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SEARCH, MONEY, AND PRICES*

BY ALOK JOHRI[1]

McMaster University, Canada

It is well known that models in which money is used as a medium of exchange to lubricate trading, frictions display multiplicity of equilibria. I show that the amount of activity varies as the value of money differs across these equilibria when production opportunities involve random fixed costs. When money has high value, trade is more profitable; therefore, there will be more agents engaged in trade relative to equilibria in which money has lower value. The higher-activity equilibria display higher production not only because more is produced and exchanged per transaction but also because more transactions occur per period. This Diamond-style result is obtained without increasing returns in the matching technology.

I. INTRODUCTION

Diamond (1982) developed an equilibrium search model, with a production sector in which production opportunities arrive randomly and a trading sector in which trading opportunities arrive randomly. While the trading decision is relatively trivial (any two agents who meet simply swap goods one for one), in the production sector opportunities have random costs, and so agents face nontrivial decisions regarding when to produce. If they choose a higher reservation production cost, they will produce and flow into the exchange sector more quickly; hence the trading sector will grow, and this leads to more trade and more consumption. Given that the arrival rate of trading partners is increasing in the size of the trading sector (increasing returns to scale, IRS, in the meeting technology), if agents choose a higher reservation production cost, then as the size of the exchange sector increases, the expected return to being in the exchange sector also will increase, which can justify choosing a higher reservation production cost.

In this way, the model generates multiple steady states, which, it has been argued, has important implications for macroeconomics.[2] In particular, the amount of

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[2] The model also displays multiple dynamic rational expectations equilibria (see Diamond and Fudenberg 1989 or Boldrin et al. 1993).

production and hence the amount of activity in the trading sector may be large or small depending on expectations. Diamond also constructed a version of the model with a cash-in-advance constraint (by assumption, traders must first exchange a good for a unit of money and then the unit of money for a good). This makes it possible to discuss macro issues in a monetary context. As in the nonmonetary version, there are multiple equilibria if and only if one assumes IRS in the meeting technology. Although these results are clearly of interest, two issues come up immediately: It is difficult to justify the cash-in-advance assumption in a model that is otherwise logically quite coherent, and it is unfortunate that the multiplicity relies on IRS, perhaps especially given recent empirical findings.[3]

In this paper I follow Kiyotaki and Wright (1991, 1993) by considering a generalized version of Diamond's model with many goods. This makes exchange nontrivial and hence leads to an endogenous role for money as a medium of exchange.[4] I also follow Shi (1995) and Trejos-Wright (1995) by endogenizing the value of money by having agents bargaining over how much output they exchange for a unit of money. Although the Shi and Trejos and Wright papers did not have an explicit production sector (agents are endowed with the ability to produce whenever they want), the model generates multiple equilibria with different positive values of fiat currency. In this paper I reintroduce a production sector with random opportunities into that model and show that the different equilibrium price levels are associated with different levels of the reservation production cost. Hence the fact that the value of fiat money is not uniquely pinned down by the fundamentals of the model is enough to induce a Diamond-style multiplicity in the amount of activity in the trading sector, even if one does *not* assume IRS in the meeting technology.[5]

I then use the model to analyze the effect of changes in the amount of money in the system, which, in the model, is equal to the number of agents with money and hence is an index of liquidity. It is shown that increasing the number of agents with money can, at least initially, reduce the equilibrium price level by increasing the amount of production and therefore the number of agents with goods for sale. While Trejos and Wright obtain this result only when bargaining occurs without search,

[3] See Pissarides (1990) for a review of empirical estimates of the aggregate matching technology.

[4] Aiyagari and Wallace (1992) and Ritter (1994) also discuss related issues in search models with fiat money. See Williamson and Wright (1994) for a model in which money helps to overcome frictions associated with private information about the quality of goods, Matsuyama et al. (1993) for an investigation of issues in international monetary theory, and Li (1995) for a discussion of the optimal inflation tax.

[5] In Shi or Trejos and Wright, the different equilibria differ in the amount of output traded in each meeting but not in the number of times agents produce, trade, or consume. In this paper the fact that the different equilibria differ in the reservation cost and hence in the number of agents in the trading sector means that they do differ in the number of times agents trade, produce, and consume. In other words, I am studying the effect on the extensive as well as the intensive margin. In a related paper (Johri 1994), I show that this extensive margin is operative even in a model where the intensive margin is shut down. That is, I use the Kiyotaki and Wright (1991, 1993) model, which has multiple equilibria with the same (exogenous) price level but with different probabilities that money is accepted, and show that these different equilibria have different reservation costs and hence different levels of activity in the trading sector.

in my structure it can be obtained irrespective of whether search is allowed while bargaining or not.

The rest of the paper is organized as follows: Section 2 discusses the environment, while the results are presented in Section 3. Concluding comments are made in Section 4, while most proofs are contained in an Appendix.

2. THE ENVIRONMENT

The economy is composed of a large number of agents who are infinitely lived. In period one, a fraction M of the agents are endowed with a unit of indivisible money (which is intrinsically valueless) and the others with differentiated nonstorable consumption goods that are perfectly divisible.[6] Not all goods are in the consumption set of every agent. The exogenously determined parameter x, $0 < x < 1$, determines the proportion of the total goods that are valued by an agent. It is convenient to let x also equal the proportion of agents that derive utility from any given good. Thus x captures the degree of specialization in production as well as consumption. Consumption of q units of any commodity yields an agent utility given by $u(q)$, where $u(0) = 0$, $u'(q) > 0$, and $u''(q) \leq 0$ if that good is part of their consumption set and zero otherwise.

Agents cannot consume their own commodities; therefore, they must trade with other agents in order to consume and derive utility. Potential trading partners are found randomly according to a Poisson process with constant arrival rate $\beta > 0$. In Diamond (1982, 1984) β is an increasing function of the number of traders in the economy; however, it is constant here, so we can focus on multiplicity of equilibria based solely on the complementarity in the acceptability of money. If there are suitable opportunities for trade, the partners bargain over the quantity of the good to be produced by the commodity trader in return for either goods or money. If and when the partners decide on how much is to be produced, production is instantaneous. When one unit of money exchanges for q units of a good, the price is implicitly defined as $p = 1/q$.

The economy can be thought of as consisting of two sectors, an exchange sector, in which trade takes place, and a production sector, in which agents search for production opportunities. At any given time all agents are in one of three states, commodity traders, money traders, or producers. Commodity traders already have obtained the ability to produce a particular type of good and are searching for trading partners who are interested in buying their good so that they can be thought of as sellers. All goods are produced with the same technology that is given by a cost function f, measured in disutility. It is assumed that $f(0) = 0$, $f'(q) > 0$, $f''(q) \geq 0$, and $f'(0) = 0$. Money traders are agents with a unit of money and are searching for commodity traders who have the ability to produce a good in their consumption set

[6] Nonstorability rules out commodity money. This also can be achieved in other ways (see Kiyotaki and Wright 1993). For models with commodity money, see Aiyagari and Wallace (1991), Kiyotaki and Wright (1989), and Oh (1989).

so that they can be thought of as buyers. Since all trades involve exchanging one's entire holding of money, there are always M money traders in the trading sector. Moreover, there are never any traders with both goods and money. The fraction of all traders who are money traders is denoted by μ. It follows that any trader located at random has money with probability μ and goods with probability $(1-\mu)$.

Agents who have traded and consumed a commodity leave the trading sector (since they now have nothing with which to trade) in search of new production opportunities. Agents cannot search for production opportunities while in possession of a commodity or of money. This search takes a random amount of time and is characterized by a Poisson process with a constant arrival rate α. Each production opportunity has an associated fixed cost $c, c \in [0, c_H]$, measured in terms of disutility that must be incurred in order to obtain the opportunity. If q^* is defined as the output level at which $u'(q^*) = f'(q^*)$, assume that $c_H < u(q^*) - f(q^*)$. Following Diamond (1982), c is drawn independently from a cumulative density function $G(c)$. Let $G(0) = 0$, $G'(\cdot) > 0$, and $G''(\cdot) \leq 0$. The fixed cost of obtaining the project is known before the implementation decision is made. Only those projects are implemented which cost less than some cutoff level c^*, where c^* is determined endogenously in equilibrium. Once agents are in possession of a production opportunity after incurring the fixed cost, they enter the trading sector as commodity traders.

It remains to specify the nature of the bargaining game that takes place between two matched traders that determines the amount q that is exchanged in each trade. This discussion is postponed until after the model with an exogenously given $q_t = Q_t$ is discussed. Agents seek to maximize expected discounted utility over their lifetimes. In the model with an exogenously specified amount to be produced, this requires them to have strategies that specify the probability with which to accept various goods and money, as well as a value for c^* taking as given other agents' strategies. I concentrate on the set of symmetric steady-state Nash equilibria. This implies that attention is restricted to the class of Nash equilibria in which (1) all commodities and all agents are treated identically and (2) all aggregate variables as well as the strategies followed by agents are constant over time.

Since commodities cannot be stored, only those goods which are in the trader's consumption set will be accepted. Hence the probability that a randomly located trader will accept a given commodity is x, and the probability that barter trade will result from a random meeting is x^2. It is profitable for agents to accept money only if they believe that other agents will accept money in return for goods.

Let Π be the probability with which a randomly located trader accepts money and $\pi \in [0, 1]$ be the representative agent's best response. In a symmetric equilibrium, $\Pi = \pi$. Also, c^* is chosen in equilibrium. Then if V_j denotes the value function of an agent in state j, where $j = p$, c, and m refer to being a producer, a commodity trader and a money trader, respectively, and $r > 0$ is the rate of time preference, the agent's Bellman equations may be written as

$$(1) \qquad rV_c = \beta(1-\mu)x^2\left[u(Q) - f(Q) + V_p - V_c\right] + \beta\mu x\left(\max_\pi\{\pi\left[V_m - V_c - f(Q)\right]\}\right)$$

$$rV_m = \max\{\beta(1-\mu)x\Pi[u(Q)+V_p-V_m], rV_p\} \tag{2}$$

$$rV_p = \alpha\int_0^{c^*}(V_c - V_p - c)\,dG \tag{3}$$

Equations (1) to (3) describe the flow return to an agent in each of the three states.[7]

In evaluating which projects should be implemented, the cutoff cost level c^* satisfies the following equality that leaves the agent indifferent between implementing the project and waiting for another project with a lower cost to arrive:

$$c^* = (V_c - V_p) \tag{4}$$

Let N_p, N_c, and N_m be the proportion of agents who are producers, commodity traders, and money traders, respectively. Note that $N_m = M$ if $\Pi > 0$ and $N_m = 0$ otherwise. In a steady state it must be true that aggregate variables are constant over time; thus the N_j's must be constant. This requires that the flow into and out of each state must be equal. It suffices to look at any one state, and here I look at the flow into and out of production. Therefore, in a steady state the following condition must hold:

$$\alpha G(c^*)N_p = \beta(1-\mu)x^2N_c + \beta(1-\mu)\Pi x N_m \tag{5}$$

Focusing attention for the moment on the case where money is valued, $\Pi > 0$ implies that $N_m = M$ and $\mu = M/(N_c + M)$. The definition of μ can be used to simplify the system. When $N_m = M$, Eq. (5) may be rewritten as a function of μ, Π, and c^* as

$$G(c^*) = \frac{\beta}{\alpha}\frac{\left[x^2M(1-\mu)^2 + x\Pi(\mu-\mu^2)M\right]}{\mu - M} \tag{6}$$

Given M, Eq. (6) defines a relationship between Π and (c^*, μ) pairs and will be referred to as the *steady-state condition* (SSC). Substituting Eq. (4) into Eqs. (1) to (3) yields

$$\begin{aligned} rV_c = {} & \beta(1-\mu)x^2[u(Q) - f(Q) - c^*] \\ & + \beta\mu x(\max_\pi\{\pi[V_m - V_c - f(Q)]\}) \end{aligned} \tag{7}$$

$$rV_m = \max\{\beta(1-\mu)x\Pi[u(Q) - c^* + V_c - V_m], rV_p\} \tag{8}$$

$$rV_p = \alpha\int_0^{c^*}(c^* - c)\,dG \tag{9}$$

[7] The return for a commodity trader (Eq. 1) is the sum of the expected gain from barter with another commodity trader and from selling goods to a trader with money. Detailed interpretations of these equations can be found in Johri (1994).

Given M and Q, Eqs. (6) to (9) implicitly define a correspondence $\Pi(\pi)$. This system of equations is similar to the fixed-price search models of money with the exception of the inclusion of fixed costs of obtaining a production opportunity and variable costs of production (see Kiyotaki and Wright 1993 or Johri 1994 for an example with fixed costs). In order to determine Q endogenously, the bargaining game must be specified; this is done in the next section. Before turning to bargaining, it is interesting to note a few results about the fixed-price structure. The preceding correspondence yields three steady-state Nash equilibria associated with $\Pi = \pi = 0, x, 1$; that is, there is one nonmonetary equilibrium associated with the general rejection of money as well as two monetary equilibria: a mixed equilibria in which money is only partially accepted as well as one in which it is fully accepted. Subtracting Eq. (7) from Eq. (8) and Eq. (9) from Eq. (7) and substituting yields an expression for c^* as a function of μ and Π that is referred to as the *cutoff rule*. Combining the cutoff rule with the steady-state condition (SSC) reveals an implicit function $c^*(M, \Pi)$, and it was shown in Johri (1994) that $c^*(M,1) > c^*(M,0) > c^*(M,x)$ for a range of positive M.

Bargaining. When two agents meet and wish to trade, they bargain taking the value associated with being a buyer (V_M), a seller (V_c), and a producer (V_p) and the value of money (Q) as given, where $P = 1/Q$ is the aggregate price level. When the buyer is a money trader and the seller a goods trader, they bargain over the quantity of the good to be supplied by the seller in return for one unit of money. The structure of the bargaining game is standard in the literature and is omitted for brevity (the details can be found in the referenced literature). Using the Nash bargaining solution, it can be shown that the equilibrium quantity exchanged q^m solves Eq. (10).[8]

$$q = \text{argmax}\left[u(q) + V_p\right]\left[V_m - f(q)\right] \tag{10}$$

Individual rationality conditions guaranteeing that neither player wishes to abandon the process and search for a new partner are discussed below.

In case two sellers meet and decide to barter, q^* is produced by both, where q^* is the q that solves $u'(q) = f'(q)$. This emerges as the solution to the bargaining problem, since both agents are in a symmetric situation with identical utility and cost functions, so they agree to produce where surplus is maximized.

Taking q^m and q^* as the quantities exchanged in the two kinds of trade, the Bellman equations of the agent can be written as

$$\begin{aligned} rV_c = {} & \beta(1-\mu)x^2\left[u(q^*) - f(q^*) + V_p - V_c\right] \\ & + \beta\mu x\left(\max_\pi\{\pi\left[V_m - V_c - f(q^m)\right]\}\right) \end{aligned} \tag{11}$$

[8] It is well known that this can be derived from strategic bargaining. Note also that this solution corresponds to bargaining without search, which implies that the threat points are zero. Bargaining with search, where threat points are the continuation values, is considered later. See Trejos and Wright (1993) for a discussion of both points.

(12) $$rV_m = \max\left\{ \beta(1-\mu)x\Pi\left[u(q^m) + V_p - V_m\right], rV_p\right\}$$

(13) $$rV_p = \alpha\int_0^{c^*} (V_c - V_p - c)\, dG$$

Nothing that has been said above guarantees that trade will take place in case the bargaining solution is not individually rational. Thus the following additional conditions are imposed that guarantee that neither side is worse off by trading than without.[9] The individual rationality (IR) constraint for the commodity trader requires the agent to be at least as well off by producing the good and switching states as he or she would be by remaining a commodity trader. A similar requirement holds for the money trader. These restrictions may be expressed compactly as

(14) $$u(Q) \geq V_m - V_c \geq f(Q)$$

where symmetry has been imposed ($q = Q$).

3. RESULTS

3.1. *Steady-State Equilibria.* Having described the model, I now turn to characterizing the set of equilibria.

DEFINITION 1. A nondegenerate symmetric steady-state Nash equilibrium is defined as a list $(V_j, N_j, c^*, \Pi, \pi, q^m, Q)$ satisfying Eqs. (10), (11) to (13), (4), and (5), $\Pi = \pi$, $q^m = Q$, $c^* > 0$, and the condition that $N_M = 0$ if $\Pi = 0$ and $N_M = M$ otherwise. If $\Pi > 0$, it is a monetary equilibrium; otherwise, it is a nonmonetary equilibrium. If Eq. (14) holds with strict inequality, the monetary equilibrium is called an *unconstrained monetary equilibrium*; otherwise, it is a *constrained monetary equilibrium.*

The nonmonetary equilibrium is characterized first. Note that there always exists a degenerate equilibrium ($c^* = 0$) in which no activity takes place. In what follows the degenerate equilibrium will be ignored.

PROPOSITION 1. *There exists a unique nondegenerate nonmonetary equilibrium, that is,* $\Pi = \pi = 0$, $\mu = N_M = 0$, $q^m = Q = 0$, *and* $c^* > 0$.

PROOF. See Appendix.

Note that this equilibrium corresponds to the nondegenerate equilibrium in Diamond (1982) if the participation externality is removed from that model. This is equivalent to constant returns to scale in the matching technology, as has been assumed in the current paper.

The economy also displays two types of equilibria in which money is always valued by all agents. The interesting feature of these equilibria is that even though money is

[9] This two-step procedure is equivalent to solving the bargaining game with players having the option to opt out of the bargaining game after rejecting an offer. See Osborne and Rubinstein (1990) for further details.

always accepted in return for goods in both, the value of money is lower in one, and this implies that a lower reservation cost c^* is chosen by agents so that production on the extensive margin will differ across equilibria.

PROPOSITION 2. *There exists a critical value $r' > 0$ such that if $r < r'$, $x < x_L$, and if $M \in [0, M_b^*]$, where $0 < M_b^* < 1$ and where $0 < x_L < 1$, there exists a constrained and an unconstrained monetary equilibrium that satisfies the conditions of Definition* 1.

PROOF. The proof of the proposition involves the use of Lemmas 1 and 2.

LEMMA 1. *For any $\mu \in (0,1)$, $c^* > 0$, $\Pi = 1$, and given the V_js, there exists an $r^{\max} > 0$ such that for any $r < r^{\max}$ there exists a unique unconstrained solution to the bargaining problem $q^m > 0$ as well as a unique constrained solution $q_1 > 0$ where $q^m > q_1$. For $r > r^{\max}$, $q^m = 0$.*

PROOF. See Appendix.

LEMMA 2. *Given a Q that satisfies Eq.* (10) *and* $\Pi = 1$, *for any $0 < M < M_b^*$ and $x < x_L$ there exists a solution $(V_j, N_j, c^*, \mu, \pi)$ satisfying $\Pi = \pi$, $c^* > 0$, Eqs.* (11) *to* (13), (4), *and* (5), *and $N_m = M$ if $r < r_L$.*

PROOF. See Appendix.

Proposition 2 can now be established. Lemma 1 establishes the existence of two nondegenerate solutions to the bargaining problem given a c^* and μ for $r < r^{\max}$. Lemma 2 establishes the existence of a unique nondegenerate solution pair (c^*, μ) given either solution unconstrained or constrained q^m for $r < r_L$. Either $r_L \leq r^{\max}$, in which case both lemmas hold for the former, or vice versa. Defining r' as the minimum of the two, both lemmas hold for $r < r'$. Define $c^*(0)$ as the level of c^* in the nonmonetary equilibrium. Given an $M \leq M_b^*$, the complete problem is a composite mapping from (c^*, μ) to (c^*, μ) space where $c^* \in [c^*(0), c_h^*]$ and $\mu \in [\mu_1, \mu_b^*]$. This is so because $T(\cdot)$ maps points in (c^*, μ) space into either q, while the SSC and the cutoff rule uniquely determine (c^*, μ) pairs given a q. Since both mappings are continuous, the composite map is continuous on a compact and convex subset of R^2. Application of Kakutani's fixed-point theorem guarantees the existence of a fixed point in either case.

3.2. *Money and Output*

COROLLARY 1. *For any $M \in [0, M_b^*]$, the unconstrained monetary equilibrium is such that $c^*(M,1) > c^*(0)$ if and only if $r < r'$ and $x < x_L$, while the constrained monetary equilibrium is such that $c^*(M,1) < c^*(0)$ for all $M > 0$.*

PROOF. It is immediate from the proof of Lemma 2 that for $M \in [0, M_b^*]$, the SSC intersects the cutoff rule in the range where its slope is positive (Fig. 1). Since the cutoff rule rises above its level in the nonmonetary economy, $c^*(0)$, in this range it follows that in the monetary equilibrium $c^*(M,1) > c^*(0)$. Since the constrained equilibrium involves the commodity trader being strictly indifferent between trading

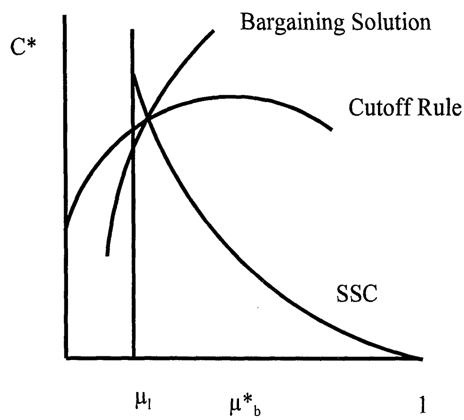

FIGURE 1

THE UNCONSTRAINED MONETARY EQUILIBRIUM.

with money or through barter, the arguments of Lemma 2's proof imply that $c^*(M,1)$ must always fall below the nonmonetary economy for $M > 0$. □

The preceding propositions establish the existence of equilibria with fully valued money and show that if the amount of money traders (or buyers) in the economy is not so large as to crowd out too many of the commodity traders (or sellers), then the equilibrium with the highest value of money involves higher production on the extensive margin as compared with the equilibrium with a lower value. This result emerges because the higher value of money makes trade more profitable, which induces producers to be willing to incur a higher cost (c^*) in order to get into the trading sector sooner. The resulting steady state can be thought of as a higher-activity equilibrium because there are more traders actively seeking to trade.

The two equilibria also differ in the amount of production on the intensive margin because more is produced and exchanged per transaction in the unconstrained (higher-value) equilibrium as compared with the constrained equilibrium, as shown by Shi (1995). This occurs because when the value of having money is high, the money trader (or buyer) has a bigger threat point and is able to squeeze out more output from the seller in return for a unit of money, which in turn justifies the high value of money. In my model, with changing levels of traders, this effect is compounded by the fact that there are more sellers, which results in a thicker market and more frequent meetings between buyers and sellers, so that total output per unit of time is higher both because more is produced in each transaction and because more transactions occur each period. Moreover, the increased value of trading due to more traders implies that q will rise even more than it would when participation was constant, since q responds positively to c^*, as will be seen below.

Notice that we have obtained results very similar to those found in Diamond (1984). The model displays the existence of two pure monetary steady states that differ in the number of traders. Moreover, these equilibria are obtained without assuming increasing returns in matching and without imposing the constraint that money be used as a medium of exchange.

An issue addressed in the literature is the effect of changing the number of money traders. Trejos and Wright (1993) consider the effect of changing M on prices, velocity, welfare, and aggregate output in a model with a fixed number of traders. Very similar results can be obtained for my structure, some of which are discussed in Johri (1994). Here I focus on the effects on production on both the intensive and extensive margin.

PROPOSITION 3. *There exists an r^* such that if $M < M_b^*$, then increasing M in the unconstrained monetary equilibrium leads to an unambiguous increase in c^* and q^m as long as $r < r^*$. If $r > r^*$, q^m falls and μ increases, while the effect on c^* is ambiguous. For $M > M_b^*$, increases in M lead to a falling c^* and q^m. In the constrained equilibrium, increases in M involve falling c^* and q^m for all parameter values.*

PROOF. See Appendix.

The preceding discussion points to an interesting feature of the model. It is possible for increases in M to result in a drop in prices (an increase in q) for very low values of M. It is interesting to note that a similar result was found in such models as that of Trejos and Wright (1995) in which the number of traders was fixed. In these models, when M increases, the probability that a seller will be able to find a buyer improves, which is just the lubrication effect of money. At the same time, however, since no increase in activity due to a higher c^* can occur, the probability of a buyer finding a seller decreases. These two effects should combine to lower the threat point of the buyer, resulting in a lower q as M increases. It is hard to understand why the declining threat point of the buyer does not result in monotonically higher prices in the Trejos and Wright model as M increases. Unlike those models, in the present model the reason q may rise is intuitively clear. The increase in c^* results in an increase in the number of traders, and this raises the probability that a money trader will be able to find commodity traders (sellers), and thus he or she is able to obtain a better price. Eventually, of course, the former effects dominate, and prices start to rise.

Further light is shed on this issue by considering a variant of the bargaining structure in which players are allowed to search while bargaining. In Trejos and Wright (1993), the search while bargaining model yields *monotonically decreasing q* as M increases. I will show that this is not the case in my model.[10] Allowing search while bargaining complicates the model considerably because now the threat points in the Nash product are the continuation values for the commodity trader and the money trader, whereas earlier they were zero. For simplicity, I modify the model to rule out the possibility of barter. With this simplification it can be shown that there is a range of M for which *q is increasing in M*. This result is summarized in the following proposition.

[10] I thank Randy Wright for encouraging me to explore this question.

PROPOSITION 4. *In the model without barter but with search while bargaining, there is a unique unconstrained monetary equilibrium with the property that there exists a* r^{**} *such that if* $M < M^* < \frac{1}{2}$, *then increasing* M *leads to an unambiguous increase in* c^* *and* q^m *as long as* $r < r^{**}$. *If* $r > r^{**}$, q^m *falls and* μ *increases, while the effect on* c^* *is ambiguous. For* $M > M^*$, *increases in* M *lead to a falling* c^* *and* q^m.

PROOF. The uniqueness of an unconstrained monetary equilibrium is a straightforward generalization of Proposition 2 in Trejos and Wright (1993). Replicating the arguments of Proposition 3 yields the result. See Appendix.

Proposition 4 makes clear the significance of movements on the extensive margin for movements on the intensive margin (changes in q) in response to increasing M. Where c^* cannot change, the only effect of an increase in M is to raise the threat point of commodity traders, as was discussed earlier. In my model, along with this effect, when c^* rises with M, there are also more commodity traders in the market, which in itself increases the bargaining power of money traders. For a certain range, the latter effect may dominate so that we see q rising with M.

4. CONCLUSION

Since agents accept unbacked fiat money as payment for goods and services only if they believe others will accept it in turn, monetary economies display multiplicity of equilibria in which the value of money varies. It is shown that two Diamond-style monetary equilibria emerge in which the number of agents in the trading sector varies endogenously. These equilibria are obtained without using increasing returns in the matching technology. In one of the equilibria, agents are willing to incur a higher fixed cost of production in order to reduce the time spent searching for production opportunities because of the higher return to trade. This higher-activity equilibrium is associated with a higher value of money as compared with the low-activity equilibrium as well as higher production on both the intensive and extensive margin.

Despite variation in the value of money, money is fully accepted in both equilibria; that is, no seller refuses money as a means of payment. It is worth pointing out that while many of these results also appear in the fixed-price version of this model, the multiplicity of equilibria in that model are associated with variation in the acceptability of money. The introduction of endogenous prices into this framework, then, has important consequences because multiplicity is obtained without variation in the acceptability of money as a means of payment. An important aspect of production is inventory accumulation, but here production is to meet demand. An extension would be to study a model in which production occurs prior to bargaining.

APPENDIX

PROOF OF PROPOSITION 1. Setting $\Pi = 0$ in Eq. (12) implies $V_m = V_p$, using $V_c - V_p = c^*$, and since $N_M = 0$, $\Pi = 0$ implies that $\mu = 0$. Then the preceding dynamic program reduces to an expression in c^* as

$$(r+\beta x^2)c^* + \alpha\int_0^{c^*}(c^*-c)\,dG = \beta x^2[u(q^*)-f(q^*)] \tag{A.1}$$

The RHS of Eq. (A.1) is positive and constant as c^* changes. The LHS is zero if $c^*=0$ and increases with c^*. Thus there must exist a positive c^* that satisfies Eq. (A.1).
We refer to this as both $c^*(0,\Pi)$ and $c^*(M,0)$ because in equilibrium all money is disposed. Other variables are determined from c^*.

PROOF OF LEMMA 1. The following expression solves the unconstrained bargaining problem:

$$T(q) = u'(q)[V_m - f(q)] - f'(q)[u(q)+V_p] = 0 \tag{A.2}$$

where V_p is constant as q changes. Substituting for V_m evaluated at $Q=q^m$, the following expression is obtained

$$T(q) = u'(q)\left\{\frac{\beta(1-\mu)x}{r+\beta(1-\mu)x}[u(q)+V_p] - f(q)\right\} \\ - f'(q)[u(q)+V_p] = 0 \tag{A.3}$$

Note that $T(0)>0$ and $T(q^*)<0$. By continuity, therefore, there exists a $q \in (0,q^*)$ such that $T(q)=0$. In order to see that this equilibrium is unique, note that while $T'(0)$ may be positive depending on the parameters of the utility function, eventually as q increases, $T'(\cdot)$ becomes negative and remains so for larger values of q. This is so because u' is decreasing while f' is increasing.

It remains to be shown that the individual rationality constraints are satisfied strictly at $Q=q^m$. Let $w(q)=u(q)-f(q)$ and $w^*=u(q^*)-f(q^*)$. Let $I(q)$ refer to the IR constraint for the commodity trader; then $I(q)=V_m - V_c - f(q)$. Substituting for $V_m - V_c$, it can be shown that the constraint is satisfied for all $q \in [q_1, q_2]$ where $q_1 < q^{\max} < q_2$. Note that the unconstrained solution $q^m < q^{\max} < q_2$; thus the upper bound on q is not violated. Also, as r increases above zero, the solution q^m to $T(q)=0$ decreases monotonically to zero while q_1 increases. Hence there is an $r^{\max}>0$ (which depends on the state of the economy c^*, μ) such that $T(q_1)=0$. For $r<r^{\max}$, the preceding implies that the unconstrained solution to $T(q)=0$ satisfies $q_1 < q^{\max} < q_2$.

To find the constrained equilibrium, we have to check $q^m(Q)$, where $Q=q_1, q_2$. If $q^m(Q)<Q$, then Q cannot be an equilibrium because the commodity trader would prefer to deviate to q^m. If the opposite is true, then it is an equilibrium because neither party wishes to deviate. First, check $Q=q_2$. At $Q=q^{\max}$, $T(q)<0$. Since $q^{\max}<q_2$, by continuity, $T(q_2)<0$, so q^m is less than $Q=q_2$, and it is not an equilibrium. Now consider $Q=q_1$ as a candidate equilibrium. We know from the preceding analysis that $T(q_1)>0$ evaluated at $Q=q_1$; therefore, it must be that the q that satisfies $T(q, Q=q_1)=0$ must be greater than q_1. Therefore, it is a constrained equilibrium. This establishes the claim.

PROOF OF LEMMA 2. First consider the unconstrained solution. By assumption, Q is such that $V_m - V_c - f(Q) > 0$. Thus the best response to $\Pi = 1$ is $\pi = 1$. Substituting Eq. (4) into Eqs. (11) to (13) and subtracting Eq. (11) from Eq. (12) and Eq. (13) from Eq. (11) and solving for $V_m - V_c - f(q)$ after substituting $\Pi = \pi = 1$ into the expressions, the dynamic programming problem can be reduced to the cutoff rule, which implicitly defines c^* in terms of μ given a q and $\Pi = 1$:

$$\text{(A.4)} \qquad rc^* = \beta(1-\mu)x^2S^* - \alpha\int_0^{c^*}(c^*-c)\,dG + \frac{\beta^2(\mu-\mu^2)x^2}{r+\beta x}[S(q) - xS^*] - \frac{r\beta\mu x}{r+\beta x}f(q)$$

where $S(q) = w(q) - c^*$ and $S(q^*) = w^* - c^*$, while the SSC is the same as before. The cutoff rule and the SSC together give the equilibrium value of μ and c^* given M and q. Consider the cutoff rule first. It implies that $0 < c^* < c_H^*$ at $\mu = 0$, which corresponds to the nonmonetary economy and $c^* = 0$ when $\mu = 1$. A necessary condition for the cutoff rule to be upward sloping is that the IR constraint of the goods trader hold with strict inequality. However, this condition is not sufficient. A necessary and sufficient condition is that the cutoff rule have a positive slope evaluated at $\mu = 0$. It can be shown that the sign of the derivative depends on

$$\text{(A.5)} \qquad -\beta x^2[S_0^*] - \frac{r\beta x}{r+\beta x}f(q) + \frac{\beta^2x^2}{r+\beta x}[S_0(q) - xS_0^*]$$

where S_0^* refers to the fact that S^* is being evaluated at $c^*(M,0)$. As $r \to 0$, this expression reduces to

$$\text{(A.6)} \qquad S_0(q) > 2xS_0^*$$

Clearly, for a small enough x, this inequality will be satisfied. Let x_L be the x for which Eq. (A.6) holds with equality. Thus, for $x < x_L$, there exist $r > 0$ such that Eq. (A.5) is positive. Let r_L refer to the highest r for which Eq. (A.5) holds for a positive x. Then the cutoff rule has a positive slope at $\mu = 0$ if and only if $r \le r_L$. If the cutoff rule is increasing at $\mu = 0$ and eventually falls to zero, there must be a μ_b^* at which the cutoff rule achieves its maximum height. Clearly, then, the cutoff rule is concave under the specified conditions in this region.

Now consider the SSC. At $\mu = 1$, $c^* = 0$, and as μ declines toward $\mu > m, c^*$ rises, thus taking on all possible values between 0 and c_H^*. The SSC is monotonic and convex. Increases in M shift the SSC to the right while leaving the cutoff rule unaffected. Thus changing M results in solutions to c^* and μ being traced out along the cutoff rule (see Fig. 1). Let M_b^* be the value of M that results in an intersection of the two curves at μ_b^*. Then for any M such that $0 < M < M_b^*$ there is a solution to the cutoff rule and the SSC. Given $N_m = M$, since μ is determined, so are N_c and N_p. V_p is determined from the value of c^* from Eq. (13) and V_m from the value of V_p from Eq. (12). Similarly, V_c is determined from Eq. (11).

Consider now the case where Q is a constrained solution to the bargaining problem. By definition, then, $V_M - V_c - f(Q) = 0$; therefore, the cutoff rule must be monotonically decreasing. From the arguments presented above, there must be an intersection of the cutoff rule and the SSC, thus determining uniquely c^* and μ as well as the V_js and N_js.

PROOF OF PROPOSITION 3. I offer the sketch of a proof; details are available on request. Note first that changing M will shift the SSC to the right, and therefore, both c^* and μ will increase along the cutoff rule. This will influence the bargaining solution q^m. Totally differentiating this expression yields

$$\text{(A.7)} \quad D^*dq = -\frac{\alpha}{r}(u' - f')G(c^*)\,dc^* + \frac{r\beta x}{[r + \beta(1-\mu)x]^2}(u + V_p)\,d\mu$$

where

$$\text{(A.8)} \quad D = \{u''[\phi(u + V_p) - f] - f''(u + V_p) + u'(u'\phi - 2f')\}\,dq$$

From Eq. (A.7) it can be shown that increases in c^* lead to increases in q^m, while increases in μ have the opposite effect. The bargaining solution curve can be defined as the locus of points in (c^*, μ) space such that q^m remains unchanged. This is obtained by setting $dq = 0$ in Eq. (A.7):

$$\text{(A.9)} \quad \frac{dc^*}{d\mu} = \frac{r\beta x}{[r + \beta(1-\mu)x]^2}[u(q_b) + V_p]\left[\frac{1}{(\alpha/r)G(c^*)(u' - f')}\right]$$

A monetary equilibrium may then be represented by the intersection of the cutoff rule, the SSC, and the bargaining solution in (c^*, μ) space as in Figure 1. Since the bargaining solution is positively sloped, the key to determining if c^* rises or falls as M increases is the relative slope of the cutoff rule and the bargaining solution at the equilibrium point. If the cutoff rule has a higher slope, then the effect of a higher c^* dominates the increase in μ so that a higher q and c^* result from an increase in M. It can be shown that there exists a r^* such that for all $r < r^*$ this condition is satisfied.

PROOF OF PROPOSITION 4. With search while bargaining, the $T(q) = 0$ that solves for the bargaining solution has a new form:

$$\text{(A.10)} \quad T(q) = u'(q)\left\{\frac{\beta(1-\mu)x}{r + \beta(1-\mu)x}[u(q) + V_p] - f(q)\right\} - f'(q)\left\{\frac{r + \beta\mu x}{r + \beta(1-\mu)x}[u(q) + V_p]\right\}$$

From the usual arguments (u' decreasing f' increasing, etc.) it is easy to see uniqueness and that $T'(q) < 0$ at the solution point. To save notation, set $\beta x > 0$, as you did in Trejos and Wright. In Proposition 3, my basic strategy was to compare the slope of the bargaining solution curve (drawn in c^*, μ space) with the slope of the cutoff rule. For a low enough r, the cutoff rule has higher slope, so we get q increasing in M along with c^*. Use the same approach here. Totally differentiating $T(q)$ yields an equation $D^* dq = A^* dc^* + B^* d\mu$.

$D = T'(q)$ is evaluated at solution point q^m, which we know is negative.

$$A = \left[u'(1-\mu) - f'(r+\mu)\right]\frac{\alpha G(c^*)}{r} \tag{A.11}$$

$$B = u'\left(u + V_p - f\right) + f'\left(u + V_p\right)$$

Now at any q^m, $T(q^m) = 0 \Rightarrow u'(1-\mu)/(r+\mu) > f'$, so $A > 0$ and $B > 0 \Rightarrow q$ rises with c^* and falls with μ. Setting $dq = 0$ gives expression for the bargaining solution curve as before. Clearly, $dc^*/d\mu > 0$ evaluated at an equilibrium point, but note that $dc^*/d\mu > 0$ as $r > 0$. The cutoff rule has the same properties as before, so by the same arguments, there is a $r^{**} > 0$ such that for $r < r^{**}$ the cutoff rule has higher slope. All the other arguments follow Proposition 3 from this point on.

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