances at any given time. Thus, exchanges are bilateral and one-for-one swaps of goods for goods or goods for fiat money. At the beginning of a search period for a particular trader, he costlessly produces (or is endowed with) one unit of a commodity. Without loss of generality, we can motivate trade by assuming agents cannot consume their own production type.[1] These agents immediately become "commodity traders" and enter into the market where they randomly contact other traders for the purpose of bilateral exchange.

Search is costly in this set-up because (1) the time spent in the search process is discounted and (2) agents must choose costly search effort (or search intensity) which determines the rate by which they make contact with other traders in the market. A commodity trader's search leads to either an exchange of his good for a desired consumption good (barter) or fiat money (monetary exchange). If a monetary exchange occurs, that agent becomes a "money trader" and must then locate commodity traders who are holding a desirable consumption good and willing to exchange it for money. The instantaneous utility associated with consumption is given by U. Once consumption occurs, the agent immediately produces another good and the search process starts over again. Let the probability that any randomly selected good will be acceptable for consumption by an individual trader be $x < 1$. Kiyotaki and Wright (1993) demonstrate that there exists a symmetric Nash equilibrium in this framework where money is acceptable in all trades. Intuitively, the acceptance of fiat money minimizes the search costs of exchange given that everyone else also

1. Relaxation of this restriction complicates the notation without altering any of the features of the model. See Kiyotaki and Wright (1993) or Burdett et al. (1995).

accepts it in exchange. For the purposes of our analysis, we will take this result of a pure monetary equilibrium as given.[2]

Let N_1 denote the fraction of agents in the economy which initially produce and hold a unit of a real commodity and N_m denote the fraction initially endowed with a unit of real money. Thus, $N_1 + N_m = 1$ and the aggregate real money supply is given by $M = N_m = 1 - N_1$. Because all exchanges are one-for-one swaps of goods and money, N_1 and N_m are by construction the steady state fractions of commodity and money traders. These fractions also denote the probability that any randomly selected trader is a commodity or money trader, respectively. Also, let the search intensity of a commodity and money trader be given by β_1 and β_m. The cost of search effort for commodity and money traders is captured by a search intensity cost function $s(\beta_i)$, $i = 1, m$, which is continuously differentiable, increasing, and convex ($s' > 0$, $s'' > 0$ and $s(0) = 0$).

From the viewpoint of an individual trader, denote β_{-1} and β_{-m} as the search effort of other commodity and money traders, respectively. The rate by which an individual commodity trader contacts other commodity traders is strictly increasing in his own search effort as well as the effort of other commodity traders. However, search by money traders increases competition for contacting other commodity traders and decreases this meeting rate. This meeting rate is also increasing in the ratio of commodity to money traders. We can express this meeting rate as $g(\beta_1; \beta_{-1}, \beta_m)G(N_1/N_m)$, where $g_1, g_2 > 0$, $g_3 < 0$, and $G' > 0$. Similarly, meetings between commodity and money traders depend (i) positively on their own search effort and the effort of agents on the other side of the market, (ii) negatively on the effort of those on the same side of the market, and (iii) on the ratio of commodity to money traders. Thus, a commodity trader contacts money traders at rate $p(\beta_1; \beta_{-1}, \beta_m)P(N_1/N_m)$ where $p_1 > 0, p_2 < 0, p_3 > 0$ and $P' < 0$; and a money trader contacts commodity traders at rate $q(\beta_m; \beta_1, \beta_{-m})Q(N_1/N_m)$ where $q_1 > 0$, $q_2 > 0$, $q_3 < 0$, and $Q' > 0$. Given symmetric equilibria where all commodity traders choose β_1 and money traders choose β_m, the aggregate transactions (matching) technology which captures the flow rate of barter and monetary exchanges is given by

$$
\begin{aligned}
f(\beta_1, \beta_m, N_m) &= N_1\{g(\beta_1; \beta_1, \beta_m)G(N_1/N_m)x^2 + p(\beta_1; \beta_1, \beta_m)P(N_1/N_m)x\} \\
&= N_1 g(\beta_1; \beta_1, \beta_m)G(N_1/N_m)x^2 + N_m q(\beta_m; \beta_1, \beta_m)Q(N_1/N_m)x\,. \quad (1)
\end{aligned}
$$

Notice that this identity implies $N_1 pP = N_m qQ$. With this, it is immediate that $p(\beta_1; \beta_1, \beta_m) = q(\beta_m; \beta_1, \beta_m) \equiv h(\beta_1, \beta_m)$, where $h_1 = p_1 + p_2 = q_2 > 0$ and $h_2 = p_3 = q_1 + q_3 > 0$, and $Q/P = N_1/N_m$. This also implies that the transactions technology will be strictly increasing in search efforts. Finally, since both commodity

2. That is, we focus specifically on pure monetary equilibria. Kiyotaki and Wright (1993) provides the underlying specification of goods and preferences that endogenizes these results. Li (1992) provides a similar demonstration with endogenous search intensities.

traders who barter and money traders must contact commodity traders, we will assume $G(N_1/N_m) = Q(N_1/N_m)$.

B. Search Equilibrium and Efficiency

Let V_1 denote the optimal value associated with the state of holding commodities and V_m the optimal value associated with being in the money-trading state. With $r > 0$ as the rate of time preference, the "stationary asset pricing" characterization of Bellman's equations, which have become standard in search theory, can be expressed as

$$rV_1 = \max_{\beta_1}\{-s(\beta_1) + g(\beta_1; \beta_{-1}, \beta_m)G(N_1/N_m)x^2U + p(\beta_1; \beta_{-1}, \beta_m)P(N_1/N_m)x(V_m - V_1)\} \tag{2}$$

$$rV_m = \max_{\beta_m}\{-s(\beta_m) + q(\beta_m, \beta_1, \beta_{-m})Q(N_1/N_m)x(U + V_1 - V_m)\}, \tag{3}$$

Equation (2) says that the flow value associated with a commodity trader is equal to the arrival rate of a successful barter exchange, gGx^2, where x^2 is the probability of a "double coincidence," times the instantaneous utility U from consumption; plus the arrival rate of money traders times the probability that his good is acceptable to the money trader, pPx, times the net gain from becoming a money trader $(V_m - V_1)$; less optimal search costs $s(\beta_1)$. Similarly equation (3) is the flow value associated with a money trader. It equals the arrival rate of commodity traders times the probability that the money trader is willing to accept a commodity trader's good, qQx, times the net gain from consumption and returning to the state of being a commodity trader $(U + V_1 - V_m)$, less the cost of optimal search effort $s(\beta_m)$. Since we will be concerned about the efficiency of pure monetary equilibria where money is acceptable in all trades, this requires that $V_m - V_1 \geq 0$ and $U + V_1 - V_m \geq 0$.

The first-order conditions associated with equations (2) and (3) are given by

$$s'(\beta_1) = g_1Gx^2U + p_1Px(V_m - V_1), \tag{4}$$

$$s'(\beta_m) = q_1Qx(U + V_1 - V_m). \tag{5}$$

Equations (4) and (5) equate the marginal cost of the search effort with the expected private marginal return from search in each state. By subtracting equation (2) from (3) we can solve for the relative value of holding money to goods as a function of search efforts:

$$\Delta \equiv V_m - V_1 = \frac{-s(\beta_m) + s(\beta_1) + [h - gx]QUx}{r + hx[Q + P]}, \tag{6}$$

We can define a *decentralized monetary equilibrium* given real money supply $M = N_m$ as a list $\{\beta_1, \beta_m, \Delta\}$ satisfying (4), (5), and (6). To address the question of the efficiency of decentralized equilibria, we need to establish a criterion by which to measure social welfare. Define steady-state aggregate welfare (or average lifetime utility of the infinitely lived representative trader) as the sum of the value associated with each state weighted by the fraction of the population in that state: $W = N_1V_1 + N_mV_m$. From (2) and (3) we see that aggregate welfare is simply the aggregate flow rate of consumption less search costs:

$$\begin{aligned} rW &= f(\beta_1, \beta_m, M) - N_1 s(\beta_1) - N_m s(\beta_m) \\ &= N_1 \hat{g} G x^2 U + N_m h Q x U - N_1 s(\beta_1) - N_m s(\beta_m)\,, \end{aligned} \tag{7}$$

where $\hat{g}(\beta_1, \beta_m) = g(\beta_1; \beta_1, \beta_m)$, $\hat{g}_1 > 0$ and $\hat{g}_2 < 0$. Thus, for an exogenously given M, the socially efficient outcome in this monetary economy is a choice of β_1 and β_m which maximizes equation (7). The first-order conditions for this social planner's problem is given by

$$N_1 s'(\beta_1) = N_1 \hat{g}_1 G x^2 U + N_m h_1 Q x U\,, \tag{8}$$

$$N_m s'(\beta_m) = N_1 \hat{g}_2 G x^2 U + N_m h_2 Q x U\,. \tag{9}$$

Equation (8) equates the aggregate marginal costs of search by commodity traders with the social marginal benefits of the consumption it generates from both barter and monetary exchanges. Similarly, equation (9) sets the aggregate marginal cost of the search by money traders to its social marginal benefits. Thus, the right-hand side of (9) takes into account that while search by money traders increases the total flow rate of consumption arising from monetary exchange, it also reduces the flow of barter trades through a congestion effect.

Equating the private and social marginal benefits from search by commodity traders in (4) and (8) and solving for Δ gives

$$\Delta = \left\{ \frac{N_1}{N_m} \left[\frac{\hat{g}_1(\beta_1, \beta_m) - g_1(\beta_1; \beta_1, \beta_m)}{p_1(\beta_1; \beta_1, \beta_m)} x \right] + \frac{h_1(\beta_1, \beta_m)}{p_1(\beta_1; \beta_1, \beta_m)} \right\} U\,. \tag{10}$$

Similarly, equating the private and social marginal benefits from search by money traders in (5) and (9) and solving for $U - \Delta$ gives

$$U - \Delta = \left\{ \frac{N_1}{N_m} \frac{\hat{g}_2(\beta_1, \beta_m)}{q_1(\beta_m; \beta_1, \beta_m)} x + \frac{h_2(\beta_1, \beta_m)}{q_1(\beta_m; \beta_1, \beta_m)} \right\} U\,. \tag{11}$$

Notice that Δ and $U - \Delta$ are simply the gains from a monetary exchange for commodity and money traders, respectively. Thus, (10) and (11) specify the share of the gains from a monetary exchange, U, which must be given to commodity and money traders so that the private incentives to search coincide with social efficiency. Equation (10) implies that the private marginal gain of search to a commodity trader from a monetary exchange, $p_1\Delta$, is set equal to the marginal benefit of allowing other commodity traders to barter (weighted by the relative difficulty of barter to monetary exchange), $(\hat{g}_1 - g_1)x(N_1/N_m)U$, plus the marginal benefit of allowing money traders to consume, h_1U. Similarly, (11) implies that the private marginal gain of search for a money trader, $q_1(U - \Delta)$, is set equal to the aggregate benefit of consumption it generates, h_2, less the cost of reducing the number of barter exchanges through a congestion effect, $\hat{g}_2x(N_1/N_m)$.

Satisfaction of both (10) and (11) are necessary and sufficient conditions for efficiency. Thus, efficiency requires that these shares of the gains from trade sum to unity:

$$x\frac{N_1}{N_m}\left\{\frac{(\hat{g}_1 - g_1)}{p_1} + \frac{\hat{g}_2}{q_1}\right\} + \frac{h_1}{p_1} + \frac{h_2}{q_1} = 1\ . \tag{12}$$

It is clear from equation (6) that the equilibrium value of the division of the surplus will be a function of the decentralized choices of β_1 and β_m. Thus, there is no internal mechanism which enforces this "sharing rule" to be consistent with (10) and (11). The individual choices of search effort by commodity traders ignore the positive effect it has on the meeting rate of other commodity traders and money traders seeking consumption, and the negative congestion effect it has on other commodity traders seeking a monetary exchange. Also, the search intensity choice of money traders ignores the benefits to commodity traders seeking monetary exchange as well as its congestion effect on both commodity traders seeking barter and other money traders. However, with an appropriate division of the gains from monetary exchange, given by (10) and (11), it is possible to internalize the direct and congestion externalities and allow decentralized equilibria to attain social efficiency.[3] We now turn to a couple of interesting examples highlighting some implications of this efficiency result.

EXAMPLE 1: *Efficiency with a Diamond (1984) Transactions Technology*

An interesting example of this efficiency condition is in the context of including endogenous search effort in the "cash-in-advance" transactions technology of Diamond (1984). Letting $g = \hat{g} = 0$, $p(\beta_1;\ \beta_{-1},\ \beta_m) = (\beta_1)(\beta_{-1})^{-b}(\beta_m)^{1-b}$, $q(\beta_m;\ \beta_1\beta_{-m}) = (\beta_m)(\beta_{-m})^{-b}(\beta_1)^{1-b}$, where $b < 1$, the efficient sharing rule given by (10) and (11) becomes $\Delta = \{\beta_1h_1/h\}U$ and $U - \Delta = \{\beta_mh_2/h\}U$. It is thus immediate that the necessary condition for efficiency given by (12) requires that the transactions technology have constant returns to scale in search intensities β_1 and β_m. Since

3. Note that the presence of congestion externalities is required for decentralized equilibria to attain social efficiency. If the only externality involved is the positive effect of search on those on the other side of the market, then efficiency requires both money and commodity traders be given the total match surplus U. This clearly violates equation (12).

$h_1 = (1 - b)h/\beta_1$ and $h_2 = (1 - b)h/\beta_m$, this necessary condition becomes $b = 1/2$. Therefore, the efficient sharing rule is given by $\Delta = U/2$ and $(U - \Delta) = U/2$. Social efficiency requires an even division of the trade surplus. This result offers some interesting parallels with previous work. While inefficiencies in this model arise from search effort, the restriction on the returns to scale of the matching technology is generally consistent with models of search externalities arising from production decisions studied by Diamond (1984) and Hosios (1990). Although there are no search externalities in Kiyotaki and Wright (1993), they also find that the Nash bargaining solution of evenly dividing the trade surplus is Pareto optimal. While search and production decisions in their model are exogenously fixed, the division of the surplus does affect the number of participants in the market and hence aggregate welfare.

EXAMPLE 2: *Some Implications for Monetary Policy*

While decentralized equilibria are inefficient relative to the social optimum, efficiency may still be possible with an appropriate choice of the equilibrium share of the trade surplus given to buyers and sellers. This example considers the role of the quantity of money in influencing this equilibrium sharing rule and aggregate welfare in the search economy. To simplify the analysis and provide more concrete results, consider a pure currency economy where only money traders expend search effort. This assumption, which has been shown to be an endogenous outcome of search-theoretic models by Burdett et al. (1995), naturally leads to a monetary economy where cash is involved on one side of every transaction without imposing an a priori cash-in-advance constraint. With this, we have $\beta_1 = 0$ and specify the meeting rate functions as $g = 0$, $p = (\beta_m)^{a-b}$, $q = (\beta_m)^a(\beta_{-m})^{-b}$, $P = (N_1/N_m)^{c-1}$, and $Q = (N_1/N_m)^c$ where $a > b > 0$, $a - b \leq 1$, and $0 < c < 1$. These functional forms imply that in the aggregate $h = q = p = (\beta_m)^{a-b}$ and the transactions technology is given by $f(\beta_m, N_m) = (\beta_m)^{a-b}(N_1)^c(N_m)^{1-c}$. The parametric restrictions on the values of a, b, and c imply that the matching function f is characterized by constant or diminishing returns to scale in β_m and constant returns in N_1 and N_m. Without loss of generality, let $x = 1$. From the optimal value functions and first-order condition for β_m, respectively, it will be sufficient to characterize the equilibrium given $M \in (0, 1)$ as a list $\{B_m,\Delta\}$ satisfying

$$\Delta = \frac{-s(\beta_m) + h(\beta_m)QU}{r + h(\beta_m)(Q + P)} \equiv F(\beta_m) \tag{13}$$

and

$$\Delta = \frac{q_1(\beta_m; \beta_m)QU - s'(\beta_m)}{q_1(\beta_m; \beta_m)Q} \equiv J(\beta_m)\ . \tag{14}$$

That is, equilibria occurs at a value of β_m^* where $F(\beta_m^*) = J(\beta_m^*) = \Delta^*$. It can be shown that $F'(\beta_m^*) < 0$ and $J'(\beta_m^*) < 0$ (see Li 1995). The pure currency version of

(7) is given by $rW = N_m\{hQ - s(\beta_m)\}$. With this the unique socially efficient level of search effort for a given $M \in (0,1)$ is given by β_m^E which satisfies $J(\beta_m^E) = U(b/a)$. From this we see that a necessary and sufficient condition for efficiency of the decentralized equilibrium is given by $F(\beta_m^*) = J(\beta_m^*) = U(b/a) = \Delta$. This condition states that the socially efficient share of the surplus given to commodity (money) traders is strictly increasing (decreasing) in the size of the congestion effect, b, relative to the direct effect of search on consumption, a. It follows immediately that too much search occurs in the decentralized equilibrium if and only if $\Delta < U(b/a)$ and too little search if and only if $\Delta > U(b/a)$.

We now investigate the effect of a change in the real money supply M on the decentralized equilibrium. Notice that since money and goods are indivisible and individuals are restricted to store only one unit of inventory at a time ($N_1 + N_m = 1$), this exercise necessarily implies that an increase in $M = N_m$ leads to a one-for-one decrease in N_1.[4] From (13) and (14) it is apparent that $\partial J/\partial M < 0$ and $\partial F/\partial M < 0$ implying that the overall effect of a change in M on equilibrium β_m and Δ will generally be ambiguous. As increasing M makes trade more difficult for money traders, there is a direct effect in reducing the marginal benefits to search and search effort. However, since contacting commodity traders is more difficult for money traders, the relative value of holding money balances to goods for any given level of β_m falls and this tends to increase search effort. Thus, the overall effect of a change in M on equilibrium β_m will generally be ambiguous. However, since money is a necessary input in the transactions process, it should be clear that welfare will be strictly increasing (decreasing) in M for M sufficiently small (large). Furthermore, we can make the following proposition regarding the optimal quantity of money.

PROPOSITION: *There exists an optimal quantity of money* M* *such that the decentralized pure currency equilibrium attains social efficiency.*

VERIFY. Substituting $Q = [(1 - M)/M]^c$ into (13) and simplifying gives us $F(\beta_m) = -s(\beta_m)(1 - M)/\{r(1 - M) + h(1 - M)^c M^{-c}\} + hU(1 - M)/\{rM^c(1 - M)^{1-c} + h\}$. Taking the limiting cases of $M \to 0$ gives us $F(\beta_m) \to U$ and $M \to 1$ gives us $F(\beta_m) \to 0$. Also, from (14) it is immediate that as $M \to 0$ we have $J(\beta_m) \to U$ while $M \to 1$ implies that $J(\beta_m) \to -\infty$. Letting Δ^* denote the equilibrium surplus given to commodity traders, we have that $\lim_{M\to 0} \Delta^* = U$ and $\lim_{M\to 1} \Delta = 0$. By continuity of F and J these results imply that for every $\Delta^* \in (0,U)$ there exists an $M \in (0,1)$. Since $U(b/a) \in (0,U)$, there will exist an optimal quantity of money M^* such that $\Delta^* = U(b/a)$.■

This example has shown that since the real quantity of money affects the difficulty of exchange in the search economy, it directly affects the relative value of holding money to goods. In particular, the optimal quantity of money M^* in this framework

4. As in Kiyotaki and Wright (1993), this welfare analysis regarding the quantity of money is very stylized. However, it is suggestive as to the nature of optimal monetary policy in more generalized settings. For example, one natural way of interpreting this mechanism is that the government extracts seigniorage revenue by purchasing commodities from commodity traders with real money balances.

corresponds to that value that sets the division of the trading surplus between buyers and sellers to its socially efficient value. The external effects from search are internalized relative to a "second-best" socially efficient allocation of a social planner which is constrained to treat M^* as exogenous. Although M^* maximizes aggregate welfare in the decentralized equilibrium, there is no guarantee that it will correspond to a global "first-best" efficient allocation of $\{M, \beta_m\}$ which maximizes aggregate welfare. Finally, note that a heuristic interpretation of this optimal policy result can be given in the context of a nominal pricing mechanism. As in Diamond (1984) and Kiyotaki and Wright (1993) nominal prices can be thought of as the outcome of a bargaining rule which divides the surplus of trade between buyers and sellers. Thus, as an alternative to directly manipulating the real money stock, there may be a role for policies in directly setting the sharing-rule and nominal prices to coincide with the optimal quantity of money.[5]

2. SUMMARY

This paper analyzes the efficiency of monetary exchange in a search-theoretic model of money. Because the optimal exchange behavior of the individual trader is based upon the private rather than social gains from trade, decentralized monetary equilibria are shown to be generally inefficient relative to the social optimum. A generalized transactions technology was considered where both commodity and money traders expend search intensity for the location of consumption goods. Search externalities arise because individuals ignore the benefits of their own search effort to agents on the other side of the market as well as the costs of congestion to agents on the same side of the market. However, for an appropriate division of the monetary trade surplus between buyers and sellers, it is possible for market equilibria to attain social efficiency. This feature also provides a role for the quantity of money in reducing the inefficiencies of decentralized trading. In the context of a pure currency economy, the optimal quantity of money is that which internalize these external effects relative to a "second-best" efficient allocation that treats the real money supply as given.

Although not identical to the shopping externalities considered by this paper, these results suggest positive and normative parallels with external effects that can arise from production decisions in search economies. For example, Diamond (1984) demonstrated that externalities associated with production willingness generally precluded efficient allocations. Given that the government cannot directly control production decisions, a second-best allocation can be achieved through the manipulation of the real money stock. An alternative interpretation of the Diamond result can also be seen in the context of specialization in the Kiyotaki-Wright (1993) model. That is, given that there is a trade-off between time spent in production and the marketability, too much specialization would likely occur in the decentralized equi-

5. Recent work extending search-theoretic models of money to include explicit nominal prices through strategic bargaining include Shi (1993) and Trejos and Wright (1993, 1995).

librium. The quantity of money that affects the relative gains from trade in the market can also promote efficiency in the production/exchange process. These results suggest the importance of the decentralized nature of monetary exchange in the evaluation of optimal policy.

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