INTERNATIONAL ECONOMIC REVIEW
Vol. 36, No. 4, November 1995

THE OPTIMAL TAXATION OF FIAT MONEY IN SEARCH EQUILIBRIUM*

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This paper investigates the link between welfare and the taxation of money in the context of a general equilibrium search model where decentralized trading motivates the use of fiat money. Since buyers with money must invest effort or search intensity to contact sellers with goods, a natural trading externality arises. Given a sufficiently productive economy, search efforts will be too low relative to social efficiency. This provides a welfare improving role for policies which tax money balances. The nature of this role is explored and its implications for optimal monetary policy discussed.

1. INTRODUCTION

It is conventional wisdom that the existence of valued fiat money arises from a world characterized by trading frictions and the lack of a centralized marketplace through which trade is coordinated. In general equilibrium models of money built on the "frictionless" competitive equilibrium paradigm, these trading frictions are approximated by either placing money directly into the utility function (Sidrauski 1967, Brock 1975, and McCallum and Goodfriend 1988) or into household finance constraints (Grandmont and Younes 1972 and Lucas 1980). The normative issue of optimal monetary policy in general equilibrium has almost exclusively been evaluated within the context of this paradigm. However, by not explicitly considering an economic environment which gives rise to money as the medium of exchange, these models overlook the important issue of how such policies affect the decentralized trading process. A more natural setting in which to consider the question of optimal monetary policy is within the context of an exchange process which is both time consuming and costly in terms of real resources.

This paper explores the role of policies which tax money balances in coordinating the inefficiencies of an economy characterized by explicit decentralized exchange. Monetary exchange is described as a search process with heterogenous agents having preferences over a set of heterogenous goods. This aspect provides the underlying motivation for the existence of a medium of exchange—alleviating trading frictions caused by the lack of a "double coincidence of wants." Formal demonstrations of this search aspect of monetary exchange are provided by Jones

* Manuscript received April 1994; revised November 1994.
[1] This paper is drawn from Chapter 3 of my Ph.D. dissertation at Northwestern University. I am grateful to my advisor Dale Mortensen for his guidance and continuous encouragement. I also thank Timothy Fuerst and Kiminori Matsuyama for their many insightful comments and valuable suggestions. This paper has benefitted from the comments of two anonymous referees, and seminar participants at Northwestern, the University of Pennsylvania, the Federal Reserve Bank of Philadelphia, and the 1994 Summer Meetings of the Econometric Society in Quebec City.

(1976), Iwai (1988), Oh (1989) and Kiyotaki and Wright (1989, 1991, 1993). Our general equilibrium set-up is a variant of the search equilibrium approach to trade frictions and money by Diamond (1982, 1984) and Kiyotaki and Wright (1991, 1993). The transactions technology of these models is modified by allowing individual traders an optimal choice of search effort. This adds the more realistic feature that individual traders can and do influence their velocity of transactions. Introducing costly investment in exchange activity implies that the trading strategies of an individual agent can have important external effects on the aggregate rate of transactions. This is because individual decisions to search are based upon the private rather than social gains to search. Thus, the number of transactions which occur in the decentralized economy will typically be too low relative to the social optimum. This type of general equilibrium search externality is similar to those explored within the context of labor markets and other dynamic economic environments (e.g., Mortensen 1982a, 1982b, Pissarides 1986, and Hosios 1990).

Whether or not this implies that the private decision of search effort is too high or too low depends upon the aggregate costs of production. Given that agents in the search economy are sufficiently productive, it is shown that individual search efforts do have a positive external effect on the rate of transactions. This feature provides a channel by which taxing fiat money can actually improve the efficiency of monetary exchange. In particular, a tax on money in our set-up causes agents holding money to optimally incur the costs of searching more intensely for consumption goods. This is similar to the impact of inflation on transactions costs in inventory-theoretic models of money (e.g., Baumol 1952, Tobin 1956, Jovanovic 1982, and Romer 1986). However, given a positive search externality, higher search effort leads to a greater aggregate flow of consumption and steady state welfare. Because taxing money balances also "crowds-out" commodity holdings both directly and indirectly (through increasing search efforts), the overall optimal tax rate on money will be directly linked to the level of productivity. Thus, contrary to the traditional view that an inflation tax is disruptive to economic activity and welfare, this paper suggests that policies which tax money holdings, such as inflation, may enhance welfare by promoting efficiency in the monetary exchange process.

Section 2 of this paper presents the basic model, characterizes equilibria with endogenous search intensity, and evaluates the efficiency of monetary exchange. Section 3 then considers the welfare implications of taxing money holdings in the search economy. Finally Section 4 concludes with a summary and possible extensions to the research.

2. THE BASIC MODEL

A. *The Economic Environment.* The economy is populated by a continuum of infinitely lived agents with total mass normalized to unity. These agents have heterogenous preferences over a large number of indivisible commodities. In particular, only a fraction $0 < x < 1$ of the total number of commodities yields positive utility $\overline{U}$ to the representative trader when consumed while the others provide zero utility. It is also assumed that preferences are symmetric over the commodity space.

That is, while the value of x is constant across individuals, no two individuals have exactly the same set of consumption goods which yield positive utility. Thus, x also reflects the probability that any given agent will be willing to consume a particular commodity type. In addition to real commodities there is also fiat money—intrinsically useless objects which offer no utility when consumed. The storage technology of each agent is restricted to holding only one unit of a real commodity or one unit of real money balances at any moment in time.

The search period for the representative trader begins on the production island where production goods arrive according to a Poisson process with exogenous arrival rate $\alpha > 0$. Thus, the costs of producing a real commodity is just the waiting time involved and α can be interpreted as a measure of productivity. Without loss of generality, we can motivate trade by assuming agents cannot consume their own production good.[2] Once a good is produced, an agent immediately becomes a "commodity trader" and must enter into a costly bilateral exchange process where the rate of contacting other traders is stochastic. These other traders are holding either a commodity or fiat money. Because of the indivisibility of commodities and real money, all exchanges are one-for-one swaps of goods for goods (a barter exchange) or goods for money (a monetary exchange).[3] If a barter trade occurs, utility is obtained from consumption and the agent returns to the production island. If a monetary exchange occurs, that agent becomes a "money trader" and must search for commodity traders who hold a desired consumption good and is willing to trade it for fiat money. When successful, money traders obtain utility from consumption and return to the production island.

Optimal search for the representative trader consists of (i) reservation strategies regarding which types of items (commodities and fiat money) are acceptable in trade, and (ii) optimal search effort. It is assumed that whatever a commodity trader is encountered, the type of good that trader is holding will be a pure independent random draw from the commodity space. Also, there is a fixed transactions cost ϵ incurred by any individual accepting a real commodity. The net utility obtained from consumption can thus be given by $U = \overline{U} - \epsilon$. These specifications are sufficient to guarantee that no good will be accepted in exchange unless it will be consumed, i.e., the trivial reservation strategy is to accept and consume only goods which yield positive utility. Intuitively, all real commodities are equally marketable on the trading island and there are no advantages to storing one good relative to another. The decision of whether or not to trade one's production good for an offer of fiat money depends upon the probability that fiat money will be acceptable to any randomly selected commodity trader. Kiyotaki and Wright (1991, 1993) formally demonstrates that, given a positive rate of time preference, there exists a unique symmetric Nash equilibrium where the best response strategy is to always accept money in trades given that it is generally acceptable by all other traders in the

[2] Relaxation of this restriction complicates the notation without altering any of the features of the model. See Kiyotaki and Wright (1993) or Burdett et al. (1993).

[3] This indivisibility assumption allows us to circumvent the more complex issue of bilateral nominal price determination. Recent works by Shi (1993) and Trejos and Wright (1993) attempt to solve this problem by incorporating strategic bargaining theory.

market. The acceptance of fiat money allows agents to circumvent the double coincidence of wants problem associated with barter and minimizes the expected search time before a successful transaction occurs. Thus, fiat money endogenously circulates as the unique medium of exchange. After the decision of accepting money as the medium of exchange is made, agents must also choose how much effort to invest in search (i.e., search intensity). First is the question of whether money traders or commodity traders or both actively expend search effort in equilibrium. In a recent paper, Burdett et al. (1993) determines endogenously which agents invest in active search and which agents passively wait for trading partners to come to them. They explicitly show that this decision is *not symmetric* and is fundamentally different for buyers and sellers. In general, equilibria arises where only money traders expend search effort if and only if the cost of search to buyers is not "too large" and the cost of search to sellers is not "too small," hence giving rise to an endogenous cash-in-advance constraint. Furthermore, because the number of commodity types directly influence these relative costs, the set of equilibria where these conditions are satisfied increases as the number of commodities becomes arbitrarily large. We will incorporate this endogenous outcome into our framework by imposing the restriction that only money traders expend search effort. These results are natural and confirm the observation that cash is typically involved on one side of every transaction in all specialized modern economies.[4]

The benefits of greater search effort for money traders is that, all else being equal, it reduces the expected search time before locating a desired consumption good. However, there is a disutility cost of search. Denote the search intensity choice of money traders as β and the cost of search intensity as $s(\beta)$, where s is continuously differentiable, increasing, and convex [$s(\cdot)$ satisfies $s', s'' > 0$, and $s(0) = 0$]. Define N_0, N_1 and N_m as the proportion of agents which are producers, commodity traders, and money traders, respectively, where $N_0 + N_1 + N_m = 1$. From the indivisibility of real money balances, the real money supply is given by $M = N_m = 1 - N_0 - N_1$. Since the production and trading islands are mutually exclusive, it will also be convenient to denote m as the fraction of agents on the trading island holding money, i.e., $m = N_m/(N_1 + N_m)$. The stochastic arrival of a successful transaction in each state follows a Poisson process with arrival rates determined by the fraction of agents holding money on the trading island, m, the search intensity choice, β, and the probability of an acceptable trade, x. With this, the arrival rate of a successful exchange of goods for money to a commodity trader is given by βmx and the arrival rate of a successful exchange of money for a consumption good to a money trader is given by $\beta(1 - m)x$. Thus, the aggregate flow of consumption in the economy, or the transactions technology, can be derived by aggregating the meeting probability across the measure of agents holding money, and is given by $f(\beta) = N_m \beta(1 - m)x$.

A policy of taxing money balances can be introduced by allowing the monetary authority to extract seignorage revenue from money traders. In particular, government agents encounter money holders according to a Poisson process with arrival

[4] Part B of the Appendix provides a simple demonstration of this feature of the model. See Burdett et al. (1993) for a more complete discussion and formalized arguments.

rate μ and confiscate their money balances. When this occurs, money traders must return to the state of production without consumption. The government also purchases goods with this tax revenue by meeting commodity traders stochastically according to a Poisson process with arrival rate δ^g and buying their goods with money. The government's budget constraint in this situation is given by setting revenues equal to expenditures or $\mu N_m = \delta^g N_1$. Solving for δ^g and using the definition of m gives $\delta^g = \mu m/(1-m)$.[5]

B. *Decentralized Search Equilibrium.* We first need to characterize the solution of the representative trader's dynamic optimization problem subject to the transactions technology and the money tax process described above. Let V_0, V_1, and V_m represent the value of an individual in the state of production, commodity trading, and money trading, respectively, and $r > 0$ be the rate of time preference. The continuous time optimal value functions can be written as

$$(1) \qquad rV_0 = \alpha(V_1 - V_0),$$

$$(2) \qquad rV_1 = \left\{ \beta m x + \frac{\mu m}{(1-m)} \right\}(V_m - V_1),$$

$$(3) \qquad rV_m = \max_{\beta \geq 0}\left\{ -s(\beta) + \beta(1-m)x(U + V_0 - V_m) + \mu(V_0 - V_m) \right\}.$$

Equation (1) states that the flow value to a producer is the arrival rate of production goods, α, times the change in states when the producer becomes a commodity trader, $V_1 - V_0$. Equation (2) indicates that the flow value associated with a commodity trader is the rate at which he is able to trade his production good for money, attained by summing the arrival rates of money traders, $\beta m x$, and government buyers, $\delta^g = \mu m/(1-m)$, times the expected net gain of becoming a money trader, $V_m - V_1$. The flow value of a money trader given by equation (3) is the sum of the gain from encountering and consuming a desired consumption good and returning to the production island, $(U + V_m - V_0)$, which occurs at rate $\beta(1-m)x$, and the net loss of having money taxed away and returning to the production island $(V_0 - V_m)$, which occurs at the money tax rate μ, less the cost of search effort $s(\beta)$. The interior first order condition for β associated with (3) is given by

$$(4) \qquad s'(\beta) = x(1-m)(U + V_0 - V_m).$$

Equation (4) equates the marginal cost of search effort with the expected marginal benefit from search. Notice that agents take m and $N_m = M$ as given in the optimal value functions. However, the aggregate the value of M, given an exogenous value of m, is solved for in steady state by the following condition:

$$(5) \qquad \alpha N_0 = \{ \beta x(1-m) + \mu \} N_m.$$

[5] This taxation process was introduced by Kiyotaki and Wright (1991) in their analysis of the robustness of pure monetary equilibria. Government expenditures are nonproductive in this set-up.

Equation (5) endogenizes M by setting the flow of agents out of the production island equals to the flow into production.[6] We can now define the steady state equilibrium of the model.

DEFINITION. *A decentralized monetary equilibrium given $m \in (0,1)$ and $\alpha > 0$ is a list $\{\beta, V_0, V_1, V_m, N_0, N_1, N_m\}$ satisfying* (1), (2), (3), (4), (5), $m = N_m/(N_1 + N_m)$ *and* $N_0 + N_1 + N_m = 1$.

To analyze the steady state properties of the decentralized monetary equilibrium it will be convenient to adopt some notation. In particular, let $\gamma^1 = \beta m x + \mu m/(1-m)$, $\gamma^m = \beta(1-m)x + \mu$, $\lambda(\beta) = \gamma^1/(r+\gamma^1)$, and $y = \alpha/(r+\alpha)$. Thus, γ^1 and γ^m are the total meeting probabilities of a representative commodity and money trader. Equations (2) and (3) can be rewritten as

$$rV_1 = \gamma^1(V_m - V_1), \tag{6}$$

$$rV_m = \max_{\beta \geq 0}\{-s(\beta) + \beta(1-m)xU - \gamma^m(V_m - V_0)\}. \tag{7}$$

Solving (1), (6), and (7) for V_m as a function of the search intensity choice, we have $V_0 = yV_1$, $V_1 = \lambda V_m$ and thus $rV_m = -s(\beta) + \beta(1-m)xU - \gamma^m(V_m - \lambda y V_m)$ or

$$V_m = \frac{-s(\beta) + \beta(1-m)xU}{r + \gamma^m(1-\lambda y)}. \tag{8}$$

Subtracting $V_0 = \lambda y V_m$ from (8) we can explicitly define a function $F(\beta)$ which gives the value of holding money relative to the production state $(V_m - V_0)$ for a given β:

$$V_m - V_0 = \frac{\psi(\beta)}{\Gamma(\beta)} \equiv F(\beta), \tag{9}$$

where

$$\psi(\beta) = \beta U - \frac{s(\beta)}{x(1-m)}, \Gamma(\beta) = \frac{1}{x(1-m)}\left\{\frac{r}{1-\lambda y} + \beta(1-m)x + \mu\right\}.$$

By rearranging the first-order condition in (4) we can define a function $G(\beta)$ which implicitly defines an optimal choice of β for a given $(V_m - V_0)$:

$$V_m - V_0 = \frac{x(1-m)U - s'(\beta)}{x(1-m)} \equiv G(\beta). \tag{10}$$

[6] As in Kiyotaki and Wright (1993) an alternative definition is to hold N_m exogenous and determine an equilibrium value of m. However, since it can be shown that for every given $N_m \in (0,1)$ there exists a unique $m \in (0,1)$, these are equivalent definitions.

With this, it will be sufficient to characterize the decentralized monetary equilibrium with exogenous money balances as a list $\{\beta, \Delta\}$ satisfying (9) and (10), where $\Delta \equiv (V_m - V_1)$. A monetary equilibrium occurs at a value of β^* where $F(\beta^*) = G(\beta^*) = \Delta^* \geq 0$.

Notice that equation (5), the definition of m, and the fact that the population is normalized to one provides three equations which uniquely define N_0, N_1, and N_m as functions of search effort and the model's exogenous parameters. Solving for N_m gives

(11) $$N_m = \frac{m\alpha}{\alpha + m[\beta(1-m)x + \mu]}.$$

Therefore, given an optimal search intensity choice satisfying (9) and (10), equation (11) determines the steady state real quantity of money. This leads to the following proposition regarding the model's steady state.

PROPOSITION 1.

(i) *There exists a unique* β_0 *such that* $F'(\beta_0) = 0$ *and* $\beta_0 = \operatorname{argmax} F(\beta)$.
(ii) *Monetary equilibria occur at* β^* *such that* $F'(\beta^*) < 0$ *and* $G'(\beta^*) < 0$.
(iii) *There exists at least one steady state equilibrium. There can exist* $2n+1$ *steady state equilibria where* $n \in \{0, 1, 2, \dots\}$.

PROOF. See the Appendix.

The basic idea behind Proposition 1 can be summarized in Figures 1 and 2. In particular, notice that (ii) implies

$$\frac{\partial \Delta^*}{\partial \beta} > 0 \quad \text{and} \quad \frac{\partial \beta^*}{\partial \Delta} > 0.$$

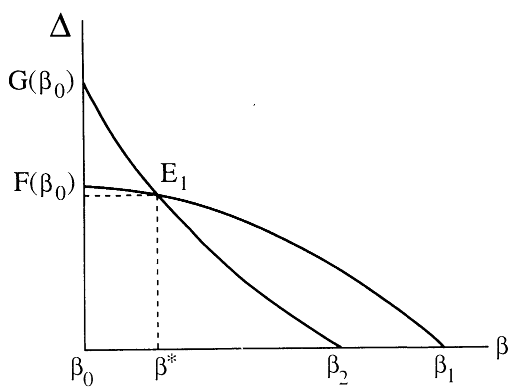

FIGURE 1

UNIQUE MONETARY EQUILIBRIUM

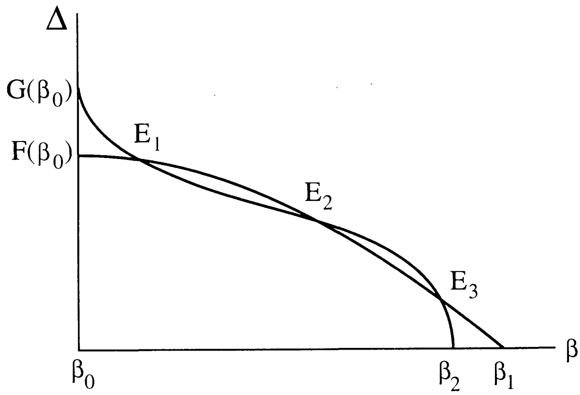

FIGURE 2

MULTIPLE MONETARY EQUILIBRIA

Intuitively, there is an externality in that a greater optimal search intensity choice by money holders increases the relative value of production to holding money. Furthermore, there is a "feedback" to this externality in that a lower relative value of production to holding money causes money holders to increase their optimal search intensity choice. Although there exists at least one steady state (E_1 in Figure 1), this result implies the possibility of an odd number of multiple steady state monetary equilibria (e.g., E_1, E_2, and E_3 in Figure 2).[7] Since the analysis of this paper will focus on the inefficiencies arising from the direct effect of the search externality, we will restrict attention to the unique equilibrium case. A sufficient condition for this is given by $G'(\beta) < F'(\beta)\ \forall \beta$.

C. *Aggregate Welfare and Social Efficiency.* In order to address the issue of efficiency we first need to define an appropriate measure of steady state welfare. We can construct a welfare function for this search economy by aggregating the value functions across the measure of agents involved in state $i = 0$, 1, and m:

$$W = N_0 V_0 + N_1 V_1 + N_m V_m. \tag{12}$$

Equation (12) can be interpreted as either aggregate welfare or the ex ante expected lifetime utility of the representative trader before his current state is revealed. Substituting (1), (6), and (7) into (12) and collecting terms gives us

$$rW = N_m\{-s(\beta) + \beta(1-m)xU\} = f(\beta) - N_m s(\beta).$$

Aggregate welfare is simply the total flow of consumption in the search economy less the aggregate costs of search effort incurred by money traders. Substituting in

[7] The fact that search frictions can generate multiple equilibria has been well documented by other works. For example, Diamond (1982, 1984) and Diamond and Fudenberg (1989) analyzes how increasing returns in the matching technology with fixed search effort can lead to multiple steady states.

the steady state money stock N_m from (11) gives

$$rW = \frac{m\alpha\{-s(\beta)+\beta(1-m)xU\}}{\alpha+m[\beta(1-m)x+\mu]} = \frac{m\alpha x(1-m)\psi(\beta)}{\alpha+m[\beta(1-m)x+\mu]}. \tag{13}$$

We now evaluate the efficiency of the decentralized equilibrium in the absence of policy intervention (i.e., $\mu = 0$). Consider a social planner's problem which chooses an efficient search intensity level β which maximizes (13). The first-order condition for this planner's problem is given by $\{\alpha + m\beta(1-m)x\}\psi'(\beta) - \psi(\beta)m(1-m)x = 0$ or

$$\psi'(\beta) = \frac{\psi(\beta)m(1-m)x}{\alpha+\beta m(1-m)x} \equiv H(\beta). \tag{14}$$

Since by definition $\psi'(\beta) = G(\beta)$, the efficient search intensity level β^E must satisfy $G(\beta^E) = H(\beta^E)$. We can now derive some properties of the social optimum and evaluate the efficiency of decentralized monetary exchange.

PROPOSITION 2. *There exists a unique solution to the social planner's problem:* $\beta^E = \text{argmax}\{rW\}$.

PROOF. See the Appendix.

PROPOSITION 3. *The efficiency of monetary exchange depends critically on the productivity parameter* α. *In particular, (i) there exists* $\alpha > 0$ *sufficiently small such that* $\beta^* > \beta^E$, *(ii) there exists a bounded* α *sufficiently large such that* $\beta^* < \beta^E$, *and (iii) there exists a unique* $\hat{\alpha}$ *such that* $\beta^* = \beta^E$.

PROOF. The social planner sets $G(\beta) = H(\beta)$ while the decentralized equilibrium sets $G(\beta) = F(\beta)$. Given that $G'(\beta) < F'(\beta) < 0$ in the decentralized equilibrium, a sufficient condition for $\beta^* < \beta^E$ is given by $F(\beta) > H(\beta)\ \forall\beta \in B = [0, \beta_1]$ where β_1 is such that $\psi(\beta_1) = 0$. This implies that $\psi(\beta)/\Gamma(\beta) > \psi(\beta)m(1-m)x/\{\alpha + \beta m(1-m)x\}\ \forall\beta \in B$. Simplifying and rearranging, we have the condition $\alpha > rm/[1-\lambda(\beta)y]$. Taking the limiting case where $\alpha \to \infty$, we have $y \to 1$ and the condition is satisfied. By continuity, there exists a bounded α such that the condition is satisfied. Similarly, a sufficient condition for $\beta^* > \beta^E$ is given by $F(\beta) < H(\beta)\ \forall\beta \in B$ or $\alpha < rm/[1-\lambda(\beta)y]$. As $\alpha \to 0$, we have that $y \to 0$ and the right-hand side of this condition equals $rm > 0$. Thus, this condition will eventually be satisfied for some $\alpha > 0$ sufficiently small. Also, notice that $\lambda(\beta)$ and hence $1/[1-\lambda y]$ is strictly increasing in $\beta \in B$. Thus, any value of $\alpha > rm/[1-\lambda(\beta_1)y]$ guarantees that $\beta^* < \beta^E$ while any $\alpha < rm$ leads to $\beta^* > \beta^E$. Since the choice of β^* and β^E always occurs along $G(\beta)$, its continuity implies that there exists a unique $\hat{\alpha} \in (rm, rm/[1-\lambda(\beta_1)y])$ such that $\beta^* = \beta^E$. □

The intuition behind Proposition 3 is as follows. The social planner seeks to maximize the aggregate flow of consumption net of search costs. From (13) we see that increasing search effort β has two distinct effects on aggregate welfare. First, it speeds up the consumption rate of all money traders while imposing the cost of search effort. Second, equation (11) also indicates that the steady state fraction of money traders and the real money supply N_m diminishes with a higher value of β. Thus, the overall effect of changing β on the aggregate flow of consumption will be ambiguous. Individual decisions ignore the benefits of greater search effort on the consumption rate of other traders as well as the cost of reducing N_m. If the productivity rate α is sufficiently low, then the latter effect dominates and search effort in the decentralized equilibrium is too high relative to social efficiency. However, if α is sufficiently large, then the former effect dominates and individual decisions of search effort are too low relative to social efficiency. In this case, the social benefits of increasing search intensity and the rate by which individual money traders contact commodity traders exceeds the costs from reducing the number of money traders. This proposition also shows that for all possible values of $\alpha > 0$ there exists a single value $\hat{\alpha}$ where these two opposing external effects exactly cancel out and the decentralized equilibrium attains social efficiency. However, there is no guarantee that the exogenous productivity rate will coincide with this value.

3. THE TAXATION OF MONEY AND WELFARE

The previous section demonstrated that the decentralized choice of search effort is generally inefficient. In particular, given a sufficiently productive economy, individual choices of search effort will be too low relative to social efficiency. We now address the issue of whether a policy of taxing real money balances by choosing $\mu > 0$ can influence the private decisions to search and thus improve aggregate welfare. Is there a role for policies which tax money balances in achieving a "second-best" allocation? From the characterization of monetary equilibria given in Proposition 1, we have the following result regarding the effect of imposing a tax on money.

PROPOSITION 4.

(i) *The equilibrium search intensity choice β^* is strictly increasing in the tax rate on money.*
(ii) *The steady state fraction of commodity and money traders are strictly decreasing in the tax rate on money.*

PROOF.

(i) Recall that the decentralized monetary equilibrium for this economy may be characterized as a β^* such that $F(\beta^*) = G(\beta^*) = \Delta^*$. Since $G'(\beta) < 0$ and $\partial G(\beta)/\partial\mu = 0$, a necessary and sufficient condition for our result is given by $\partial F(\beta_m)/\partial\mu < 0$. This is immediately verified by differentiating (9) with respect to μ:

$$\frac{\partial F(\beta)}{\partial\mu} = -\frac{\psi(\beta)}{\Gamma(\beta)^2(1-m)x}\left\{\frac{ry}{(1-\lambda y)^2}\frac{\partial\lambda}{\partial\mu} + 1\right\} < 0,$$

since $\partial\lambda/\partial\mu = rm/\{(1-m)(r+\gamma^1)^2\} > 0$. Therefore, β^* is strictly increasing in μ. Also it can be verified that since F is convex in μ and as $\mu \to \infty$, $F(\beta) \to 0$ and $\partial F/\partial\mu \to 0$, β^* will also be concave in μ.

(ii) This result is immediate from equation (11) and result (i) of this proposition. □

Proposition 4, result (i) says that money holders find it optimal to increase their search efforts because a higher tax rate on money increases the probability of having money confiscated, and thus the desire to find consumption goods. Taxing money causes it to become a "hot potato" and increases the incentive for money holders to get rid of their real balances. It also implies that for an arbitrarily large tax rate on money, equilibrium search effort asymptotically approaches a positive and bounded value. Proposition 4, result (ii) indicates that the imposition of a tax on money balances "crowds-out" commodity and money holdings through two channels. First, there is the direct effect of reducing the number of steady state money traders through the confiscation of money. Second, there is an indirect effect through increasing search effort and flow of money traders who successfully implement a transaction and leave the trading island. Thus, the aggregate stock of commodities and real quantity of money are strictly decreasing in the tax rate on money. From these observations, we have the following proposition regarding the optimal tax rate on money which maximize aggregate welfare (13) in the decentralized equilibrium.

PROPOSITION 5.

(i) *There exists a unique optimal tax rate on money* $\mu^* \geq 0$. *Furthermore, there exists a productivity rate* α *sufficiently large such that it will be strictly positive.*

(ii) *When the optimal tax rate on money* μ^* *is positive, it is strictly increasing in the productivity rate* α.

PROOF. See the Appendix.

These results are a direct consequence of the efficiency of monetary exchange and the impact of taxing money on search efforts. In particular, Proposition 4 concluded that if α is very small, then search effort in the decentralized equilibrium is too high relative to social efficiency. Given that taxing money balances increases equilibrium search efforts, the optimal tax rate on money will obviously be zero. However, if α is sufficiently large, then search effort will have a positive external effect on the aggregate rate of consumption. In this situation policies which tax money balances may actually promote efficiency by encouraging investment in search intensity. Furthermore, given $\mu^* > 0$, result (ii) implies that economies characterized by higher productivity rates α also possess a higher optimal money tax rate. This is because a higher arrival rate of goods on the production island acts to mitigate the effect of the money tax in "crowding-out" steady state commodity and real money holdings. This strengthens the overall positive benefits of greater search effort. A very special case of the proof to Proposition 5 arises when we take $\alpha \to \infty$. In this limiting case, there are no costs of production and no crowding-out effects from taxing money balances. As a result, welfare will be strictly increasing in the tax rate on money.[8]

4. SUMMARY AND EXTENSIONS

This paper has explored the role of taxing money balances as a means of improving the allocative efficiency of monetary economies with explicit decentralized trading. Since buyers with money in this model must invest effort or search intensity to contact sellers with goods, a natural trading externality arises. Each individual trader chooses an optimal search intensity based upon the private gains to exchange activity rather than the social gains. For a sufficiently productive search economy, the external effects from search effort will be positive and this provides a welfare improving role for policies which tax money balances. This role emanates directly from the ability of such policies to increase search efforts and the aggregate rate of transactions. That is, the search externality provides a role for government in subsidizing search activity through taxing "nonsearch." Although these results are formally derived in a specific model, it should be clear that this type of trading externality would naturally arise in any reasonable model where transactions costs are involved in the exchange process.

These results are also suggestive of the nature of optimal monetary policy in actual monetary economies. In particular, an important example of a policy which can be interpreted as a tax on money holdings is the inflation tax. Similar to the effect of inflationary taxation, the tax on money in our search model increases the costs of holding money, crowds-out real commodities in the form of seignorage revenue, and reduces the real quantity of money. Although the price level is fixed, the model is able to approximate several important features of actual inflation. Given this interpretation, our results suggests that, contrary to more traditional general equilibrium models of money which confirm the Friedman (1969) rule of optimal deflation, the optimum quantity of money may involve some steady state money growth and inflation. To further explore this issue models which relax the assumption of both indivisible money and goods will be necessary so that nominal prices and inflation can be analyzed more explicitly. By considering nominal price determination as the outcome of strategic bargaining problems, recent works by Shi (1993) and Trejos and Wright (1993) take the first step towards this direction.

There are also a couple of possible extensions to the current model. First is the issue of the global dynamics of these monetary economies and the multiplicity of steady state equilibria. Studying the global dynamics of these economies permits a more complete analysis of questions such as the existence of periodic "limit cycles" and self-fulfilling "sunspot" equilibria in the case of multiple steady states.[9] A natural implication of such an extension is a role for taxing money balances in

[8] Of course, this special case assumes that there are no additional costs to the government of imposing taxes on money. Otherwise, the optimal tax rate of money will continue to be bounded.

[9] These types of questions have been addressed by Matsuyama (1991) in the context of industrialization and Mortensen (1991) in the context of endogenous aggregate fluctuations. They have also been studied in a nonmonetary version of the Kiyotaki-Wright model by Boldrin, Kiyotaki, and Wright (1993).

coordinating across multiple steady state equilibria. Second, one of the central questions in the analysis of welfare and inflation is its impact on the steady state capital stock. Introducing inventory accumulation in our model is particularly interesting because of its emphasis on optimal exchange behavior given an explicit transactions technology by which inventories are converted into desired consumption goods. The implications of taxing money in such a model is pursued in Li (1994).

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APPENDIX

A. *Proofs to Propositions.*

PROOF OF PROPOSITION 1. (i) Notice that $F(0)=0$ and there exists β_1 satisfying $s(\beta_1)=\beta_1(1-m)xU$ such that $F(\beta_1)=0$. Thus, it will be sufficient to show that $F(\beta)$ is strictly concave for all β such that $F'(\beta)>0$. Differentiating $F(\beta)$ yields

$$F'(\beta)=\frac{\psi'(\beta)-F(\beta)\Gamma'(\beta)}{\Gamma(\beta)}. \tag{A1}$$

Dividing by $F(\beta)$ and using $F(\beta)=\psi(\beta)/\Gamma(\beta)$ gives us

$$\frac{F'(\beta)}{F(\beta)}=\frac{\psi'(\beta)}{\psi(\beta)}-\frac{\Gamma'(\beta)}{\Gamma(\beta)}<\frac{1}{\beta},$$

since $\psi(\beta)$ is strictly concave in β, $\lambda'(\beta)>0$, and $\Gamma'(\beta)/\Gamma(\beta)>0$. Therefore, there exists a unique β_0 where $F'(\beta_0)=0$ and $\beta_0=\operatorname{argmax} F(\beta)$.

(ii) Notice that $\psi'(\beta)=G(\beta)\ \forall\beta$ and the equilibrium value of search effort β^* must satisfy $G(\beta^*)=F(\beta^*)$. Substituting these into (A1) gives

$$F'(\beta^*)=\frac{G(\beta^*)[1-\Gamma'(\beta^*)]}{\Gamma(\beta^*)}<0,$$

since $\Gamma'(\beta)=ry\lambda'(\beta)/\{x(1-m)[1-\lambda(\beta)y]^2\}+1>1$. Also, we have that $G'(\beta)=-s''(\beta)/x(1-m)<0$ for all β.

(iii) The properties of $F(\beta)$ include that $F(0)=0$, there exists a unique $\beta_0=\operatorname{argmax} F(\beta)$, and there exists a unique $\beta_1>0$ satisfying $s(\beta_1)=\beta_1(1-m)xU$ such that $F(\beta_1)=0$. We also know that equilibria must occur at points where $F'<0$. The properties of $G(\beta)$ include $G(0)=U>0, G'<0$, and there exists a bounded and unique $\beta_2>0$ satisfying $s'(\beta_2)=(1-m)xU$ such that $G(\beta_2)=0$. From this we have that a necessary and sufficient condition for the existence of steady state monetary equilibria is given by $\beta_2<\beta_1$. This is immediately verified by noting that $s'(\beta_2)=s(\beta_1)/\beta_1<s'(\beta_1)$ by convexity of $s(\cdot)$ in β. Thus, there exists at least one

steady state equilibria and if there are multiple equilibria then the number of equilibria must be odd. □

PROOF OF PROPOSITION 2. Notice that $H(0)=0$ and there exists β_1 satisfying $s(\beta_1)=\beta_1 x(1-m)U$ such that $H(\beta_1)=0$. Differentiating $H(\beta)$ gives

$$\text{(A2)}\quad H'(\beta)=\psi'(\beta)\left\{\frac{m(1-m)x}{\alpha+\beta m(1-m)x}\right\}-\psi(\beta)\left\{\frac{m(1-m)x}{\alpha+\beta m(1-m)x}\right\}^2.$$

Dividing by $H(\beta)$ and using its definition given by (14) yields

$$\frac{H'(\beta)}{H(\beta)}=\frac{\psi'(\beta)}{\psi(\beta)}-\frac{m(1-m)x}{\alpha+\beta m(1-m)x}<\frac{1}{\beta},$$

since $\psi(\beta)$ is strictly concave in β. This implies $H(\beta)$ is strictly concave in β when $H'(\beta)>0$ and there exists a unique β such that $H'(\beta)=0$ and $\beta=\operatorname{argmax} H(\beta)$. The solution to (14) requires that $G(\beta^E)\equiv\psi'(\beta^E)=H(\beta^E)$. Substituting this into (A2) and simplifying gives $H'(\beta)=m(1-m)x\{G(\beta)-H(\beta)\}/\{\alpha+\beta m(1-m)x\}=0$. Therefore, there is a unique $\beta^E=\operatorname{argmax}\{rW\}=\operatorname{argmax} H(\beta)$. □

PROOF OF PROPOSITION 5. Differentiating (13) with respect to μ gives us the first order condition for the optimal tax rate on money:

$$\text{(A3)}\quad \alpha+m[\beta(\mu)(1-m)x+\mu]=\frac{\psi[\beta(\mu)]}{\psi'[\beta(\mu)]}m\left\{(1-m)x+\frac{1}{\beta'(\mu)}\right\}.$$

Denote the left-hand side of (A3) as $\omega(\mu,\alpha)$ and the right-hand side as $\phi(\mu)$. Note that $\omega(\mu,\alpha)>(<)\phi(\mu)$ implies that $\partial W/\partial\mu>(<)\ 0$. Differentiating each with respect to μ gives

$$\frac{\partial\phi}{\partial\mu}=-\frac{m\psi(\beta)\beta''(\mu)}{\psi'(\beta)\beta'(\mu)^2}+\left\{1-\frac{\psi(\beta)\psi''(\beta)}{\psi'(\beta)^2}\right\}[x(1-m)\beta'(\mu)+1]>0,$$

$$\frac{\partial\omega}{\partial\mu}=m[x(1-m)\beta'(\mu)+1]>0,$$

since $\psi(\beta^*)/\psi'(\beta^*)\geq 0$, $\psi''(\beta)<0$ and $\beta''(\mu)<0$. By comparing the two expressions above, it is immediate that $\partial\phi/\partial\mu>\partial\omega/\partial\mu$. At $\mu=0$ we have $\omega(0,\alpha)=\alpha+\beta(0)m(1-m)x>0$ and $\phi(0)=\psi[\beta(0)]m[(1-m)x+1/\beta'(0)]/\psi'[\beta(0)]>0$. Taking the limiting case of $\alpha=0$ gives us $\omega(0,0)<\phi(0)$ since this implies that

$$\beta(0)m(1-m)x\beta'(0)<\frac{\psi[\beta(0)]}{\psi'[\beta(0)]}m[(1-m)x\beta'(0)+1],$$

and this inequality is always true since by concavity of ψ, $\psi[\beta(0)]/\psi'[\beta(0)]>\beta(0)$. Thus, there exists $\alpha>0$ sufficiently small such that $\partial W/\partial\mu<0$ and $\mu^*=0$. A necessary and sufficient condition for there to exist a unique $\mu^*>0$ is given by $\omega(0,\alpha)>\phi(0)$. Since ω is strictly increasing in α there exists a bounded α

sufficiently large such that this condition will be satisfied. When this is the case it is immediate that the value of μ^* satisfying $\omega(\mu, \alpha) = \phi(\mu)$ is strictly increasing in α. □

B. *Endogenizing the Cash-in-Advance Constraint in a Simple Example.* The arguments provided in this simple example are essentially those contained in Burdett et al. (1993). Let $\alpha \to \infty$ so that the arrival rate of a commodity is instantaneous. Suppose that search effort for goods and money holders, β_1 and β_m, can only take on two possible values: $\beta_1, \beta_m \in \{0, \beta\}$. Let $s_i(\beta_i) \geq 0$ be the cost of search effort β_i where $s(0) = 0$. We will address the following question: if money holders invest in search, i.e., $\beta_m = \beta$, when will it pay for commodity traders to be passive and choose $\beta_1 = 0$? Let n denote the fraction of commodity traders who choose to search and $x \in (0,1)$ represent the probability that any trader in the economy is willing to consume a particular good. Denoting the optimal value associated with searching commodity traders, passive commodity traders, and search money traders as V_1, J_1, and V_m, these may be expressed as

$$\text{(A4)} \quad rV_1 = \beta(1-m)x^2[U + \max(V_1, J_1) - V_1] + \beta mx(V_m - V_1) - s_1(\beta),$$

$$\text{(A5)} \quad rJ_1 = \beta n(1-m)x^2[U + \max(V_1, J_1) - J_1] + \beta mx(V_m - V_1),$$

$$\text{(A6)} \quad rV_m = \beta(1-m)x[U + \max(V_1, J_1) - V_m] - s_m(\beta).$$

These equations embody the assumption that the arrival rate of an agent who is searching depends upon only own search effort and the probability of a successful transaction. However, the arrival rate for a passive commodity trader depends upon the fraction of other traders who are searching (n for other commodity traders and 1 for money traders) times their respective search effort. For example, the arrival rate for barter exchanges when passive is the search effort of *other* commodity traders who search (β) times the fraction of commodity traders who search (n). The $\max(V_1, J_1)$ terms in these value functions reflect the fact that anytime the trader returns to holding goods, the decision of whether to search or not must be made. From these, two results are immediate. First, if $n = 1$, then its clear that $J_1 > V_1$ which implies that no commodity traders search. This leads to $n = 0$ and a contradiction. Thus, it will never be optimal for all commodity traders to search. Second, if $s_1(\beta) > (1-m)x^2U$ and $s_2(\beta) < (1-m)xU$, then all commodity traders are passive. Verification of this is straightforward. Setting $V_1 < J_1$ in (A4) through (A6) implies that $[J_1 - V_1] = [s_1(\beta) - (1-m)x^2U]/(r + (1-m)x^2U) > 0$ if and only if $s_1(\beta) - (1-m)x^2U > 0$. However, a monetary equilibrium exists iff $V_m > J_1$ and this condition only holds when $s_m(\beta) < (1-m)xU$. Given that these conditions are satisfied, an endogenous cash-in-advance constraint arises.

REFERENCES

BAUMOL, W. J., "The Transactions Demand for Cash: An Inventory-Theoretic Approach," *Quarterly Journal of Economics* 66 (1952), 545–556.

BROCK, W., "A Simple Perfect Foresight Monetary Model," *Journal of Monetary Economics* 1 (1975), 133–150.

BOLDRIN, M., N. KIYOTAKI, AND R. WRIGHT, "A Dynamic Equilibrium Model of Search, Production and Exchange," *Journal of Economic Dynamics and Control* 17 (1993), 723–758.

BURDETT, K., M. COLES, N. KIYOTAKI, AND R. WRIGHT, "Buyers and Sellers," CARESS Working Paper, University of Pennsylvania, 1993 (forthcoming in *American Economic Review*).

DIAMOND, P., "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy* 90 (1982), 881–895.

———, "Money in Search Equilibrium," *Econometrica* 52 (1984), 1–20.

——— AND D. FUDENBERG, "Rational Expectations Business Cycles in Search Equilibrium," *Journal of Political Economy* 97 (1989), 606–620.

FRIEDMAN, M., *The Optimum Quantity of Money and Other Essays* (Chicago: Aldine, 1969).

GRANDMONT, J. AND Y. YOUNES, "On the Role of Money and the Existence of Monetary Equilibrium," *Review of Economic Studies* 39 (1972), 355–372.

HOSIOS, A., "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies* 57 (1990), 279–298.

IWAI, K., "The Evolution of Money: A Search Theoretic Foundation of Monetary Economics," CARESS Working Paper No. 88-03, University of Pennsylvania, 1988.

JONES, R. A., "The Origin and Development of Media of Exchange," *Journal of Political Economy* 84 (1976), 757–775.

JOVANOVIC, B., "Inflation and Welfare in the Steady State," *Journal of Political Economy* 90 (1982), 561–577.

KIYOTAKI, N. AND R. WRIGHT, "On Money as a Medium of Exchange," *Journal of Political Economy* 97 (1989), 927–954.

——— AND ———, "A Contribution to the Pure Theory of Money," *Journal of Economic Theory* 53 (1991), 215–235.

——— AND ———, "A Search-Theoretic Approach to Monetary Economics," *American Economic Review* 83 (1993), 63–77.

LI, VICTOR E., "Inventory Accumulation in a Search-Based Monetary Economy," *Journal of Monetary Economics* 34 (1994), 511–536.

LUCAS, R. E. JR., "Equilibrium in a Pure Currency Economy," in J. Karekan and N. Wallace, eds., *Models of Monetary Economies* (Minneapolis: Federal Reserve Bank of Minneapolis, 1980).

MCCALLUM, B. AND M. GOODFRIEND, "Theoretical Analysis of the Demand for Money," *Economic Review*, Federal Reserve Bank of Richmond (1988), 16–24.

MATSUYAMA, K., "Increasing Returns, Industrialization, and Indeterminancy of Equilibrium," *Quarterly Journal of Economics* 106 (1991), 617–650.

MORTENSEN, D. T., "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review* 72 (1982a), 968–979.

———, "The Matching Process as a Noncooperative Bargaining Game," in J. J. McCall, ed., *The Economics of Information and Uncertainty* (Chicago: University of Chicago Press, 1982b).

———, "Equilibrium Unemployment Cycles," working paper, Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1991.

OH, S., "A Theory of a Generally Acceptable Medium of Exchange and Barter," *Journal of Monetary Economics* 23 (1989), 101–119.

PISSARIDES, CHRISTOPHER A., "Search Intensity, Job Advertising, and Efficiency," *Journal of Labor Economics* 2 (1984), 123–143.

ROMER, D., "A Simple General Equilibrium Version of the Baumol-Tobin Model," *Quarterly Journal of Economics* 100 (1986), 663–685.

SHI, S., "Money and Prices: A Model of Search and Bargaining," working paper, the University of Windsor, 1993 (forthcoming in *Journal of Economic Theory*).

SIDRAUSKI, M., "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review* 57 (1967), 534–544.

TOBIN, J., "The Interest-Elasticity of the Transactions Demand for Cash," *Review of Economics and Statistics* 38 (1956), 241–247.

TREJOS, A. AND R. WRIGHT, "Search, Bargaining, Money, and Prices," CARESS working paper, University of Pennsylvania, 1993 (forthcoming in *Journal of Political Economy*).