The Payments System, Liquidity, and Rediscounting

By SCOTT FREEMAN *

In an economy where fiat money serves both as a medium of exchange and the means by which debts are cleared, it is shown that nonoptimal equilibria of constrained liquidity may arise. Optimality may be restored by temporary expansions of the monetary base (e.g., an active central-bank "discount window"). (*JEL* E52, E58, E43)

This paper models the payments system, the arrangement that arises to facilitate the repayment of debt. In particular, the model captures the following basic features of the payments system: (i) people make some purchases with debt; (ii) debts are repaid with fiat money; and (iii) there is an active market in second-hand debt (i.e., at least some debt is cleared through third parties).

Interest in the payments system is heightened by observations that suggest it does not necessarily operate flawlessly. The payments system has seemed constrained by a lack of liquidity, or by expensive access to liquidity. For example, Bray Hammond (1957 p. 11) reports that the settlers of British North America complained constantly that they lacked the currency (but not the wealth) to pay their debts and taxes: "... there were lawsuits and writs against 'good honest housekeepers' who had property enough and the will to pay their debts but could not raise the money—the reason being that there was none."

In a later example, a strong demand for currency, indicated by seasonally high short-term interest rates, was common at harvest time in the United States before 1914. In the words of Milton Friedman and Anna J. Schwartz (1963 p. 292), "That seasonal movement was very much in the minds of the founders of the System and was an important source of their belief in the need for an 'elastic' currency." This seasonal variation diminished notably with the establishment of the Federal Reserve System. Friedman and Schwartz continue, "The seasonal pattern in currency outside the Treasury was widened, the seasonal pattern in call money rates narrowed. The System was almost entirely successful in the stated objective of eliminating seasonal strain."[1] Before the establishment of the Federal Reserve System, therefore, agents seemed often constrained by a lack of liquidity, or by expensive access to liquidity, constraints that were noticeably loosened by a change in the central banking system. The announced intentions of the Federal Reserve Act "... to furnish an elastic currency, to afford means of rediscounting commercial paper, ..." are consistent with the spirit of the Real Bills Doctrine, which advocated allowing the stock of money to fluctuate to meet the needs of trade.

The goals of this paper therefore go beyond a basic model of the payments system to a model in which agents may be occasionally constrained by a lack of liquidity. The model will be asked to suggest the conditions under which these episodes may occur and to explain why they may diminish in severity as the result of a central bank's interventions. The model, with its microeconomic foundations, will be used to evaluate the welfare implications of elastic currency policies like those suggested by the Real Bills Doctrine. The model will also

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[1] Friedman and Schwartz (1963 pp. 292–94) summarize evidence from several sources on seasonal movements before and after the establishment of the Federal Reserve.

be used to suggest measures needed to maintain price-level stability under an elastic currency regime, a concern of the critics of the Real Bills Doctrine.

Most models of money based on microeconomic foundations have featured money's role as a medium of exchange. The role of money as a medium of exchange has been illustrated in overlapping-generations models following Paul A. Samuelson (1958) and spatial-separation models following Robert M. Townsend (1980). In these models, money is used to acquire the consumption goods desired through a pair of spot market trades: an agent trades goods for money and then money for the goods he desires.

However, goods are also often acquired in exchange for debt, in particular for a promise to pay fiat money at a future date. The coexistence of private debt with a demand for fiat money was developed in models by Thomas J. Sargent and Neil Wallace (1982), Toshihide Mitsui and Shinichi Watanabe (1989), Townsend (1989), and Stacey L. Schreft (1992), among others, but in these models debts are repaid effortlessly and without any special need of fiat money. Dan Bernhardt (1989) offers a model in which the repayment of debt is more problematic so that fiat money purchases usefully supplement the bilateral exchange of debt, but fiat money is still not essential to the repayment of debt. Fiat money does play an essential role in the bilateral repayment of debt in Freeman and Guido Tabellini (1992). Freeman (1996) extends that model to examine the role of private banks in the central clearing of debt, with a particular examination of the potential for an inflationary overissue of private bank notes.

The interaction of bank liquidity problems and panics is examined in Bruce Champ et al. (1996) (see also James McAndrews and William Roberds [1995]). Their model features coexisting demands for bank deposits and currency and seasonal fluctuations in the demand for currency. The liquidity problems in their model result from random shocks to the demand for liquid currency when it is already stretched by a high seasonal demand. These shocks may set off panics when the exhaustion of bank reserves is foreseen. A currency premium may result then from a sufficiently severe panic. The model is motivated in particular by the financial panics that took place under the National Banking Act, generally starting in agricultural areas during the harvest, when currency demand was the greatest.

The model presented here builds on Freeman (1996) to consider a payments system with the features outlined above: purchases made with debt, debt settled with a final payment of fiat money, and an active essential market for the resale of bilateral debt. Because agents are spatially separated, private debt is incurred between two parties and can only be redeemed with fiat currency at a central clearing area. When the arrival of creditors and debtors at the central clearing area is not perfectly synchronized, there emerges a resale market in which debt is sold to third parties by those in immediate need of currency. If the amount of currency available at the central clearing area is insufficient to clear debts at their par value, the equilibrium will be called liquidity-constrained.

With its essential, active resale market for debt and the possibility of liquidity-constrained equilibria, the model offers a guide to optimal central-bank rediscounting policy. It is shown that an equilibrium is not optimal if risk-free debt sells at a discount. These liquidity-constrained equilibria, with their distortionary impact on credit markets, can be eliminated by policies that permit an elastic currency: the monetary authority must temporarily supply enough currency to clear all debts at par. This temporary injection of fiat money may take the form of a discount window offering central-bank loans equal to the nominal amount of debt presented to it. Once all debts are cleared, the optimal rediscounting policy requires that the central-bank loans be repaid with fiat money, which is then removed from circulation in order to return the fiat-money stock to its initial level, thereby maintaining a constant price level.

I. The Basic Model

A. *The Environment*

A large even number I of outer islands are arranged in pairs around a central island. Each pair contains both of two types of islands,

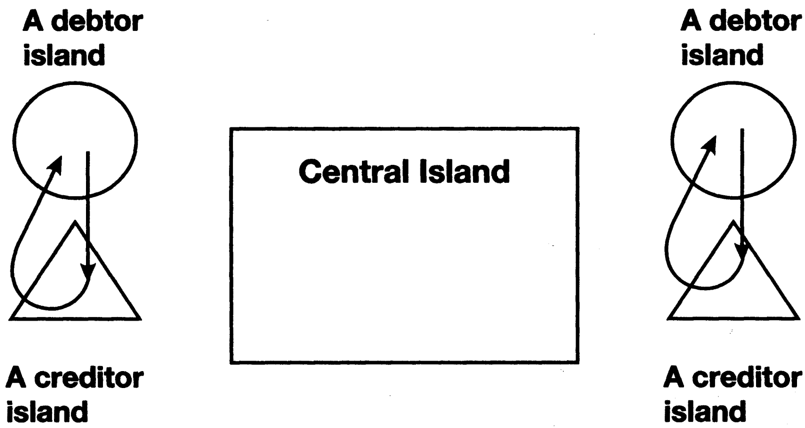

FIGURE 1. PATTERN OF TRAVEL IN THE MODEL: WHEN YOUNG, DEBTORS VISIT CREDITORS (ISSUING DEBT); THEY RETURN (AND SELL GOODS TO OLD CREDITORS FOR FIAT MONEY)

which will be called "creditor" and "debtor" islands (in anticipation of their equilibrium trading behavior). On each island, N two-period-lived agents are born in each period $t \geq 1$. In the first period each island also has N agents (the initial old) who live only in the first period. For simplicity, N is normalized to 1.

Each agent born on a debtor island (each "debtor") is endowed at birth with x units of a nonstorable good specific to his island (and with nothing when old). Agents wish to consume the goods of both debtor and creditor islands when young and nothing when old. The utility of a debtor is given by the function $v(d_{xt}, d_{yt})$, where d_{xt} and d_{yt} respectively represent his consumption of debtor and creditor island goods. At the beginning of the period, young debtors travel to the creditor island with which they are paired, where they may consume creditor island goods. They return later in the period.

Each agent born on a creditor island ("creditor") is endowed at birth with y units of a nonstorable good specific to his island (and with nothing when old). He wishes to consume the good of his home island when young (c_{yt}) and of debtor islands when old ($c_{x,t+1}$). No other consumption is desired. The utility of a creditor is given by the function $u(c_{yt}, c_{x,t+1})$. Both utility functions $u(\cdot, \cdot)$ and $v(\cdot, \cdot)$ are additively separable,[2] strictly increasing and concave in each argument, continuous, and continuously differentiable, with indifference curves that do not cross the axes.

When old, agents from each island travel to the central island. Old creditors then continue on to a randomly selected debtor island where they may trade with young debtors. The old creditors are evenly divided among debtor islands, each creditor with an equal chance of going to any given debtor island. The actual destination is not known until arrival. They arrive at their final destinations after all travel by the young has been completed. The pattern of travel is illustrated in Figures 1 and 2 (with equilibrium behavior—to be explained later—described in parentheses in the legends).

Arrival at the central island takes place in two stages. In the first, all old creditors and a fraction λ of old debtors arrive ($0 \leq \lambda \leq 1$). At the end of the first stage, a fraction $1 - \alpha$ of the old creditors leave for their final destination, while the rest stay until the end of the second stage. The remaining $1 - \lambda$ debtors

[2] Separability is not essential to any of the model's results but ensures that consumption goods are normal and adds greatly to mathematical tractability and notational ease.

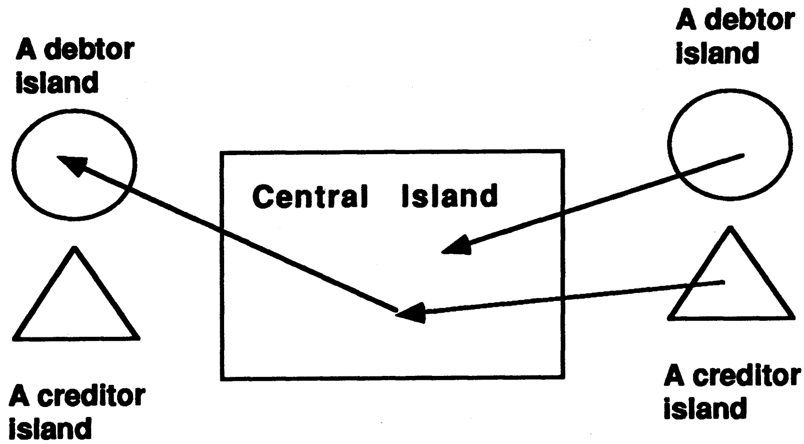

FIGURE 2. PATTERN OF TRAVEL IN THE MODEL: WHEN OLD, ALL AGENTS VISIT THE CENTRAL ISLAND (DEBTS ARE REPAID WITH FIAT MONEY); OLD CREDITORS SCATTER TO DEBTOR ISLANDS (WHERE THEY TRADE FIAT MONEY FOR GOODS)

arrive in the second stage. All creditors face the same chances of leaving early or late, and all debtors face the same chances of arriving early or late. Each learns his arrival or departure time as soon as he turns old but not before. The sequence of travel can be summarized as shown in Table 1.

All agents are able to issue unfalsifiable IOU's that identify the issuer. A legal authority exists on the central island that can enforce agreements between parties currently on that island. No such authority exists to enforce agreements at agents' final destinations.

There exists on the central island a monetary authority able to issue fiat money, which is noncounterfeitable, unbacked, intrinsically useless, and costlessly exchanged. This authority issues an initial stock of M dollars to each initial old creditor.

B. *Trading Patterns*

To consume when old, creditors must bring something of value to the debtor islands. Fiat money will be accepted by young debtors if it helps them to acquire the goods they desire. If it is accepted in equilibrium, fiat money serves as a "medium of exchange."

The young debtors wish to consume goods from creditor islands but own no goods valued by the young creditors that can be offered in immediate direct exchange. Nor do the debtors have any money at the time of this visit. The only thing a debtor can offer creditors is a promise to pay a sum of money in the next period on the central island. The young debtor will acquire this money by selling some of his endowment to those bringing money to the island later in the period. See Figures 1 and 2 for illustrations of the trading pattern.

In this monetary equilibrium, both fiat money and debt are valued. Money serves not only as a medium of exchange, but also as the means by which debts are cleared. Money is essential in this model for the clearing of debts and the existence of a credit market; without valued money, equilibrium debt equals zero.

If all debtors and creditors arrive on the central island at the same time ($\lambda = 1$), the clearing of debts can take place without difficulty because the dollars promised to creditors equal the dollars brought in by debtors. Suppose instead that the arrivals of creditors and debtors are not so conveniently synchronized: let $\lambda < 1$. Then, in the first stage, less money arrives from debtors than is needed to pay off all creditors. Those creditors leaving early will offer to sell their debt to those leaving later, who will be on the central island when the remaining borrowers arrive to redeem their debt in stage 2. The nominal amount of debt that can

TABLE 1—THE SEQUENCE OF TRAVEL WITHIN A PERIOD

Step	Young	Old
1	Young debtors visit neighboring islands	All old creditors and λ old debtors visit the central island
2		$1-\alpha$ old creditors leave the central island
3		$1-\lambda$ old debtors visit the central island
4	Young debtors return from the neighboring islands	Remaining (α) old creditors leave the central island
5	Young debtors may trade with arriving old creditors	All old creditors arrive at debtor islands

be redeemed in this resale market is limited by the size of the cash balances on the central island in the first stage. If this is insufficient to cover the shortfall, creditors will be forced to sell their debt at a discount.

C. *Equilibrium*

Let p_{xt} and p_{yt} respectively represent the dollar price of a good on a debtor island and a creditor island at t. The budget constraints of a debtor born at t may be written in nominal terms as follows:

$$(1) \qquad xp_{xt} = d_{xt}p_{xt} + m_t$$

$$(2) \qquad m_t = h_t$$

$$(3) \qquad p_{yt}d_{yt} = h_t$$

where m_t denotes the debtor's nominal demand for currency, and h_t denotes the nominal value at t of his indebtedness. Combined, these budget constraints yield

$$(4) \qquad x = d_{xt} + \frac{p_{yt}}{p_{xt}} d_{yt}.$$

The resulting first-order condition for utility maximization is

$$(5) \qquad \frac{v_x}{v_y} = \frac{p_{xt}}{p_{yt}}.$$

Let ℓ_t represent the nominal value at t of a creditor's loans to debtors. Let q_t represent the par value of nominal debt purchased by those leaving late. Let $\rho_{t+1} \leq 1$ represent the discounted nominal value of one dollar of that debt at time $t+1$. One can interpret $1/\rho_{t+1}$ as the short-run (gross) interest rate. Consumption when old of those leaving late and its marginal utility will be marked with a star (i.e., $c^*_{x,t+1}$).

The budget constraints of a creditor born at t may now be written in nominal terms as follows:

$$(6) \qquad yp_{yt} = c_{yt}p_{yt} + \ell_t \qquad \text{(when young)}$$

$$(7) \qquad \rho_{t+1}(1-\lambda)\ell_t + \lambda\ell_t = c_{x,t+1}p_{x,t+1} \qquad \text{(for the old leaving early)}$$

$$(8) \qquad \ell_t + (1-\rho_{t+1})q_t = c^*_{x,t+1}p_{x,t+1} \qquad \text{(for the old leaving late).}$$

By rate of return equality, the net nominal interest rate of debt will equal zero in equilibrium, the rate of return of fiat money. The budget constraints have already taken this into account. Creditors leaving late also face the liquidity constraint

$$(9) \qquad \lambda\ell_t - \rho_{t+1}q_t \geq 0$$

which states that the nominal value of debt purchased by a late-leaving creditor, $\rho_{t+1}q_t$, is limited by the cash balances available to a creditor at the end of the first stage, $\lambda\ell_t$.

Letting u_x and u_y denote respectively the derivatives of $u(\cdot, \cdot)$ with respect to consumption of good x and y,[3] and letting μ denote the Lagrangian coefficient for the liquidity constraint (9), the resulting first-order maximization conditions with respect to ℓ_t and q_t can be written as

$$(10)\quad -\frac{u_y}{p_{yt}} + (1-\alpha)[\rho_{t+1}(1-\lambda)+\lambda] \times \frac{u_x}{p_{x,t+1}} + \alpha\frac{u_x^*}{p_{x,t+1}} + \mu\lambda = 0$$

and

$$(11)\quad (1-\rho_{t+1})\alpha\frac{u_x^*}{p_{x,t+1}} - \mu\rho_{t+1} = 0.$$

These can be combined as

$$(12)\quad -\frac{u_y}{p_{yt}} + (1-\alpha)[\rho_{t+1}(1-\lambda)+\lambda] \times \frac{u_x}{p_{x,t+1}} + \alpha\left[1+\lambda\left(\frac{1}{\rho_{t+1}}-1\right)\right] \times \frac{u_x^*}{p_{x,t+1}} = 0.$$

If the liquidity constraint (9) is not binding ($\mu = 0$), then IOU's are not discounted when sold [$\rho_{t+1} = 1$, from (11)]. In this case, $c_{x,t+1} = c^*_{x,t+1}$, and (12) simplifies to

$$(13)\quad \frac{u_y}{p_{yt}} = \frac{u_x}{p_{x,t+1}}.$$

If the liquidity constraint (9) is binding ($\mu > 0$), IOU's are discounted when sold [$\rho_{t+1} < 1$ from (11)]. In this case, $c_{x,t+1} < c^*_{x,t+1}$ and $u_x > u_x^*$. In this way a liquidity-constrained equilibrium increases the risk faced by creditors, by making late-leavers better off than early-leavers.[4]

To find ρ_{t+1}, note that the clearing of the resale market for loans requires that

$$(14)\quad \alpha q_t = (1-\alpha)(1-\lambda)\ell_t.$$

If the liquidity constraint (9) is binding, this implies that

$$(15)\quad \rho_{t+1} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda}{1-\lambda}\right)$$

which implies that $\rho_{t+1} < 1$ if and only if $\alpha + \lambda < 1$; that is, the number of early-leaving creditors $(1 - \alpha)$ exceeds the number of early-arriving debtors (λ). Notice that the existence of a liquidity-constrained equilibrium depends uniquely on α and λ. High values of α and λ imply a high degree of overlap in central-island meetings and thus a low need for additional fiat money to clear loan markets. The parameter λ measures the fraction of debt that will be redeemed with fiat money that arrives early enough to help pay off the $1 - \alpha$ creditors leaving early. In this way λ measures the early supply of fiat money on the central island, and $1 - \alpha$ measures the need for fiat money in the early clearing of debt. The parameter α measures the fraction of creditors who do not need to be paid off immediately (before all debtors have arrived), which can be interpreted as the size of the banking sector relative to the total population of creditors, the fraction of creditors free to purchase the debts of others. As an example one might expect that when long-distance communications were difficult, credit markets within a city would have higher values of λ and α than would isolated rural credit markets.

Equilibrium also requires that the markets for goods, loans, and currency clear; therefore, a lack of liquidity may have general equilibrium effects in these markets. The conditions for the clearing of the markets for the goods of creditor and debtor islands are, respectively,

$$(16)\quad y = d_{yt} + c_{yt}$$

$$(17)\quad x = d_{xt} + c_{xt}.$$

[3] Recall the simplifying assumption that $u(\cdot, \cdot)$ is additively separable.

[4] The same temporary shortage of fiat money that prevents early-leavers from receiving the full value of their debt prevents insurance arrangements that would have *ex post* late-leavers compensate early-leavers.

The clearing of the initial markets for loans requires

$$(18)\qquad \ell_t = h_t$$

and the clearing of the market of goods for currency requires

$$(19)\qquad m_t = M$$

which using (1) implies

$$(20)\qquad p_{xt} = \frac{M}{x - d_{xt}}.$$

Note that the nominal price of x (essentially the price level) adjusts to equate the demand for currency to its supply (20).

If $\alpha + \lambda < 1$ so that $\rho_{t+1} < 1$, one can use (15) to eliminate ρ_{t+1} and thus simplify the creditor's budget constraints (7) and (8) and first-order condition (12), respectively, to

$$(21)\qquad \frac{\lambda}{1-\alpha}\ell_t = c_{x,t+1}p_{x,t+1} \quad \text{(old leaving early)}$$

$$(22)\qquad \frac{1-\lambda}{\alpha}\ell_t = c^*_{x,t+1}p_{x,t+1} \quad \text{(old leaving late)}$$

$$(23)\qquad -u_y\left(y - \frac{\ell_t}{p_{yt}}\right)\frac{p_{x,t+1}}{p_{yt}} + \lambda u_x\left[\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{\ell_t}{p_{x,t+1}}\right)\right] + (1-\lambda)u^*_x\left[\left(\frac{1-\lambda}{\alpha}\right)\left(\frac{\ell_t}{p_{x,t+1}}\right)\right] = 0.$$

A simple representation of a liquidity-constrained stationary equilibrium with $h_t = h$, $\ell_t = \ell$, $p_{xt} = p_x$, and $p_{yt} = p_y$ for all t is available by recasting the equilibrium conditions (5) and (23) in terms of the real supply of credit $L \equiv \ell/p_y$, the real demand for credit $H \equiv h/p_y$, and the relative price $P \equiv p_x/p_y$:

$$(24)\qquad -v_x\left(x - \frac{H}{P}\right) + Pv_y(H) = 0$$

$$(25)\qquad -u_y(y-L)P + \lambda u_x\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{L}{P}\right) + (1-\lambda)u^*_x\left(\frac{1-\lambda}{\alpha}\right)\left(\frac{L}{P}\right) = 0.$$

The first-order conditions (24) and (25) allow H and L to be graphed as implicit functions of P, as in Figure 3. Application of the implicit-function theorem to (24) and (25) will verify that H is always increasing in P, while L is decreasing in P if relative risk aversion (as measured by $-c_x u_{xx}/u_x$) is less than 1. The intersection of the two satisfies the market-clearing condition $H = L$. Notice that because the interest rate is fixed by the rate of return to fiat money, it is the relative price of goods that adjusts to clear the market for loans. (This occurs because loans are uniquely used to purchase the goods of another island.)

Application of the implicit-function theorem to (25) also reveals that

$$(26)\qquad \frac{\partial L}{\partial \lambda} = \frac{\left(1 + \frac{c^*_x u^*_{xx}}{u^*_x}\right)u^*_x - \left(1 + \frac{c_x u_{xx}}{u_x}\right)u_x}{Pu_{yy} + \left(\frac{\lambda^2}{1-\alpha}\right)\left(\frac{1}{P}\right)u_{xx} + \frac{(1-\lambda)^2}{\alpha P}u^*_{xx}}.$$

In the particular case of constant relative risk aversion ($-c_x u_{xx}/u_x = \sigma$), one finds that $\partial L/\partial\lambda$ agrees in sign with $(u_x - u^*_x)(1-\sigma)$. Because $u_x < u^*_x$, one finds that $\partial L/\partial\lambda$ is positive for $\sigma > 1$ and negative for $\sigma < 1$.

Figure 3 depicts market clearing for a high value of λ (graphed as L) and for a low value of λ (graphed as L^*) assuming constant relative risk aversion with $\sigma < 1$. (The assumption that $\sigma < 1$ ensures that the substitution effect

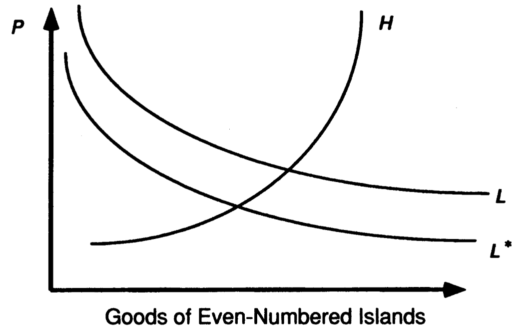

FIGURE 3. THE EFFECT OF A BINDING LIQUIDITY CONSTRAINT ON THE CREDIT MARKET

dominates the wealth effect.) In this case if the liquidity constraint is binding (a low value of λ), creditors reduce their supply of credit, which alters relative prices in their favor, to the detriment of borrowers.

Anticipations of constrained liquidity have real effects in this economy because currency is needed to clear debt.[5] An anticipated equilibrium of constrained liquidity hinders the operation of credit markets by increasing the risk to creditors that they will consume little when old because they must sell their debt at a severe discount. (That the equilibrium of constrained liquidity features increased risk is an implication of the simplifying assumption that all creditors are *ex ante* identical. One might just as easily assume that creditors know at birth whether they will leave late or leave early. In this case a lack of liquidity still benefits late-leavers at the expense of early-leavers. The substitution effect will induce early leavers to reduce their lending and induce late leavers to increase theirs.)

Notice that a lack of liquidity can occur even when fully anticipated; no aggregate shocks are required. The lack of liquidity is not the result of a financial panic, whether based on fundamentals or extrinsic uncertainty. It may be a long-run, even steady-state, phenomenon. (Liquidity problems that result from panics are examined in McAndrews and Roberts [1995] and Champ et al. [1996].)

As the quantity theory predicts, the nominal prices of goods adjust to clear the local markets in which money is exchanged for goods [see equation (20)]. However, the adjustment of nominal prices does not simultaneously clear the resale market for loans [see equation (14)]. It is the resale price of debt (the short-term nominal interest rate) that adjusts to clear the financial market for second-hand debt.

The essence of isolation, however, is not purely geographic. A place is isolated from another if contracts written in one cannot be enforced in the other. Low values of λ and α indicate that only a small fraction of debt can be redeemed directly in the presence of the enforcement authority. This forces a large fraction of debt to be redeemed indirectly, through the resale market for debt. When this fraction becomes large relative to available cash balances, debt sells below par.

II. Policy Implications

A. *Optimality*

To evaluate the welfare properties of the monetary equilibrium, I set as a benchmark the stationary allocations that maximize a weighted average of the expected steady-state utility of those born on debtor and creditor islands:[6]

$$(27)\quad \theta v(d_x, d_y) + (1-\theta) \times \{\alpha u(c_y, c_x^*) + (1-\alpha)u(c_y, c_x)\}$$

with $0 < \theta < 1$ and subject to the feasibility constraints

$$(28)\quad y = d_y + c_y$$

$$(29)\quad x = d_x + (1-\alpha)c_x + \alpha c_x^* .$$

[5] Liquidity constraints that are unanticipated at the time that goods are lent affect only the distribution of wealth among creditors, benefiting late leavers at the expense of early leavers.

[6] This is of course equivalent to maximizing debtor utility subject to a constraint that creditor utility is no less than some constant.

I call these "golden-rule" allocations. The first-order conditions for this golden rule require the equality of marginal rates of substitution:

$$u_x = u_x^* \tag{30}$$

and

$$\frac{u_y}{u_x} = \frac{v_y}{v_x}. \tag{31}$$

The equilibrium satisfies these optimality conditions only in the case of no discount on debt ($\rho_{t+1} = 1$). When debt sells below par ($\rho_{t+1} < 1$), creditors who stay late on the central island benefit at the expense of those who leave early, increasing the risk faced *ex ante* by creditors.[7]

B. *Policy Options*

I have shown that an equilibrium with a binding liquidity constraint is not optimal. What then can be done to relax this constraint? Suppose that the central-island monetary authority (or "central bank") is now authorized to issue and lend fiat money equal to the nominal amount of debt presented by any of the late-leaving creditors (call them "bankers").[8] This central-bank loan must be repaid with fiat money upon the arrival on the central island of the late-arriving borrowers. Let r_{t+1} denote the gross nominal interest rate charged on this central-bank loan within period $t + 1$. The fiat money with which the central-bank loan is repaid is removed from circulation.

Arbitrage will induce bankers to borrow from the central bank until $1/\rho_{t+1} = r_{t+1}$. By choosing r_{t+1}, the central bank can determine the extent to which the liquidity constraint binds. The lower the rate of interest at the discount window ($r_{t+1} \geq 1$), the more currency bankers will borrow and supply to the early-leaving creditors. Setting $r_{t+1} = 1$ eliminates the effects of the liquidity constraint altogether.

This simple "discount window" policy removes the liquidity constraint by allowing bankers as much fiat money as they need to purchase the debt of the early-leaving creditors at par ($\rho_{t+1} = 1$). This allows the economy to reach the golden-rule allocation. The temporary issue of fiat money is not inflationary when central-bank loans must be repaid within the period, because the total stock of fiat money is the same at the end of the period as it was at the beginning. Under this repayment requirement, there is no conflict between the provision of liquidity by the central bank and any price level or inflation targets it may have.

III. Fluctuations in Liquidity

A. *Seasonal Liquidity Demands*

Consider an economy in which λ alternates in value such that $\lambda_t = 1$ (all debtors arrive early) in even periods and $\lambda_t < 1 - \alpha$ (there are fewer debtors than creditors in the first stage) in odd periods. This may be interpreted as an economy with a seasonal agricultural sector that is active in credit markets only at harvest (odd periods). The economy is assumed to have a low value of λ at harvest because the agricultural sector is scattered and separated from the contract enforcement easily available in financial centers. Endowments and all parameters other than λ are assumed not to fluctuate.[9]

If λ affects equilibrium decisions, a fluctuating value of λ must introduce fluctuations into equilibrium prices and consumption patterns. When the liquidity constraint is binding, $u_x < u_x^*$, implying that λ appears nontrivially in the creditors' first-order condition (23). This indicates that the value of λ may affect the real supply of credit, ℓ_t/p_{yt}, and through this, equilibrium prices and consumption

[7] It is easy to imagine other sources of welfare losses. Suppose, for instance, that creditors can choose whether to leave late but must pay a cost to do so. Then a low value of ρ_{t+1} will encourage more creditors to leave late, thus increasing the total costs paid by those who leave late.

[8] An obvious requirement for the workability of this policy is that the central bank is able to identify the bankers, or the debt they present, as creditworthy.

[9] For a model that focuses more directly on seasonal fluctuations, see Champ et al. (1996).

patterns.[10] In this way, seasonal fluctuations in liquidity may bring about seasonal fluctuations in prices and consumption patterns, even if there are no fluctuations in preferences or the production technology. A central-bank discount-window policy that maintains $\rho_{t+1} = 1$ would free the economy from these liquidity-driven fluctuations in real credit.

B. *Stochastic Liquidity Demand*

Consider now an economy in which the rate of the arrival of debtors at the central island is an identically and independently distributed random variable, λ_t. If the realized value of λ_{t+1} is not known in advance (i.e., at t), the supply and demand for real credit at t (and thus the relative price P at t) will be unaffected by the realization of λ_t (although they are affected by the distribution of λ_{t+1}). By the same reasoning, the demand for money and thus nominal prices will be unaffected by the realization of λ. The appropriate agent first-order conditions (24) and (25) become

$$(32)\qquad -v_x\left(x - \frac{H}{P}\right) + Pv_y(H) = 0$$

and

$$(33)\qquad -u_y(y - L)P + E\left\{\lambda_{t+1}u_x\left(\frac{\lambda_{t+1}}{1-\alpha}\right)\left(\frac{L}{P}\right) + (1-\lambda_{t+1})u_x^*\left(\frac{1-\lambda_{t+1}}{\alpha}\right)\left(\frac{L}{P}\right)\right\} = 0.$$

The independence of nominal prices from fluctuations in the short-run interest rate has important implications for monetary policy targets. Price-level stability is assured even when there are fluctuations in the degree to which the liquidity constraint binds. A monetary authority looking only for price-level fluctuations would therefore fail to notice and correct a lack of liquidity. It is a high value of the short-run nominal interest rate (debt that sells below par) that reveals to the central bank a liquidity-constrained equilibrium.

IV. Rediscounting under Default Risk

The rediscounting task of the central bank has been kept easy so far by the assumption that all debt is repaid with perfect certainty. In reality of course, private debt bears a default risk that may or may not be observed by the central bank. I will now examine the modifications to optimal rediscounting policy in the presence of default risk.

Assume that only a fraction γ of all debtors repay their debt; the others are able to avoid going to the central island, where repayment is enforced. A fraction $\lambda < \gamma$ of the debtors arrive early, $\gamma - \lambda$ arrive late, and $1 - \gamma$ fail to arrive at all. Each debtor knows at birth whether or not he will go to the central island, but creditors are unable to determine in advance which debtors will not repay their debts.

In this case, the problem of "honest" debtors remains the same [equations (1)–(5)], and "dishonest" debtors will borrow like honest debtors to avoid revealing their type.[11] Creditors, however, must now take into account the probability of default, which will affect the interest they charge on loans, changing their budget constraints (7) and (8) to

$$(34)\qquad \rho_{t+1}(1-\lambda)R_t\ell_t + \lambda R_t\ell_t = c_{x,t+1}p_{x,t+1}$$

(when old leaving early)

$$(35)\qquad R_t\ell_t + (1-\rho_{t+1})q_t = c^*_{x,t+1}p_{x,t+1}$$

(when old leaving late)

[10] The direction of this effect will depend on creditors' relative risk aversion. For the case of relative risk aversion equal to the constant σ, the supply of real credit is increasing in λ for $\sigma < 1$, decreasing for $\sigma > 1$, and unchanged for $\sigma = 1$.

[11] It is assumed that the total debt issued by each debtor is observable, forcing dishonest debtors to mimic honest debtors.

where R_t is the gross nominal interest rate of a unit of debt. With the introduction of default risk, the expected returns of nominal debt and fiat money will be equal when

$$(36) \qquad R_t = 1/\gamma.$$

Creditors leaving late also face the liquidity constraint

$$(37) \qquad \lambda R_t \ell_t - \rho_{t+1} q_t \geq 0$$

which states that the nominal current value of debt purchased by a late-leaving creditor, $\rho_{t+1} q_t$, is limited by his available cash balances, $\lambda R_t \ell_t$. In addition, the clearing of the resale market for loans requires

$$(38) \qquad \alpha q_t = (1-\alpha)(1-\lambda) R_t \ell_t$$

which can be used to restate the liquidity constraint as

$$(39) \qquad \frac{\lambda\alpha}{(1-\alpha)(1-\lambda)} \geq \rho_{t+1}.$$

The sale of debt by early-leaving creditors to late-leaving creditors is affected by the introduction of default risk. A debt that has not been redeemed at the central island by the end of the first stage (the fraction $1-\lambda$ of all debt) may either be delayed or may be the result of default (the fraction $1-\gamma$ of all debt). Therefore, the probability that a debt will be repaid given that it has not been redeemed by the end of the first stage is

$$(40) \qquad 1 - \frac{1-\gamma}{1-\lambda} = \frac{\gamma-\lambda}{1-\lambda} \equiv \rho^*.$$

It follows that the resale value of a debt promising R_t dollars is $\rho^* R_t$ if the liquidity constraint (39) is not binding. (Late-arriving debt sells at an extra risk-based discount because defaults are indistinguishable from late-arriving good debts. If early and late arriving debt had the same default rate, there would be no additional discount on late-arriving debt.)

If the liquidity constraint (39) is binding, it must be that

$$(41) \qquad \rho_{t+1} = \frac{\lambda\alpha}{(1-\alpha)(1-\lambda)} < \frac{\gamma-\lambda}{1-\lambda} = \rho^*.$$

In this case it may be useful to decompose the resale value of the debt as follows

$$(42) \qquad \rho_{t+1} = \left(\frac{\rho_{t+1}}{\rho^*}\right)\rho^*$$

where ρ^* represents the risk-based discount on late-arriving debt and ρ_{t+1}/ρ^* is a further discount that results when the liquidity constraint is binding. If again r_{t+1} denotes the gross nominal interest rate charged on a risk-free loan of currency within period $t+1$, arbitrage will require that r_{t+1} equals the promised return on each dollar of late-arriving (risky) debt, $1/\rho_{t+1}$, multiplied by its probability of being repaid, ρ^*:

$$(43) \qquad r_{t+1} = \frac{\rho^*}{\rho_{t+1}}.$$

Perfect diversification (which is possible if each creditor is a small part of the market) will ensure that there is no randomness in the total return to late-leaving creditors.

The liquidity constraint is binding when

$$(44) \qquad \lambda\alpha < (1-\alpha)(\gamma-\lambda).$$

As before, the liquidity constraint is more likely to be binding for low values of λ and α, representing a low degree of market synchronization. From (44), one also sees that a high rate of repayment, γ, makes it more likely that the liquidity constraint will be binding because it raises the default-based price of second-hand debt, ρ^*, and thus the amount of currency needed to purchase the outstanding debt.

Optimal central-bank policy must be a bit more sophisticated in the presence of default risk. To stop the liquidity constraint from binding in equilibrium, the central bank must be prepared to supply temporarily enough cash at the discount window to remove the liquidity constraint, permitting the price of second-hand debt to rise to $\rho^* = (\gamma - \lambda)/(1 - \lambda)$. It can do so without actually knowing the repayment rate, γ, by standing ready to lend whatever currency is desired by late-leaving "bankers" for the purpose of purchasing debt from the early-leaving lenders; that is, the central bank will elastically lend $\rho_{t+1}R_t$ dollars when presented with evidence of debt promising R_t dollars from debtors who have not arrived early at the central island. If private bankers are kept responsible for the uncontingent repayment of central-bank loans and must themselves face all default risk, it is the private evaluation of this risk that will determine the equilibrium price of second-hand debt.

To make the liquidity constraint nonbinding ($\rho_{t+1} = \rho^*$), note from (43) that the central bank must lend currency at the default-free nominal interest rate $r_{t+1} = 1$. Bankers will then borrow from the central bank to purchase more late-arriving debt as long as the actual price of second-hand debt is less than ρ^*. The removal of the liquidity constraint thus allows arbitrage to move the price of second-hand debt to ρ^*, a price uniquely determined by the debt's probability of default.

In the presence of default risk, there arises an important distinction between a policy of central-bank lending to private bankers and a policy in which the central bank itself purchases debt in an open-market operation. In the latter case, the central bank bears the burden of default. If, as one might reasonably assume, the central bank has less information about the default rate than do private agents, it might pay too much for the debt it purchases.

The direct purchase of outstanding debt by the central bank will work as well as a discount window only if the central bank is able to determine ρ^*, the fair price of second-hand debt when the liquidity constraint does not bind. One such case is when the central bank directly observes the values of λ and γ that determine ρ^* as accurately as private agents observe them. This will be even more difficult if these vary over time. If instead private agents observe these values more accurately than does the central bank, the central bank can only infer ρ^* indirectly from market activity. To infer that the optimal price has been paid, the central bank must be able to observe that creditors are making positive but unconstrained purchases of debt (that, at the price paid by the central bank, creditors voluntarily hold positive amounts of both purchased debt and fiat money). If it purchases all debt, the central bank cannot be sure that it is paying the optimal price; if it purchases less than the full amount, it may not know whether it has purchased enough to remove the constraint. Therefore, just to verify that the central bank has offered the optimal price has considerable informational requirements.

Note that this information is merely what is required to check the optimality of its price from market activity after the fact. Market activity, however, responds to central-bank actions. Therefore, unless the central bank can make a series of nonbinding offers to learn the demand curve, the central bank cannot use this information in advance.[12] For the central bank to know *in advance* that its offered price is optimal requires it to know the actual values of the parameters (λ and γ) that determine the optimal price, ρ^*. For this reason it seems that optimal central-bank debt purchases require significantly more information than does a policy of central-bank lending to private bankers presenting evidence of debt.

V. Conclusion

The implications of this paper's model economy follow from the dual role played by money. Fiat money is needed both to purchase goods and to repay debt. As a result, the real stock of currency, determined by the demand for money to purchase goods, may be insufficient to permit the unconstrained clearing of credit markets. The selling of debt at a discount indicates a nonoptimal equilibrium.

The model of this paper therefore suggests that the optimal central-bank policy includes the elastic provision of a stock of fiat money. Central-bank loans that temporarily increase the stock of central-bank money permit the

[12] The central bank might learn ρ^* from the results of central-bank purchases in a sequence of periods, but only if the parameters underlying ρ^* do not change over time.

clearing of debt at par (or at its risk-adjusted price), thus restoring economic efficiency. Therefore, the two roles of money require two distinct central-bank policies: the central bank must not only choose the end-of-period fiat money stock but must also provide within-period central-bank loans sufficient for the clearing of debt unconstrained by a need for liquidity.

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