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A finite horizon monetary economy

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Abstract

Conventional wisdom says that there is no demand for money in an economy of finite duration. In this article we show that this is not necessarily the case. We use a random matching model in which the function of money is to mediate exchange by alleviating the double coincidence of wants problem. Towards the end economic activity decreases but this does not affect the ability of money to speed up trade.

Key words: Finite duration; Endogenous money
JEL classification: E00

1. Introduction

Conventional wisdom says that in a finite duration economy there will be no demand for money (e.g., Gale, 1982; Hahn, 1982). Nobody will accept money in exchange in the last period, since it is accepted only if it can be spent later. Given that money is not useable in the last period, rational agents will not accept it one period before the last period, and the familiar backward induction argument tells us that money will not be accepted in any point of time. This is called the terminal problem. The reasoning is clear and compelling when agents are assumed to behave rationally. Even if the problem is regarded as a curiosity empirically, it is theoretically a disturbing issue. It implies that the use of money hinges on the world going on indefinitely. In addition, in order to address

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monetary issues economists have to use either infinite horizon models or impose the use of money exogenously.

There are several ways to avoid the terminal problem. A common approach is to impose cash-in-advance constraints or positive terminal balances (for references see Faust, 1989). This is clearly a somewhat unsatisfactory way to bypass the problem, since then the use of money is not a choice variable. Another approach is to assume that the agents do not know when the world will end even though it is finite. It is known that the world will end in finite time for certain if during every period there is a positive fixed probability that the world will end. Any model of infinite duration with discounting can be regarded as such a finite economy if the discount factor is interpreted as a probability that the current period is not the last. It is well-known that endogenous money is perfectly possible in models of this kind (e.g., Kiyotaki and Wright, 1990, 1991). Faust (1989) has offered a solution in continuous time. The model is finite in the sense that the economy ceases to exist after some finite date T but there is no last period. In this model money serves as an insurance against the cost of illiquidity. Towards the end its value decreases rapidly.

The terminal problem comes up only if the use of money is a choice variable. Then it resembles many other situations in which finite duration causes similar unravelling problems. The most familiar instance is probably the finitely repeated prisoner's dilemma. However, the finite horizon monetary economy has potentially more structure, because money typically performs many functions. It is possible that finiteness of the economy affects different functions differently. Money as a unit of account is obviously unaffected, while money as a store of value cannot survive. The backwards induction argument applies to the latter function because in the last period money cannot retain any value. Consider circumstances in which the need for money emanates from the difficulties of barter and its function is to speed up trade. A well-known instance is when money alleviates the problem of the double coincidence of wants. It seems possible that the success of money does not depend on the infinite duration of the economy.

That this is the case is demonstrated in a finite, discrete time version of the models of Kiyotaki and Wright (1990, 1991). The model features a monetary economy without the obligatory use of money in transactions. The function of money is to alleviate the double coincidence of wants problem. There is an unambiguous last period with the final moment to trade and nothing after that. When the end of the world approaches, one would expect this fact to affect adversely the agents' willingness to produce and engage in other costly activities. The model shows that towards the end economic activity decreases, but this does not affect the usefulness of money. The success of money in a finite horizon economy is then determined by what functions of money are the most important ones.

From another point of view one can think that finite duration offers a new criterium to differentiate between various functions of money. A medium of exchange necessarily serves as a store of value, too. But the most important function of money or its rationale in a particular economy can only be discovered by considering the corresponding economy without money. In overlapping generations models the function of money is to serve as a store of value, while in models with the double coincidence of wants problem (e.g., Kiyotaki and Wright, 1990, 1991) the function of money is to mediate exchange. Correspondingly, finite duration of the economy renders monetary equilibria impossible in the first case but in the latter case money can survive.

The article is organised as follows. In Section 2 we present the model, and in Section 3 we show that a monetary equilibrium does exist in a finite duration economy. Concluding remarks are in Section 4.

2. The model

2.1. Population

There is a continuum of agents uniformly distributed on a circle of circumference two. Their number is normalised to one and they are indexed by the points on the circle.

2.2. Production

During any given period agents can produce a randomly determined good by incurring a cost $c > 0$ in utility. The goods are realised simultaneously with the investment c. The agents can produce only after they have consumed. The commodities are differentiated, uniformly distributed on a circle of circumference two and indexed by the points on the circle. All goods are indivisible.

2.3. Money

There is a good from whose consumption nobody can derive positive utility. It is called fiat money. Money comes in units of size one and is indivisible.

2.4. Endowments

Some agents are endowed with money and its total amount in the economy is $M, 0 < M < 1$. If everybody has decided to produce there are $1 - M$ agents with a good for trade. Agents can hold at most one unit of money or one unit of a commodity at a time. Money and commodities cannot be held simultaneously. If goods are not consumed, they can be carried from one period to the next.

2.5. Preferences

Agents derive positive utility from a randomly selected good with probability x. More specifically, agent i derives utility U from a commodity z as along as $|z - i| \leq x$ and zero utility if $|z - i| > x$, $x \in (0, 1)$. The distance $|z - i|$ between a randomly selected commodity z and agent i's index is measured along the circumference of the circle and is uniformly distributed on $[0, 1]$. The agents never consume their own production good. Formally the utility function of agent i is

$$u(z) = \begin{cases} U & \text{if} \quad |z - i| \leq x, \\ 0 & \text{if} \quad |z - i| > x. \end{cases} \tag{1}$$

2.6. Time

The economy proceeds in discrete time starting in an arbitrary period. At the end of period T the world ends. This is called the END. Periods are designated according to the time when they end. For instance, the last period, which is called period T, begins at time $T - 1$ and ends at time T. All agents discount the future and have a common discount factor δ, $0 < \delta < 1$. Within one period the order of things is the following: Matching, trading, eating, and production.

2.7. Trading

In the beginning of every period the agents are randomly matched in such a way that everybody meets exactly one agent. When two agents meet they can exchange the objects in their possession. This happens only if both of them are willing to do so. There is a transaction cost $\varepsilon > 0$ in utility for accepting a good. The mediating role of money is taken to the extreme and it is assumed that accepting money is costless. Two agents with a unit of money are indifferent between exchanging and holding on to their original units. If an agent decides not to produce, he will withdraw from the trading process, which means that he will not participate in the random matching.

2.8. Agents' objective

The agents can be thought to be playing a noncooperative game. The strategy of agent i consists of the decision whether to produce at time t, the probability $\gamma_t^i(z_1, z_2)$ that he will exchange object z_1 for object z_2 at time t, and the decision whether to consume or continue to store object z at time t. Kiyotaki and Wright (1990) call $\gamma_t^i(z_1, z_2)$ a trading strategy and it is part of the agents' overall strategy. Agents maximise their expected life time utilities, given other players' strategies.

2.9. Equilibrium

Strategies generate a distribution of agents in various states and in equilibrium it is assumed to be known. An equilibrium is a set of strategies and the resulting distribution, such that the strategies are best responses to the other agents' strategies and consistent with rational expectations.

The use of money and the existence of a monetary equilibrium in the infinite horizon model are based on the fact that accepting money speeds up the transaction process. The probability of trading without money is of the order x^2, the double coincidence of wants, and with money of the order $2(1-M)Mx$. Money functions as a medium of exchange that partly overcomes the difficulties of barter.

Some of the features of the model warrant motivation and clarification. In Section 2.5 it is also possible to think of x as the proportion of agents that derives positive utility from a randomly selected commodity. If x is unity, tastes or commodities are not differentiated at all. In Section 2.6 not all actions are relevant to all agents. For instance, if an agent has not managed to trade he already possesses a good and production decisions are irrelevant to him. In Section 2.7 it can be thought that accepting a consumption good is a costly activity, since the quality of the good has to be ascertained. Fiat money, however, is by definition of uniform quality. An alternative interpretation is that ε represents carrying costs and that money is easy to carry. Those agents who possess neither money nor a commodity do not participate in the random matching. One can think that both consumption and production take place at each agent's home and that production costs c comprise or include the pain of carrying a good to the location in which random matching takes place. On the other hand, if an agent already possesses a good for trade or a unit of money, there is no need to go home but he can stay at the location of random matching and wait for the next period.

3. Monetary equilibrium in the finite horizon economy

The existence of equilibrium is proved constructively. First we make a guess of the agents' strategies, and then we show that they, indeed, constitute an equilibrium. However, complete proofs are not presented, since they would be very similar to those in Kiyotaki and Wright (1990, 1991). There are several equilibria in this model. There is always an equilibrium in which money is not used at all. As the aim is not to solve the model completely but to illustrate the connection between finiteness of the economy and monetary exchange, we concentrate on demonstrating that there exists a symmetric equilibrium in which all agents accept money.

First we note that no objects are ever disposed of. All agents, whether they are money holders or commodity holders, try to acquire a consumption good. Disposing of things implies that they are left with nothing, since by Section 2.2 they cannot produce. To show that a monetary equilibrium exists agents have to accept money with probability one. As in Kiyotaki and Wright (1990) attention is restricted to equilibria where the agents' *trading strategies* are of the form

$$\gamma_t^i(z_1, z_2) = \begin{cases} a_g(|z_2 - i|) & \text{if} \quad z_1,\ z_2 = \text{real commodities}, \\ a_m(|z_2 - i|) & \text{if} \quad z_1 = \text{fiat money},\ z_2 = \text{real commodity}, \\ 1 & \text{if} \quad z_1 = \text{real commodity},\ z_2 = \text{fiat money}. \end{cases} \tag{2}$$

The strategies are identical for all agents, since we focus on a symmetric equilibrium. We also assume that the probability of agent i accepting good z depends only on the distance $|z - i|$, that money is accepted with probability one, and that the trading strategies do not depend on time. For simplicity, the argument, i.e., the distance between the good offered and the agent's index, is left out of the trading strategies, and the probability of trading a good for a good is denoted by a_g and the probability of trading money for a good by a_m. Kiyotaki and Wright (1990) prove that the best strategy of agent i is to accept a good from which he derives positive utility with probability one and other goods with probability zero. Formally expressed,

$$a_g = a_m = \begin{cases} 1 & \text{if} \quad |z - i| \leq x, \\ 0 & \text{if} \quad |z - i| > x, \end{cases} \tag{3}$$

where z is the commodity offered to agent i. Because of the symmetry of the model all commodities are equally useful for exchange purposes. Accepting a commodity from which an agent derives zero utility does not affect his chances to trade in the future but involves a transaction cost.

The decision to produce is not stationary. Towards the END there will be a period in which agents without a good decide not to produce, since their chances to make a successful trade are not high enough. When the END is close, there will be few agents with a good for trade and a lot of agents with a unit of money, as those who decide not to produce withdraw from the trading process. In these circumstances money still functions as a medium of exchange. The only change is the reduction in the amount of transactions. However, there is no reason not to accept money. It will be accepted until the END.

The rest of this section consists of determining the time at which production ceases and proving that the above strategies constitute an equilibrium.

3.1. Production decision

The decision problem of agent i is analysed with the help of value functions. They are evaluated after consumption and before the production decision. The aim is to derive the condition for production to cease in period $T-n$, $n=0,1,\ldots$. Denote the value functions of an agent without a good, with a good in stock, and with a unit of money in stock by V_0, V_g, and V_m, respectively. Agents never have a good they can consume in stock, since it is optimal to consume immediately. As in Kiyotaki and Wright (1990) it can be shown that V_g is independent of the good z in stock as long as $|z-i|>x$. The only thing the agents care about is the distance between their index and the commodity offered to them. Because of the symmetry of the model the probability of trade does not depend on the commodity they hold. The value functions of agent i in the three possible states are given below. The superscript refers to the period in which the functions are evaluated. Let $n=1,2,3,\ldots$.

$$V_0^{T-n} = \max\{-c+\delta[(1-M)[x^2(U+V_0^{T-n+1}-\varepsilon)+(1-x^2)V_\pi^{T-n+1}] + \max_\pi M[x\pi V_m^{T-n+1}+(1-x\pi)V_\pi^{T-n+1}]];\ \delta V_0^{T-n+1}\}, \tag{4}$$

$$V_g^{T-n} = \delta[(1-M)[x^2(U+V_0^{T-n+1}-\varepsilon)+(1-x^2)V_g^{T-n+1}] + \max_\pi M[x\pi V_m^{T-n+1}+(1-x\pi)V_g^{T-n+1}]], \tag{5}$$

$$V_m^{T-n} = \delta[(1-M)x(U+V_0^{T-n+1}-\varepsilon)+(1-M)(1-x)V_m^{T-n+1} + MV_m^{T-n+1}]. \tag{6}$$

In the above formulation it is assumed that up to period $T-n$ everybody else but agent i produces for certain. When this does not happen, the value functions have to be slightly modified. The first value function (4) says that, if agent i is without a good in period $T-n$, he has to make a choice whether to produce or not. If not, then he will enter period $T-n+1$ with no good. The value of this option is δV_0^{T-n+1}. If he decides to produce, he incurs a cost c in period $T-n$. Because everybody else has decided to produce, he meets with probability $1-M$ a commodity trader who accepts the commodity of agent i with probability x. Agent i accepts the good offered to him with probability x. Thus, they trade with probability x^2. The value of this option is $\delta(U+V_0^{T-n+1}-\varepsilon)$. With probability $1-x^2$ there will be no trade and the agent enters the next period with a good he has produced in period $T-n$. The value of this option is

δV_g^{T-n+1}. With probability M agent i meets an agent with a unit of money. This agent accepts the commodity offered with probability x, and agent i maximises with respect to the probability of accepting money, π. With probability $(1 - x\pi)$ agent i does not accept money or his partner does not accept the good offered. Everything that takes place in period $T - n + 1$ is discounted with factor δ.

The interpretation of the second value function (5) is almost identical. The agent has a good in stock and there is no need to produce. In the last value function (6) the agent has a unit of money in stock. With probability $1 - M$ agent i meets a commodity trader and accepts the good offered to him with probability x. His partner accepts money for certain. With probability $1 - x$ agent i does not accept the commodity offered and enters the next period holding money. With probability M agent i meets another money holder and both of them enter the next period as money holders. As long as everybody produces in period $T - n$ the decision problem of an agent is captured by the above equations. For the moment we assume that $\pi = 1$ and later we verify that this, indeed, is the case. In other words, it will be shown that, if everybody else accepts money with probability one, then the best response of any agent is to accept money with probability one.

The above formulation of the value functions is not quite adequate to derive the condition for production to cease in period $T - n$. When production ceases in period $T - n$, the number of agents in the trading process changes because those who decide not to produce stay home. Then the chances to acquire a consumption good after period $T - n$ decrease every period. Thus, it is not profitable to produce in any period after $T - n$. Consequently, the number of traders in the trading process keeps changing until the END.

To keep track of the changing number of traders we introduce two functions. Consider function Φ,

$$\Phi\colon [0,1] \to [0,1], \quad \Phi(y) = y\left[1 - \frac{y}{y+M}x^2 - \frac{M}{y+M}x\right],$$

where x is the proportion of products from which agent i can derive positive utility and M is the number of money holders. Function Φ tells how many agents hold a good next period, if their number in this period is y and nobody produces any more. Suppose that period $T - n$ is the first period in which agents without a good decide not to produce. There are $(1 - M)(1 - M)x^2 + (1 - M)Mx$ of this kind of agents (the number of agents who exchanged a good for a good plus the number of agents who exchanged a unit of money for a good). They withdraw from the trading process. The number of agents with a good in period $T - n + 1$ matching is $(1 - M)[1 - (1 - M)x^2 - Mx] = \Phi(1 - M)$. In period $T - n + 2$ of trading there will be $\Phi(\Phi(1 - M)) = \Phi^2(1 - M)$ agents with a good and in period $T - n + k$ of trading there will be $\Phi^k(1 - M)$ agents with a good, $k = 1, \ldots, n$. The number of agents with a unit of money remains M during every period. Since in the sequel the argument of Φ as well as Φ^k is $1 - M$, it will be left out. Consider another function λ,

$$\lambda: N \to [0, 1], \quad \lambda(k) = \Phi^k + M.$$

It states the total number of agents in period $T - n + k$ of trading. With this notation the probabilities of meeting an agent with a good and with money in period $T - n + k$ can be expressed as $\Phi^k \lambda^{-1}(k)$ and $M\lambda^{-1}(k)$, respectively. k indicates how many periods ago the agents decided to cease production.

We start the analysis of the production decision from the last period.

Period T

Suppose that in every period before the last everybody has decided to produce. Then the agents with no good choose not to produce in the last period, as it is the very last thing they can do before the END. Production would cost $c > 0$ but without any time to trade there is nothing to gain. This means that $V_0^T = V_m^T = V_g^T = 0$. The best any agent in any state can do in the last period is worth zero in utility. It has to be remembered that the value functions are evaluated after consumption has taken place but before the production decision.

Period T − 1

Consider agent i who has consumed in period $T - 1$ and thus has no good. Assume that until period $T - 1$ everybody has always found it profitable to produce and assume that in the current period everybody else without a good decides to produce. As $V_m^T = V_g^T = 0$, the decision problem of agent i is captured by

$$V_0^{T-1} = \max\{-c + \delta(1 - M)x^2(U - \varepsilon), 0\}. \tag{7}$$

The first argument of the max function expresses the utility of agent i in case he decides to produce. The value of the decision not to produce is zero. From (7) it is clear that, if

$$c > \delta(1 - M)x^2(U - \varepsilon) \tag{8}$$

holds, then nobody will produce. Producing when others do not is even more unprofitable than when everybody else produces.

Period T − 2

Assume that everybody has decided to produce up to period $T - 2$. Assume also that (8) holds and that everybody produces in period $T - 2$. (8) implies that $V_0^{T-1} = 0$ holds. Consider the decision problem of agent i without a good. He has to decide whether to produce and he knows that in period $T - 1$ nobody produces. His decision problem is described by the following value function:

$$V_0^{T-2} = \max\{-c + \delta[(1-M)[x^2(U + V_0^{T-1} - \varepsilon) + (1-x^2)V_g^{T-1}] + M[xV_m^{T-1} + (1-x)V_g^{T-1}]];\ \delta V_0^{T-1}\}$$

$$\Rightarrow$$

$$V_0^{T-2} = \max\{-c + \delta[(1-M)[x^2(U - \varepsilon) + (1-x^2)V_g^{T-1}] + M[xV_m^{T-1} + (1-x)V_g^{T-1}]];0\}. \tag{9}$$

Because of (8), we get the other relevant value functions easily:

$$V_g^{T1} = \delta\lambda^{-1}(1)\Phi x^2(U - \varepsilon), \tag{10}$$

$$V_m^{T-1} = \delta\lambda^{-1}(1)\Phi x(U - \varepsilon). \tag{11}$$

Substituting (10) and (11) in (9) we get an explicit expression for V_0^{T-2}:

$$V_0^{T-2} = \max(-c + \delta x^2(U - \varepsilon)[1 - M + \delta\lambda^{-1}(1)\Phi[(1-M)(1-x^2) + M + M(1-x)]];0\}. \tag{12}$$

Thus, there will be no production in period $T-2$ if

$$c > \delta x^2(U - \varepsilon)[1 - M + \delta\lambda^{-1}(1)\Phi[(1-M)(1-x^2) + M + M(1-x)]]. \tag{13}$$

Condition (13) implies condition (8). This result guides the investigation of the general case in which $n = 3, 4, \ldots$.

Period $T-n$

It is shown that the larger c is, the earlier the agents choose not to produce. Suppose that everybody produces in period $T-n$, and in period $T-n+1$ production ceases. In this case denote by W^{T-n} the value function of an agent with a good at the end of period $T-n$. Thus, the value functions denoted by W are conditional on everybody producing in the current period (indicated by the superscript), but on nobody producing in the next period. W^{T-n} is of course equivalent to V_g^{T-n} (conditional on the above production decision), but the latter can mean different things in different contexts depending on when production ceases.

Consider two cases:

Case 1. Everybody who has a production decision to make produces for the last time in period $T - n$, after which production ceases.

Case 2. Everybody who has a production decision to make produces for the last time in period $T - n + 1$, after which production ceases.

The agents can be in three different states when the value functions are evaluated. Either they have a good in stock, have a unit of money in stock, or have nothing in stock. Those who have nothing in stock and decide not to produce go off the random matching. The transition probabilities from one state to another are the same in period $T - n + 1$ in Case 1 and in period $T - n + 2$ in Case 2. In other words, the probability of obtaining a consumption good and the probability of obtaining a unit of money are the same. This reasoning applies to all subsequent periods as well. The transition probabilities in period $T - n + k$ in Case 1 and in period $T - n + k + 1$ in Case 2 are identical, $k = 1, \dots, n - 1$. Because in Case 1 production ceases one period earlier than in Case 2 there is one more period for obtaining a consumption good in Case 1, when things are considered at the time that production ceases. This shows that

$$W^{T-n} = W^{T-n+1} + \delta^n \lambda^{-1}(n-1)\Phi^{n-1}[F(n-1)x^2 + G(n-1)x](U - \varepsilon) > W^{T-n+1}, \tag{14}$$

where $\delta^n \lambda^{-1}(n-1)\Phi^{n-1}[F(n-1)x^2 + G(n-1)x]$ is the expected utility from consuming in the extra period in Case 1. $F(n-1)$ denotes the probability that after $n - 1$ periods the agent still holds the same good as of the end of period $T - n$, i.e., during this time he has met only agents who do not desire his good or whose goods he does not desire. $G(n-1)$ denotes the probability that by the beginning of the last period the agent has managed to exchange his good for money, but has not managed to exchange this money for his consumption good.

Assume that one period before the last the agent has the same good he had in period $T - n$, which takes place with probability $F(n-1)$. In the last period his probability of meeting an agent with a good is $\lambda^{-1}(n-1)\Phi^{n-1}$. With probability x^2 both desire each other's good and they trade. In all other cases the agent receives utility zero in the last period. If the agent possesses a unit of money in the end of the penultimate period, which happens with probability $G(n-1)$, he meets an agent with a good he desires with probability x and they trade. In all other cases the agent receives utility zero in the last period.

It is not necessary to derive the exact formulae of F and G. For our purposes it is enough to note that both of them are decreasing functions of n. By (14) W^{T-n} is increasing in n. There is an upper bound $\delta(U-\varepsilon)$ for W^{T-n}, and thus W^{T-n} converges to some value W^* as n approaches infinity. Based on the above analysis we get:

Proposition 1. *The larger c is, the earlier the agents cease to produce. If $c > W^*$ holds, there will be no production at any point in time. For values of c less than W^* there is some point in time when production ceases and economic activity starts decreasing. If $W^{T-n-1} \geq c > W^{T-n}$ this happens in period $T-n$.*

3.2. Decision to accept money

It remains to show that $\pi = 1$ is the optimal response of agent i to other agents' strategies. In other words, it is optimal to always accept money. We get the following result:

Proposition 2. *As long as x is less than one it is always profitable to accept money. If $W^{T-n-1} \geq c > W^{T-n}$, it is profitable to produce in periods $T-n-k$, $k = 1, 2, \ldots$, and there exists a monetary equilibrium.*

Proof. Suppose that $W^{T-n-1} \geq c > W^{T-n}$. The condition says that production ceases in period $T-n$. After that only those who have a unit of money or a good in stock participate in trading.

An agent who possesses a good at the end of period $T-n$ is expected to accept money according to the suggested equilibrium strategy. We check the strategy against one time deviations. Suppose that he considers deviating in period $T-n+1$ by not accepting money. After this he is expected to follow the suggested equilibrium strategy. For the deviation to be profitable the following condition has to hold:

$$V_g^{T-n+1} > V_m^{T-n+1}. \tag{15}$$

In words, the agent has to be better off with a good than with a unit of money in period $T-n+1$. After the deviation the agent follows the suggested equilibrium strategy, which states that money is always accepted. This implies that

$$V_g^{T-n+2} \leq V_m^{T-n+2}. \tag{16}$$

It will be shown that these two conditions cannot hold simultaneously. From (15) it follows that

$$\delta\lambda^{-1}(2)\{\Phi^2x^2(U-\varepsilon)+\Phi^2(1-x^2)V_g^{T-n+2}+MxV_m^{T-n+2}+M(1-x)V_g^{T-n+2}\}$$

$$>\delta\lambda^{-1}(2)\{\Phi^2x(U-\varepsilon)+\Phi^2(1-x)V_m^{T-n+2}+MV_m^{T-n+2}\}$$

$$\Rightarrow$$

$$\Phi^2x[V_g^{T-n+2}-(U-\varepsilon)]+\Phi^2(V_g^{T-n+2}-V_m^{T-n+2})$$

$$+M(V_g^{T-n+2}-V_m^{T-n+2})>0. \tag{17}$$

In (17) the second and third terms are nonpositive by (16). For (17) to hold, the first term has to be positive. Because production ceases in period $T-n$, in the best possible case an agent with a good meets another agent with a good and each desires what the other has. Then the agent's expected utility with discounting is $\delta(U-\varepsilon)$, which is an upper bound to V_g^{T-n+2}. Substitute the upper bound for V_g^{T-n+2} to approximate the term upwards. This gives $\Phi^2x(\delta-1)(U-\varepsilon)$ as an upper bound for the first term. It is negative since $\delta<1$. This shows that (17) cannot hold.

It can be concluded that, if it is profitable to deviate and not to accept money in period $T-n+1$, then it is profitable to do the same thing in period $T-n+2$. Clearly the same analysis can now be applied to period $T-n+2$ and so on up to period $T-1$. Profitable deviation by not accepting money in any period after $T-n$ implies that it is profitable to do so in every successive period, too. But this implies that in period $T-1$ it has to be profitable not to accept money. Formally $V_g^{T-1}>V_m^{T-1}$. But $V_g^{T-1}=\delta\Phi^n\lambda^{-1}(n)x^2(U-\varepsilon)$ and $V_m^{T-1}=\delta\Phi^n\lambda^{-1}(n)x(U-\varepsilon)$ and the above inequality cannot hold as $x<1$. Thus, it is not profitable not to accept money in any period after $T-n$ inclusive. The above analysis applies to periods $T-n-k$, $k=1,2,\ldots$, as well. ■

3.3. Equilibrium characteristics

The monetary equilibrium in the finite horizon model shares most of the features of the monetary equilibrium in the infinite horizon model. When there is production and the economy is far from the END, everything functions exactly as in the corresponding infinite horizon model. In both models it is possible to determine the optimal amount of money, i.e., the value of M that maximises the social welfare. This value depends on x, and in a finite horizon model also on the time at which the welfare is evaluated. This can be seen, e.g., from (8). The more money holders there are, the more probable it is that production stops. Far from the END the life time utilities of the agents are approximately the same as in the infinite horizon model, since the speed of exchange, rather than the time when production ceases, determines the agents' welfare. Thus, the optimal amount of money is also approximately the same.

The monetary equilibrium in the finite horizon economy is characterised by the time that production stops. The stopping time, of course, depends on the duration of the economy T. But regardless of T production always ceases the same number of periods before the T. The larger the production costs are, the earlier the agents stop production.

4. Conclusion

It has been shown that the terminal problem does not necessarily arise in a model where the specific function of money is to alleviate the double coincidence of wants problem. Money is a generally accepted medium of exchange that is costless to accept. If production costs are sufficiently high, there is a period after which agents do not produce any more. Agents without goods cease to be economically active. Those who still have a good or a unit of money trade as before. The closeness of the END in no way reduces the ability of money to overcome the double coincidence of wants problem. Of course, towards the END the proportion of agents with money to those with a good increases as the amount of agents in the trading process decreases.

Since the consumption of own production goods yields zero utility the agents are willing to give up their holdings for nothing or for a unit of money in the final period. In the random matching only those with a good or a unit of money are present. Thus, in the final period there needs to be only a single coincidence of wants. If an agent can expect to gain nothing from an object, he might, as well, give it up for free.

The model is open to some objections. For instance, introducing an arbitrary positive cost of accepting money destroys the result. This is not so surprising as almost everything in the economy is known for certain. The costlessness of accepting money is required since everybody knows precisely when the END will come. It can also be argued that the result hinges on everybody getting zero utility from the consumption of his own production good. However, for the conclusion to hold it is enough that some positive measure of agents derive zero utility from their own production good. Of course, the assumptions on the order of agents' actions, in particular that those with no good or money do not participate in the random matching, play an important role. Still, we think that the model illuminates an interesting difference between the medium of exchange role of money and the store of value role of money as far as finiteness of the economy is concerned. Common sense suggests waning economic activity towards the end of the world. The closeness of the END does not affect the use of money since its rationale is in exchange, not in production, and exchange continues until the END.

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