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Asymmetry, imperfectly transferable utility, and the role of fiat money in improving terms of trade

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Abstract

We modify the Kiyotaki and Wright (1991, J. Economic Theory 53, 215–235; 1993, Amer. Econom. Rev. 83, 63 77) framework so that there is a universal double coincidence of wants in all barter matches. We also introduce divisible service sidepayments into the model and allow agents to bargain over bundles of goods, services and money in bilateral matches. In asymmetric matches, the agent that values the other's good more dearly will typically have to make a substantial service sidepayment to complete the bargain. When sidepayments transfer utility imperfectly, the general equilibrium is inefficient. Agents barter too much. When barter is inefficient, a robust monetary equilibrium may exist which improves welfare. Both robust monetary equilibria and welfare-improving monetary equilibria require asymmetric matches, imperfectly transferable utility, and monetary exchange yields better expected terms of trade than barter. In contrast to other search models, money does not speed up trade.

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1. Introduction

In a series of influential papers, Kiyotaki and Wright (1989, 1991, 1993) have developed a search-theoretic approach to monetary economics. This approach

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articulates how Jevons' classic double-coincidence problem can yield the existence of valued money as a rational expectations equilibrium.[1] Recently, Williamson and Wright (1994) have shown that the search approach can generate valued money in an environment in which private information generates a recognizability problem. Examination of both problems yields the following three central results: (1) the nonmonetary equilibrium is inefficient, (2) there exist monetary equilibria which are robust in the sense that agents strictly prefer to accept fiat money in exchange for goods, and (3) there is a welfare-improving role for fiat money.[2] In contrast, when both problems are absent, Williamson and Wright (1994) find that none of the three results obtain.

This paper also develops a model in which neither the double-coincidence problem nor the recognizability problem are present. However, we are able to obtain results (1)–(3) by introducing three new ingredients into the Kiyotaki and Wright framework. First, we change the preferences for goods such that there is a universal double coincidence of wants in all barter matches but where some matches involve an asymmetric valuation of goods. Second, we allow all agents to produce divisible services as sidepayments to supplement goods exchange. Third, we allow agents to bargain over bundles of goods, services and money in bilateral matches.

The new ingredients yield three specific elements that are necessary for results (1)–(3) to obtain in our model: (i) asymmetric matches, (ii) services transfer utility imperfectly, and (iii) monetary exchange yields better expected terms of trade than barter. In the absence of any of these elements, a monetary equilibrium may exist but it is not robust or welfare improving. Elements (i)–(iii) reveal that money can be a superior mechanism for transferring utility between traders by yielding better expected terms of trade than barter.

In the conclusion, we argue that elements (i)–(iii) are indicative of general features of economies that generate results (1)–(3). For money to yield better terms of trade, barter must yield terms of trade that can be improved upon. In particular, in positive surplus matches, the terms of trade must not transfer utility perfectly. We argue that this is often the case in asymmetric matches. When the asymmetry is sufficiently severe, the terms of trade may be such that barter either does not occur, or if it does, utility is transferred imperfectly. Then using money may improve an agent's trading opportunities. Generally, money is

[1] This search approach builds on Jones' (1976) adaptive expectations search model. See Kiyotaki and Wright (1993) and Shi (1997) for references to papers using the search approach. Ostroy and Starr (1990) provide a survey of transaction money models.

[2] Of course, both problems have been used to motivate results (1)–(3) in general equilibrium models in which Walrasian markets are decentralized. The absence of the coincidence of wants is modelled in Samuelson's (1958) overlapping generations model and Townsend's (1980) turnpike model. Bryant (1980) generates valued money in an overlapping generations model with private information.

more highly valued the better are its expected terms of trade, and highly valued money is welfare improving as it reduces inefficient barter.

In our specific model, barter always occurs because of the double coincidence of wants. Nevertheless, the terms of trade lead to the imperfect transfer of utility. In asymmetric matches, the agent that wants the other's good more dearly, termed the buyer, will typically have to forego services in addition to their own good to complete the bargain. We model services so that at the margin the disutility of providing the service is greater than the utility of consuming it. Therefore, services transfer utility imperfectly. With services transferring utility imperfectly and the terms of trade forcing the buyer to produce services, the general equilibrium is inefficient. The inefficiency arises because all agents have an equal chance of being randomly assigned a buyer in their next match. Hence, they would be better off if less services were produced on average. This barter inefficiency arises in the general equilibrium of the model even though individual bargains may be efficient.

When barter is inefficient, a robust monetary equilibrium may exist. Agents strictly prefer to hold money because without it an agent faces the possibility of bartering at a disadvantage in an asymmetric match where the trading partner does not value his good as dearly. In equilibrium money traders get much better expected terms of trade by foregoing fewer services than buyers in asymmetric matches. In contrast to other search models, money does not improve liquidity by speeding up trade.

The terms of trade depend on the value of holding money which is endogenous to the model. The Nash solution yields money traders better terms of trade the greater is the value of holding money, a feature which yields multiple equilibria. In addition to expectations, the value of money depends on the surplus weights which divide the bargaining power.

Money is welfare improving when the economy satisfies elements (i)–(iii) and money improves the expected terms of trade sufficiently. Increasing the money supply has two effects. First, it increases welfare by reducing the inefficient production of services. Second, it reduces welfare by reducing the number of good traders. We show that the first effect may dominate the strong negative second effect in the absence of other frictions.

Shi (1995) and Trejos and Wright (1995) also develop search-theoretic money models in which relative prices are determined in bargaining. In these models, the price level is determined by the amount of the seller's services that are exchanged for the buyer's indivisible unit of money. As in other search models, fiat money is valued because it speeds up the rate at which agents are matched to sellers who are willing to trade, a feature we deliberately eliminate in our model. Our paper differs from these papers in three other ways. First, we include storable goods as well as services in the model. Second, we allow money traders to produce services which makes their offers more flexible. Third, we model asymmetric barter matches, an essential element for all our results.

The paper proceeds as follows. Section 2 lays out the model without services and discusses some key issues. Section 3 details services and bargaining, and then derives the nonmonetary equilibrium. Monetary equilibria are derived in Section 4, and Section 5 examines the optimal quantity of money. Section 6 establishes that elements (i)–(iii) are essential for results (1)–(3) even if terms of trade are chosen arbitrary. Section 7 concludes.

2. The basic model without services

The analysis of the model without services is straightforward and provides the benchmark with which to compare our results to Williamson and Wright's (1994) complete information model. They find that money is valued in a model where all goods are valued symmetrically in all matches. However, money is not robust in the sense that agents are indifferent to accepting it in equilibrium, and money is always welfare reducing. We show that their results generalize to the case where goods are valued asymmetrically in some matches.

2.1. Description

The basic model is similar to the Kiyotaki and Wright (1991, 1993) models except that all goods are valued and there are zero transaction costs in matches.

2.1.1. Agents and the production of goods

The economy has a continuum of infinitely-lived agents indexed by i on the unit interval. Each agent i can produce a storable good i in indivisible units of 1. As in Kiyotaki and Wright (1993), we assume that goods are produced instantaneously at zero disutility to avoid having to detail production decisions. An agent can only hold one unit in inventory at a time and holding a good precludes producing another good.[3] Agents that hold a good in search are called *good traders*.

2.1.2. Money

Initially, a fraction M of the agents are randomly chosen and endowed with a unit of money. Those endowed with money may either keep it or dispose of it. Then any agent who does not have money can produce. If all agents dispose of their money, the economy is called a *nonmonetary economy*.

As is standard, money is a storable item which is neither produced nor consumed. Agents can only hold one unit of a storable item in inventory, so they

[3] Instead of limiting inventory to one unit as in Kiyotaki and Wright (1991), we could use the alternate assumption in Aiyagari and Wallace (1991) and Kiyotaki and Wright (1993) that agents cannot produce before they consume another agent's good.

cannot hold both money and a good simultaneously. Agents cannot produce money and cannot produce goods while they hold money. Agents that hold money in search are called *money traders*.

2.1.3. *Preferences over goods*

Agents derive utility from consuming *all* goods other than their own production good. Each agent partitions goods into two groups: *preferred goods* and *mediocre goods*. Consuming a preferred good yields an agent utility $u > 0$; whereas, consuming a mediocre good yields utility $bu > 0$, with $0 < b < 1$. Thus, mediocre goods are valued but not as highly as preferred goods.

Each agent's set of preferred goods represents a proportion $x \geqslant 0$ of all storable goods, and mediocre goods comprise the remaining $1 - x$ of goods (as the own production good is of measure 0). In addition, each good is a preferred good of an equal proportion x of agents and the mediocre good of the remaining $1 - x$ of agents. Heterogeneity in the model is such that each agent and good can be treated symmetrically.

2.1.4. *Matching*

Time is continuous and continues forever. Agents are matched pairwise and randomly according to a Poisson process with constant arrival rate $\beta > 0$. There are three types of matches between good traders. A match in which both agents have the other's preferred good is called a *preferred good match* (PGM). A *mediocre good match* (MGM) is when both agents have the other's mediocre good. An *asymmetric good match* (AGM) is when only one agent has the other's preferred good. AGMs are a natural reflection of asymmetric tastes.[4] As matching is random, AGMs occur when $0 < x < 1$.

There are two types of matches between money traders and good traders. A *preferred good-money match* is when a money trader is matched with a good trader who has his preferred good. A *mediocre good-money match* is when a money trader is matched with a good trader who has his mediocre good.

2.1.5. *Exchange, consumption and surplus*

After exchange, the match is immediately dissolved. As all goods are valued, agents have a dominant strategy to consume any good they receive in exchange.

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[4] In Kiyotaki and Wright (1993) a subset of goods yield utility u and other goods are not consumed so that there is only a coincidence of wants in matches involving the most wanted items. In contrast, an agent's valuation of a good varies continuously in Kiyotaki and Wright (1991) according to the distance that good is from his location on a circle. Matches with a coincidence of wants with respect to the most wanted items occur with probability zero. Nearly all barter matches are asymmetric in the sense that agents derive different utilities from consuming the other's good. However, not all matches involve goods being valued at least at the cost of production.

Consumption is instantaneous. The agent can then produce a new good instantaneously and search for a new match.

Let V_i denote the value of search while holding good i. Once matched, the surplus for this agent is the difference between his payoff from trade and his reservation utility, $S_i = U_i - \bar{U}_i$. The reservation utility is the value of leaving the match without trading, $\bar{U}_i = V_i$. Like Kiyotaki and Wright (1991, 1993), we impose the following restriction which allows us to concentrate on equilibria in which all goods are equally liquid.

Restriction 1. The expected utility of searching with any good i is the same, $V_i = V_g$ for all i, so that the reservation utility is the same for all good traders, $\bar{U}_i = V_g$.

Unlike Kiyotaki and Wright, we assume that there is no transaction cost to exchanging goods in a match. Hence, there is always a positive surplus from barter. In a preferred good match (PGM) each agent trades his production good for his preferred good. After trading each agent instantly consumes and then produces. Hence, each agent receives surplus $u + V_g - V_g = u$ from trade. Similarly, in a MGM each agent receives surplus bu from trade. In an AGM the agent who consumes the preferred good (called the buyer) receives surplus $S_b = u$ and the agent who consumes the mediocre good (the seller) receives surplus $S_s = bu$. Notice that without services the total surplus in an AGM is necessarily divided asymmetrically and that as $b \to 0$ the buyer gets all the surplus.

The nonmonetary economy displays a *universal double coincidence of wants* in all bilateral matches in the sense that trading for the partner's good, consuming it, and replacing the production good in inventory yields a positive surplus. Barter occurs in all matches.

2.2. The nonmonetary economy without services

First consider an equilibrium in which money is never accepted in trade. In this equilibrium all those endowed with money dispose of it as they are better off bartering. The value function for a good trader in the nonmonetary economy (denoted by superscript n) is:

$$rV_g^n = \beta[x^2u + x(1-x)(S_b + S_s) + (1-x)^2bu] \quad (1)$$

where r is the discount rate. All agents encounter each other at rate β and trade in all matches. The probability that the match is a PGM is x^2 in which case the agent receives surplus u. The probability that the match is an AGM is $2x(1-x)$, in which case the agent has an equal chance of being a buyer or a seller. The probability of a MGM is $(1-x)^2$.

Substituting the surplus values $S_b = u$ and $S_s = bu$ into Eq. (1) yields the equilibrium value $\beta[x + b(1 - x)]u/r$ for a good trader. As all agents are good traders, expected utility is $Z^* = \beta[x + b(1 - x)]u/r$. Notice that since goods are exchanged in all matches, a planner constrained only by the random nature of matching could not do better for the representative agent. The constrained optimum also involves agents exchanging goods in all matches and yields expected utility Z^*. The following proposition summarizes.

Proposition 1. Without services, a unique nonmonetary equilibrium exists. The equilibrium achieves the constrained optimum Z^*, as agents exchange goods in all matches.

2.3. The monetary economy without services

Can money be valued in an economy where barter achieves the constrained optimum? We now show that money can be valued, though it plays no useful role.

Let V_m denote the value of search while holding money. Once matched, the surplus for a money trader is the difference between the match payoff and the reservation utility. As before, the reservation utility is the value of leaving the match without trading. Therefore, for a money trader the reservation utility is V_m. A good trader in a preferred good-money match receives surplus $S_g = V_m - V_g \equiv Y$ from trade, and a money trader receives surplus $S_m = u + V_g - V_m = u - Y$. Similarly, a money trader in a mediocre good–money match receives surplus $S_m^M = bu - Y$ and the good trader receives surplus $S_g^M = Y$. Individual rationality requires that the surpluses be non-negative for trade to occur.

We consider only pure monetary equilibria where money is not discarded and is accepted with probability 1. Then the value functions for good traders and money traders respectively satisfy

$$rV_g = \beta(1 - M)[x^2 u + x(1 - x)(S_b + S_s) + (1 - x)^2 bu]$$

$$+ \beta M[xS_g + (1 - x)S_g^M], \tag{2}$$

$$rV_m = \beta(1 - M)[xS_m + (1 - x)S_m^M]. \tag{3}$$

The value function for the good trader is simply altered to include encounters with money traders. A good trader encounters a money trader at rate βM. A proportion x of these encounters involve preferred goods and the remaining $1 - x$ involve mediocre goods. Conversely, a money trader encounters a good trader at rate $\beta(1 - M)$. A proportion x have his preferred good and the remaining his mediocre good. Formal derivation of equations similar to Eqs. (2) and (3) can be found in Kiyotaki and Wright (1991).[5]

Monetary trades require nonnegative surplus, $S_g = S_g^M = Y \geqslant 0$. Thus, a monetary equilibrium is a joint solution of Eqs. (2) and (3) which satisfies $Y \geqslant 0$. Suppose $Y \geqslant 0$ and money always trades for both preferred and mediocre goods. Substituting the surplus values into Eqs. (2) and (3) and subtracting one equation from the other yields a single equation in variable Y which has a unique solution $Y = 0$. We have the following proposition.[6]

Proposition 2. Without services, a unique monetary equilibrium exists for which no agent discards money. Money is always accepted in exchange for a good, and money traders and good traders do equally well, $V_m = V_g = (1 - M)\beta[x + b(1 - x)]u/r = (1 - M)Z^*$.

Though a monetary equilibrium exists, it is not robust in the sense that good traders are indifferent to trading for money but nevertheless always trade for money. Williamson and Wright (1994) derive a similar result in their complete information case. They show that when there is a symmetric double coincidence of wants, $x = 1$, a monetary equilibrium exists in which $V_m = V_g$. We find that this result generalizes to $x \leqslant 1$.

With money, expected utility of the representative agent (before money is distributed) is $Z = MV_m + (1 - M)V_g = (1 - M)Z^*$. Hence, the optimal money supply is $M^o = 0$. As in Williamson and Wright (1994), the nonmonetary equilibrium dominates the monetary equilibrium when there is no private information.

2.4. Some key issues

The model without services has several interesting features. First, in AGMs the buyer gets the bulk of the surplus. Second, the nonmonetary equilibrium is

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[5] Eqs. (2) and (3) ignore the probabilistic decisions of agents to accept the trade. For example, if on average preferred good traders accept money with probability Π_g and a particular money trader accepts his preferred good with probability π_m, the first term on the RHS of Eq. (3) must be modified to $\beta(1 - M)x\Pi_g \max_{\pi_m} \pi_m[0, S_m]$.

[6] The proof of the proposition is straightforward and is available from the authors on request. It can be shown that if money trades with less than probability one, no $Y \geqslant 0$ exists that satisfies both value functions. The proposition generalizes to nonpure monetary equilibria.

The proposition also generalizes to the case where there is a positive cost of producing a good. Then, agents never discard money so the proposition describes the unique monetary equilibrium. However, given our assumption of costless production, there exists a continuum of monetary equilibria according to the proportion of agents that do not dispose of their money endowment, m, $0 \leqslant m \leqslant M$. In all these equilibria $V_m = V_g = Z = (1 - m)Z^*$. The more agents that discard money the greater is welfare.

optimal. Third, the monetary equilibrium is not robust in the sense that good traders are indifferent to receiving it in trade, $V_m = V_g$. Fourth, money serves no useful role and is welfare reducing.

Introducing services into the economy allows flexible terms of trade and allows us to examine the robustness of the above results. We find the following. First, in AGMs the bargaining terms of trade are typically less favorable to the buyer. Second, this results in the nonmonetary equilibrium being inefficient. Third, this inefficiency is necessary for monetary equilibria to be robust in the sense that $V_m > V_g$. Fourth, money plays a useful role by improving the terms of trade and may be welfare enhancing.

3. Services, bargaining, and barter

We now introduce another technology for transferring utility between agents which we call services. Services are divisible and we allow agents to bargain over bundles of indivisible goods and divisible services.

3.1. Services

All agents can produce, trade and consume services (or equivalently, perishable goods). An agent is capable of instantaneously producing any amount of divisible units of a service at a constant unit cost of $c > 0$. The service can only be consumed by other agents, and q units of services yields the consumer utility $v(q) = c(1 - e)q$, where $0 \leqslant e < 1$. We say that *services transfer utility imperfectly* when $e > 0$ and perfectly when $e = 0$. Unless otherwise stated, we will assume that $e > 0$ so that $v'(q) = c(1 - e) < c$. This avoids the exchange of services for services.[7] However, services may be produced in order to transfer more utility to the trading partner to purchase the partner's good.

3.2. The constrained optimum

Consider the rule a planner would choose to maximize the expected utility of the representative agent constrained only by the exogenous matching pattern. The planner would have agents produce, then swap and consume goods in all matches because of the double coincidence of wants. On the other hand, the

[7] Our results are not attributable to services being undesirable per se, $v'(0) = c(1 - e) < c$ for $e > 0$. In a note available upon request we show that the inefficiency remains under a more general specification in which the marginal utility of services declines: $v'(0) > c$ and $\lim_{q \to \infty} v'(q) \to c(1 - e) < c$. In this case it is optimal to exchange some services in all matches. Nevertheless, the inefficiency remains because the bargaining terms of trade drive marginal utility of services below their marginal cost. Too many services are exchanged in equilibrium.

planner would forbid the exchange of services in matches because the production cost exceeds the consumption utility. Since an agent meets other agents at rate β and the proportion x of agents have preferred goods and $1-x$ have mediocre goods, an agent's expected utility under the planner is $Z^* = \beta[x + b(1-x)]u/r$. The constrained optimum is the same as in the model without services.

Proposition 3. The constrained optimum expected utility is Z^*. The constrained optimum requires that agents swap goods in each match but never provide services.

3.3. Bargaining in matches

The terms of trade in each bilateral exchange (including monetary trades) is given by the generalized Nash bargaining solution. This mechanism is the natural choice as agents are matched bilaterally and, consistent with the Nash program, it is supported by cooperative and noncooperative games. The solution maximizes the weighted product of the surpluses from exchange.

$$\max_{q_i, q_j} S_i^{\omega} S_j^{1-\omega} \quad \text{s.t. } S_i \geqslant 0,\ S_j \geqslant 0,\ q_i \geqslant 0,\ q_j \geqslant 0,$$

where ω, $0 \leqslant \omega \leqslant 1$, is the weight on agent i's surplus. We allow the weight ω to differ depending on the match type. As before, agent i's surplus is $S_i = U_i - \bar{U}_i$, where the reservation utility corresponds to the value of continuing to search with the item in inventory. If agent i is a good trader $\bar{U}_i = V_g$. If agent i is a money trader $\bar{U}_i = V_m$. The strategic game that supports this specification is detailed in Engineer and Shi (1996).[8]

3.3.1. The surplus frontiers

In an asymmetric good match (AGM) we distinguish between good traders. We dub the agent that wants the preferred good the *buyer* and the agent that has that good the *seller*. We choose this language because the buyer typically will have to trade services in addition to his own good to purchase the seller's good. Let q_b (q_s) be service payments paid by the buyer (seller) to the seller (buyer). As

[8] In the sequential game the surplus weight ω corresponds to the probability of agent i moving first in each round of bargaining. The reservation utility depends on agents encountering other agents between bargaining rounds and leaving the existing match if they encounter a desireable new trading partner. Trejos and Wright (1995) refer to this as search while bargaining. The above Nash solution corresponds to the limit as the period of delay between bargaining rounds becomes small. See Binmore (1987), Osborne and Rubinstein (1990) and Wolinsky (1987) for discussions of noncooperative bargain solutions.

all agents enter and exit matches holding goods, the surpluses only involve the utility from trade within the period. The surpluses are:

$$S_s = bu + c(1-e)q_b - cq_s, \qquad S_b = u + c(1-e)q_s - cq_b. \tag{4}$$

Solving for q_s and q_b as functions of S_s and S_b yields:

$$q_s = \frac{[(1-e)+b]u - (1-e)S_b - S_s}{ce(2-e)} \quad \text{and}$$

$$q_b = \frac{[1+(1-e)b]u - (1-e)S_s - S_b}{ce(2-e)}. \tag{5}$$

Thus, $q_s \geqslant 0$ and $q_b \geqslant 0$ are equivalent to

$$S_s \leqslant [(1-e)+b]u - (1-e)S_b \text{ and } S_b \leqslant [1+(1-e)b]u - (1-e)S_s. \tag{6}$$

These constraints form the surplus frontier drawn in Fig. 1. They intercept at the kink (bu, u). At the kink no services are produced. The production of services expands the set of feasible trades. As services transfer utility imperfectly, $e > 0$, trades on the surplus frontier corresponds to no more than one agent producing services, $q_b q_s = 0$. The kink is the only point which maximizes the sum of the surpluses.

The frontier in a PGM can be derived as a special case of the AGM where $b = 1$. The kink is at (u, u) and the frontier is symmetric. The surplus frontier in a MGM is identical to the PGM except that the frontier is shifted inward so that the kink is at (bu, bu).

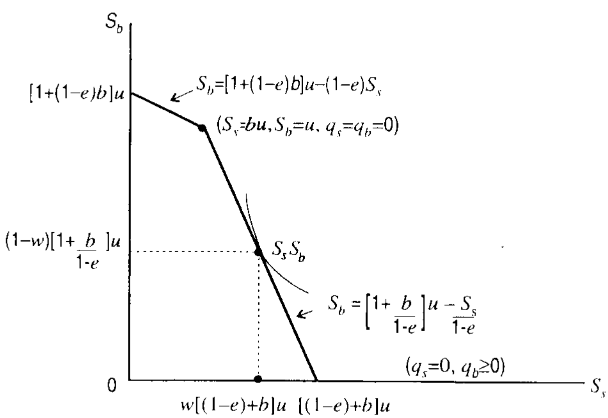

Fig. 1. Feasible set in an asymmetric good match.

3.3.2. Bargaining outcomes

In both PGMs and MGMs, agents are in a position symmetric to their trading partner. Hence, we weight each agent's surplus equally, $\omega = \frac{1}{2}$. Since the level curve, $S_i^{1/2}S_j^{1/2} = constant$, is symmetric in S_i and S_j and the surplus frontier is symmetric, the Nash product is maximized at the kink. The quantities of services produced at the kink are $(q_i, q_j) = (0, 0)$. Symmetric matches yield symmetric outcomes in which no services are traded.

In an AGM agents are in a nonsymmetric situation. We allow the bargaining weight to vary and use the generalized Nash solution:

$$\max_{(S_s, S_b)} \{S_s^\sigma S_b^{1-\sigma}:\ S_s \geqslant 0,\ S_b \geqslant 0,\ \text{Eq. (6)}\}.$$

where σ, $0 \leqslant \sigma \leqslant 1$, is the weight on the seller's surplus in an AGM. Define

$$\underline{\sigma} \equiv \frac{(1-e)b}{1+b(1-e)} < \frac{1}{2} \quad \text{and} \quad \bar{\sigma} \equiv \frac{b}{(1-e)+b} > \underline{\sigma}.$$

Lemma 1. *The bargaining solution in an asymmetric good match (AGM) is as follows*:

σ	S_b	S_s	
$[0, \underline{\sigma})$	$[1+b(1-e)](1-\sigma)u$	$\dfrac{1+b(1-e)}{1-e}\sigma u$	
$[\underline{\sigma}, \bar{\sigma}]$	u	bu	(7a)
$(\bar{\sigma}, 1]$	$\left[1+\dfrac{b}{1-e}\right](1-\sigma)u$	$[(1-e)+b]\sigma u$	

and

σ	q_b	q_s	
$[0, \underline{\sigma})$	0	$\dfrac{\sigma+(1-\sigma)b(1-e)}{c}u$	
$[\underline{\sigma}, \bar{\sigma}]$	0	0	(7b)
$(\bar{\sigma}, 1]$	$\dfrac{\sigma(1-e)-b(1-\sigma)}{c(1-e)}u$	0	

Proof. (i) $\sigma < \underline{\sigma}$. Consider the following maximization problem:

$$\max_{(S_s, S_b)} \{S_s^\sigma S_b^{1-\sigma} \text{ s.t. } S_b \leqslant [1+b(1-e)]u - (1-e)S_s,\quad S_s \geqslant 0,\ S_b \geqslant 0\}.$$

That is, disregard for a moment the constraint $q_s \geqslant 0$ (the first one in Eq. (6). The maximization delivers the values S_s and S_b listed in Eq. (7). Then it can be shown that at the maximization point $q_s > 0$.

(ii) $\sigma \in (\bar{\sigma}, 1]$. Similarly, solve the maximization problem including the constraint for $q_s \geqslant 0$ but discarding the constraint $q_b \geqslant 0$ (the second one in Eq. (6)). The solution yields the values in Eq. (7). At the maximization point $q_b > 0$.

(iii) $\sigma \in [\underline{\sigma}, \bar{\sigma}]$. The Nash product is maximized at the kink formed by the constraints given by Eq. (6). At the kink $q_b = q_s = 0$ and the values for S_m and S_g immediately follow. □

What we refer to as our *main case* is drawn in Fig. 1. There the seller has sufficient bargaining power, $\sigma > \bar{\sigma} \equiv b/(1 - e + b)$, to extract services from the buyer, $q_b > 0$. This is the natural case to concentrate on when there is a substantial asymmetry, that is, b is small. Note as $b \to 0$, $\bar{\sigma} \to 0$ so that the main case encompasses the entire range.

3.4. The nonmonetary equilibrium and welfare

Consider an equilibrium in which money is never accepted in trade. In this equilibrium all those endowed with money dispose of it as they are always better off bartering. As before, the expected utility for a good trader in the nonmonetary economy, V_g^n, is given by Eq. (1). The equilibrium value of V_g^n, denoted $V_g^n(N)$, is just given by substituting the values of S_s and S_b from the above bargaining solution, Eq. (7), into the value function, Eq. (1).

Proposition 4. *A nonmonetary equilibrium always exists in which money is discarded and all agents exchange goods with each agent they meet in search. Services trade if and only if* $\sigma \notin [\underline{\sigma}, \bar{\sigma}]$. *The value of holding a good in search is* $V_g^n = V_g^n(N)$.

For the main case, $\sigma > \bar{\sigma}$, the equilibrium value of search is

$$rV_g^n(N) = \beta\left[x^2 + x(1-x)\left(1 + \frac{b}{1-e}\right)(1 - e\sigma) + (1-x)^2 b\right]u.$$

Comparing this to the constrained optimum reveals an inefficiency:

$$\frac{Z^* - V_g^n(N)}{Z^*} = \frac{(1-x)ecq_b}{RZ^*} = \frac{ex(1-x)[\sigma(1-e) - b(1-\sigma)]}{(1-e)[x + b(1-x)]} > 0,$$

where $R \equiv r/\beta x$. The inefficiency arises from the buyer making service payments, $q_b > 0$, and the inefficiency is the greatest when the seller has all the bargaining power, $\sigma = 1$. Notice that after substituting q_b that R does not enter the welfare ratio. Thus, the inefficiency does not disappear when search is infinitely fast. This is because agents always enter into AGMs. The alternative of waiting for a better partner entails foregoing the surplus from trading in AGMs and would lower utility.

More generally, the inefficiency from barter in a nonmonetary economy for arbitrary values of q_b and q_s is

$$Z^* - V_g^n = \beta x(1-x)ec(q_b + q_s)/r. \tag{8}$$

Inefficiency arises from service payments in AGMs when services transfer utility imperfectly. If asymmetric matches are absent ($x = 0$ or $x = 1$) no services are traded. If services are exchanged, there is no inefficiency if services transfer utility perfectly ($e = 0$).

Proposition 5. *The nonmonetary economy is inefficient if and only if all of the following three elements are present*: (*i*) *asymmetric goods matches*, $0 < x < 1$; (*ii*) *services transfer utility imperfectly*, $e > 0$; *and* (*iii*) *service payments*, $q_b + q_s > 0$.

The three features we have introduced in the model are all essential for barter inefficiency.[9] Thus, whether the inefficient transfer of utility emerges in equilibrium is an endogenous outcome that depends on the specific trading mechanism. Under Nash bargaining $\sigma \notin [\underline{\sigma}, \bar{\sigma}]$ results in the trade of services. We have the following proposition.

Proposition 6. *The nonmonetary equilibrium is inefficient if and only if* (*i*) $0 < x < 1$; (*ii*) $e > 0$; *and* (*iii*) $\sigma < \underline{\sigma}$ *or* $\sigma > \bar{\sigma}$, *that is, either buyers or sellers have sufficient bargaining power to extract service sidepayments from their trading partners.*

In asymmetric matches, the Nash solution can quite naturally yield service payments and thereby induce inefficiency. This is because it maximizes the weighted product of the surpluses in each match rather than the sum of the surpluses. Relative to a straight swap of goods, bargaining reduces the sum of the surpluses by $ce(q_b + q_s)$ in each AGM.

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[9] Inefficiency can arise for $b = 1$. However, there must still be an asymmetry in AGMs. For the main case above, we can have $b = 1$ as long as $\sigma > \bar{\sigma} = 1/(2-e) > 1/2$. Here there is symmetry in the valuation of goods. Nevertheless, the equilibrium is inefficient because of the asymmetry in bargaining power.

It is important to note that the inefficiency does not refer to bargaining in individual matches. Given V_g, the bargaining outcome in any match is efficient as it lies on the match's utilities possibility frontier. Rather, the inefficiency arises in the general equilibrium from the exchange of too many services when services transfer utility imperfectly. As each agent has an equal chance of being a buyer or a seller in their next match, the exchange of too many services lowers expected utility.

Finally, from the foregoing it is clear that a social contract to eliminate service sidepayments is desirable. However, such a contract is not enforceable in our environment. Thus, we get the apparent paradox that expanding the bargaining set in bilateral matches can yield an inferior general equilibrium outcome.[10]

4. Monetary equilibrium

In Section 2.3 we found that money could be valued in the model without services. However money played no useful role and the equilibrium was not robust in the sense that agents were indifferent to holding money. There the terms of trade were fixed.

Now we let either the money trader and/or the good trader make service sidepayments. The question we now pose is: Can a robust monetary equilibrium ($V_m > V_g$) exist in an economy with a universal double coincidence of wants in goods when service sidepayments are possible? We show that Nash bargaining can induce a robust demand for fiat money when it plays a useful role in improving the terms of trade for money traders.

4.1. Bargaining in money matches

The bargaining solutions between good traders is unaffected by the presence of money in the economy and so we only have to examine matches between money and good traders.

Consider preferred good-money matches. Let q_m be the quantity of services produced by the money trader and q_g be the quantity of services produced by the good trader. A trade involves money and q_m for the good and q_g. The good trader consumes q_m and becomes a money trader. The money trader consumes the good received and q_g and then produces to become a good trader. This trade

[10] This result is similar in flavor to Hart (1975) who finds that increasing the number of markets can be welfare reducing.

yields surplus $S_{\rm m}$ to the money trader and a surplus $S_{\rm g}$ to the good trader:

$$S_{\rm m} = u + c(1-e)q_{\rm g} - cq_{\rm m} - Y \quad \text{and} \quad S_{\rm g} = c(1-e)q_{\rm m} - cq_{\rm g} + Y, \tag{9}$$

where $Y \equiv V_{\rm m} - V_{\rm g}$. Both $V_{\rm g}$ and $V_{\rm m}$ and hence Y are taken as exogenous by agents. Solving for $q_{\rm m}$ and $q_{\rm g}$ and imposing $q_{\rm m} \geqslant 0$ and $q_{\rm g} \geqslant 0$ implies

$$S_{\rm m} \leqslant u - (1-e)S_{\rm g} - eY \quad \text{and} \quad S_{\rm g} \leqslant (1-e)u - (1-e)S_{\rm m} + eY. \tag{10}$$

These constraints are drawn in Fig. 2. As in Fig. 1, the kink is the only point which maximizes the sum of the surpluses.

Now consider mediocre good–money matches. The surpluses from trade are

$$S_{\rm m}^{\rm M} = bu + c(1-e)q_{\rm g}^{\rm M} - cq_{\rm m}^{\rm M} - Y \quad \text{and} \quad S_{\rm g}^{\rm M} = c(1-e)q_{\rm m}^{\rm M} - cq_{\rm g}^{\rm M} + Y. \tag{11}$$

The frontier is similar to Fig. 2 except that the kink is at $(Y, bu - Y)$.

Good traders in money matches and sellers in AGMs are in similar asymmetric positions – both types of agents have the item with greater consumption utility. The following restriction ensures that different bargaining weights are not generating our results.

Restriction 2. The weight on the good trader's surplus in a match with a money trader is the same as the weight on the seller's surplus in an AGM, $\omega = \sigma$, $0 \leqslant \sigma \leqslant 1$.

The Nash solution for a preferred good–money match is given by

$$\max_{(S_{\rm g}, S_{\rm m})} \{S_{\rm g}^{\sigma} S_{\rm m}^{1-\sigma}\colon S_{\rm g} \geqslant 0,\ S_{\rm m} \geqslant 0,\ \text{Eq. (10)}\}.$$

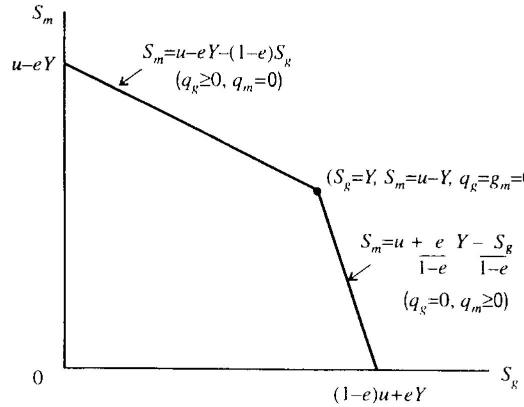

Fig. 2. Feasible set in a monetary match.

Lemma 2. The bargaining solution in preferred good–money matches is as follows:

Y	S_m	S_g	
$\left[0, \dfrac{(1-e)\sigma u}{1-e\sigma}\right)$	$\dfrac{(1-\sigma)[(1-e)u+eY]}{(1-e)}$	$\sigma[(1-e)u+eY]$	
$\left[\dfrac{(1-e)\sigma u}{1-e\sigma}, \dfrac{\sigma u}{1-e+e\sigma}\right]$	$u-Y$	Y	
$\left(\dfrac{\sigma u}{1-e+e\sigma}, \dfrac{u}{e}\right]$	$(1-\sigma)(u-eY)$	$\dfrac{\sigma(u-eY)}{1-e}$	(12a)
$>\dfrac{u}{e}$	no trade	no trade	

and

Y	q_m	q_g	
$\left[0, \dfrac{(1-e)\sigma u}{1-e\sigma}\right)$	$\dfrac{(1-e)\sigma u-(1-e\sigma)Y}{c(1-e)}$	0	
$\left[\dfrac{(1-e)\sigma u}{1-e\sigma}, \dfrac{\sigma u}{1-e+e\sigma}\right]$	0	0	(12b)
$\left(\dfrac{\sigma u}{1-e+e\sigma}, \dfrac{u}{e}\right]$	0	$\dfrac{(1-e+e\sigma)Y-\sigma u}{c(1-e)}$	
$>\dfrac{u}{e}$	0	0	

Proof. For $Y > u/e$, there is no trade as no nonnegative S_m and S_g satisfy Eq. (10). Following the proof to Lemma 1, it can be shown: if

$$Y \in \left(\frac{\sigma u}{1-e+e\sigma}, \frac{u}{e}\right],$$

the solution lies on the segment of the surplus frontier to the right of the kink in Fig. 2; if

$$Y \in \left[0, \frac{(1-e)\sigma u}{1-e\sigma}\right),$$

the solution is on the frontier to the left of the kink, and if

$$Y \in \left[\frac{(1-e)\sigma u}{1-e\sigma}, \frac{\sigma u}{1-e+e\sigma}\right]$$

the solution is at the kink. □

Remark 1. Mediocre good-money matches are a special case of Lemma 2 in which bu is substituted for u. Money is not traded for mediocre goods when $Y > bu/e$.

4.2. *Equilibrium*

Our main objective is to demonstrate the existence of a monetary equilibrium and examine its welfare-improving properties. It greatly simplifies the analysis to concentrate on finding equilibria in which money only trades for the preferred good. This restriction is not restrictive when b is sufficiently small. Then our analysis captures almost all robust monetary equilibria.

Restriction 3. The value of searching with money relative to searching with a good is sufficiently high that money is never traded for mediocre goods, but low enough that it is always traded for preferred goods, $bu/e < Y < u/e$.

Under Restriction 3, the value functions, Eqs. (2) and (3), reduce to the following:

$$rV_{\mathrm{g}} = \beta(1-M)[x^2u + (1-x)^2bu + x(1-x)(S_{\mathrm{b}} + S_{\mathrm{s}})] + \beta MxS_{\mathrm{g}}, \tag{13}$$

$$rV_{\mathrm{m}} = \beta(1-M)xS_{\mathrm{m}}. \tag{14}$$

The value functions reveal that holding money does not increase the probability of an agent trading for the preferred good. This is in contrast to other search models of money. In our model, good traders not only encounter agents with their preferred good in PGMs but also as buyers in AGMs. Hence, the rate at which a good trader is able to barter for his preferred good is $\beta(1-M)[x^2 + x(1-x)] = \beta(1-M)x$. This is the same rate that a money trader encounters agents with whom they want to trade. Thus, if money is valued, it is valued not because it speeds up the procurement of the preferred good but because it improves the terms of trade of the money trader. In fact, since the good trader barters also in MGMs and as a seller in AGMs his total rate of trading for goods is far greater.

Neither does increasing M increase the aggregate rate of acquisition of preferred goods. The probability of a randomly selected agent procuring their preferred good is $Mx + (1-M)x^2 + (1-M)x(1-x) = x$. In contrast, in the Kiyotaki and Wright model the chance is $Mx + (1-M)x^2 = x^2 + Mx(1-x)$, which increases with M. Thus, if money is welfare improving in our model it is due to the improved terms of trade.

The analysis is restricted to symmetric stationary pure strategy equilibria.

Definition 1. A monetary equilibrium (in which money traders only purchase preferred goods) is a joint solution of Eqs. (7), (12)–(14) for q_b, q_s, q_m, q_g, V_m, and V_g where $bu/e < Y < u/e$.

The examination of monetary equilibrium simplifies to finding a fixed point for Y. Subtracting Eq. (13) from Eq. (14) and using the definitions of S_b, S_s, S_m and S_g yields:

$$Y = \frac{c[1-e(1-M)]q_g - c(1-eM)q_m - (1-M)(1-x)\left[\dfrac{bu}{x} - ce(q_b+q_s)\right]}{1+R}. \tag{15}$$

Lemma 3. *A monetary equilibrium in which money traders only trade for preferred goods* ($Y > bu/e$) *exists only if the seller has sufficient bargaining power,* $\sigma > \bar{\sigma} \equiv b/(1-e+b)$, *to extract services from the buyer,*

$$q_b = \frac{\sigma(1-e) - b(1-\sigma)}{c(1-e)}u > 0 \quad \text{and} \quad q_s = 0.$$

Proof. Suppose not, $q_b = 0$. Then $q_s \geqslant 0$ and the conditions for $S_s \geqslant 0$ and $S_g \geqslant 0$ imply $cq_s \leqslant bu$ and $q_g \leqslant (1-e)q_m + Y/c$. Substituting these inequalities into Eq. (15) yields

$$[R + e(1-M)]Y \leqslant -(1-M)(1-x)bu\left(\frac{1}{x} - e\right) - e(1-M)(2-e)cq_m < 0.$$

Hence $Y < 0$ if $q_b = 0$. From Eq. (7) $q_b > 0$ and $q_s = 0$ if and only if $\sigma > \bar{\sigma}$. □

This lemma indicates that for highly valued money, barter must involve a service payment to the seller (our main case). Substituting q_b and q_s into Eq. (15) yields:

$$Q = f(Y) \equiv \frac{(1+R)Y - (1-M)(1-x)[e\sigma - b[\frac{1}{x} + e\frac{1-\sigma}{1-e}]]u}{c[1-e(1-M)]}. \tag{16}$$

where

$$Q \equiv q_g - \frac{1-eM}{1-e(1-M)}q_m$$

is a new variable used to replace both q_g and q_m.

The bargaining outcomes for q_m and q_g can also be used to express Q in terms of Y because q_m and q_g only depend on Y in Lemma 2:

$$Q = Q(Y) \equiv \begin{cases} \dfrac{1-eM}{1-e(1-M)} \dfrac{(1-e\sigma)Y-(1-e)\sigma u}{c(1-e)} & \text{if } Y \in \left[0, \dfrac{(1-e)\sigma u}{1-e\sigma}\right), \\ 0 & \text{if } Y \in \left[\dfrac{(1-e)\sigma u}{1-e\sigma}, \dfrac{\sigma u}{1-e\sigma}\right], \\ \dfrac{[1-e+e\sigma]Y-\sigma u}{c(1-e)} & \text{if } Y \in \left(\dfrac{\sigma u}{1-e+e\sigma}, u/e\right], \\ \text{no trade} & \text{if } Y > u/e. \end{cases}$$

A monetary equilibrium is a solution to $Q(Y) = f(Y)$. Once Y is found, Lemma 2 and the definition of Q can be used to find the equilibrium values for q_m and q_g.

Fig. 3 plots $f(Y)$ and $Q(Y)$. The intersections E^s and E^w are defined as follows:

$$E^s\text{: } q_m = 0,\ q_g = 0, \quad Y = \frac{(1-M)(1-x)}{1+R}\left[e\sigma - b\left[\frac{1}{x} + e\frac{1-\sigma}{1-e}\right]\right]u;$$

$$E^w\text{: } q_m = \frac{\sigma u}{c} - \frac{1-e\sigma}{1-e}\cdot\frac{Y}{c},\ q_g = 0,$$

$$Y = (1-e)\frac{(1-eM)\sigma - (1-M)(1-x)\left[e\sigma - b\left[\dfrac{1}{x} + e\dfrac{1-\sigma}{1-e}\right]\right]}{(1-e\sigma)(1-eM) - (1-e)(1+R)}u.$$

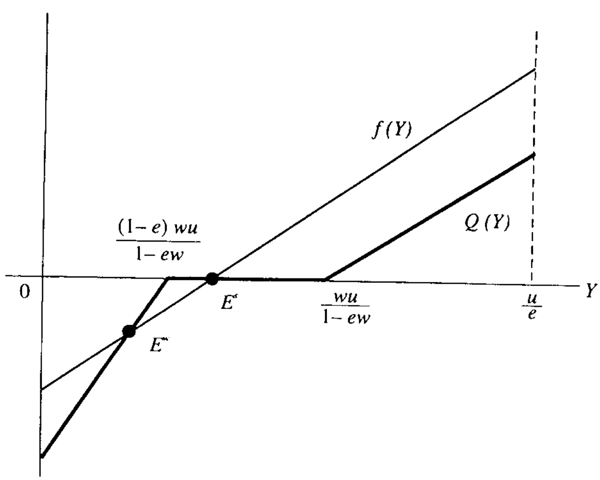

Fig. 3. Existence of monetary equilibria.

where $R \equiv r/\beta x$. In E^s no services are rendered by the money trader whereas in E^w service payments are positive. Hence, the relative value of holding money as opposed to a good, Y, is greater in E^s. In E^w, $q_m \geqslant 0$, if and only if

$$b \leqslant b^* \equiv \frac{e\sigma(1-e\sigma)(1-M)(1-x)-(1-e)\sigma(1+R)}{(1-M)(1-x)(1-e\sigma)\left[\dfrac{1}{x}+\dfrac{e(1-\sigma)}{1-e}\right]}. \tag{17}$$

This expression turns out to be a condition for the existence of a monetary equilibrium. It is satisfied for $\bar{\sigma} < \sigma < 1, 0 < x < 1$, and e sufficiently close to 1.

Proposition 7. A monetary equilibrium (in which money traders only purchase preferred good) exists if and only if $b \leqslant b^*$, in which case E^s and E^w are monetary equilibria.

Proof. First, we show that $b \leqslant b^*$ is the relevant restrliction on b. There are three other conditions restricting b. Lemma 3 requires $\sigma > \bar{\sigma} \equiv b/(1-e+b)$ or equivalently $b < (1-e)\sigma/(1-\sigma)$. Restriction 3 $Y > bu/e$ implies upper bounds for b with respect to E^s and E^w:

$$b^{s*} \equiv \frac{(1-M)(1-x)e^2\sigma}{1+R+e(1-M)(1-x)\left[\dfrac{1}{x}+\dfrac{e(1-\sigma)}{1-e}\right]},$$

$$b^{w*} \equiv$$

$$\frac{e(1-e)\sigma[1-eM-e(1-M)(1-x)]}{(1-e\sigma)(1-eM)-(1-e)(1+R)-e(1-M)(1-x)\left[e(1-\sigma)+\dfrac{1-e}{x}\right]}.$$

It can be shown that $b^* < b^{s*} < (1-e)\sigma/(1-\sigma) < b^{w*}$, so that if $b \leqslant b^*$ is satisfied the other conditions are also satisfied.

Second, we show that there is a fixed point for Y if and only if $b \leqslant b^*$. The following features of Fig. 3 can be directly verified:

1. There is no intersection between $f(Y)$ and $Q(Y)$ which gives a positive Q for $Y \leqslant u/e$.
2. The intersection between $f(Y)$ and the horizontal axis lies in $(0, \sigma u/(1-e\sigma))$. This intersection is on the left of $(1-e)\sigma u/(1-e\sigma)$ if and only if $b > b^*$.
3. As $Q(0) < f(0) < 0$, the intersection E^w exists if and only if $b \leqslant b^*$, and E^s exists if and only if $b \leqslant b^*$. Hence E^w and E^s are the only equilibria. When

$b = b^*$, point E^w coincides with point E^s on the horizontal axis and the two equilibria coincide. □

Corollary 1. *The condition for a monetary equilibrium, $b \leqslant b^*$, implies:*

$$e > \frac{1-\sqrt{1-\sigma}}{\sigma} > \frac{1}{2}, \qquad x < \frac{2e-1-e^2\sigma}{e(1-e\sigma)},$$

$$M < 1 - \frac{1-e}{e(1-x)(1-e\sigma)}, \qquad R < \frac{e(1-e\sigma)(1-M)(1-x)}{1-e} - 1.$$

Proof. The condition for R comes directly from $b^* > 0$, which represents the lower bound of b^*. For that restriction to be satisfied by some positive R, in turn, M has to satisfy the condition in the Corollary. For the condition on M to be satisfied by some number in (0,1), the condition on x in the Corollary must hold. For the condition on x to be satisfied by some number in (0, 1), $e > (1-\sqrt{1-\sigma})/\sigma$. □

As long as $\sigma \in (\bar{\sigma},1)$, there is a parameter region in which a monetary equilibrium exists. If a monetary equilibrium exists, then equilibrium E^s coexists with E^w. The monetary equilibria have different terms of trade, indicated by $q_m(E^s) = 0 < q_m(E^w)$, and relative values of holding money, $Y(E^s) > Y(E^w)$. As both favor E^s, we refer to E^s as the *strong equilibrium* and refer to E^w as the *weak equilibrium*. It can be readily shown that $S_m(E^s) > S_m(E^w)$ and $S_g(E^s) > S_g(E^w)$ so that both money traders and good traders are better off in E^s. Money traders are able to secure better terms of trade when money is highly valued, and money is more valuable in equilibrium when it improves the money trader's terms of trade.[11]

The following proposition follows immediately from Eqs. (13) and (14) and the fact that the surpluses in monetary trades are greater in the strong equilibrium.

Proposition 8. *The strong equilibrium E^s Pareto dominates the weak equilibrium E^w.*

[11] Interestingly, the flexible price models of Shi (1995) and Trejos and Wright (1995) also display an even number of robust monetary equilibria which coexist and differ in the purchasing power of money. This suggests that multiple monetary equilibria are a feature of search and bargaining money models. Burdett and Wright (1998) find multiple equilibria in other search models with constant returns to scale matching.

In both monetary equilibria, agents strictly prefer to hold money as it improves their expected terms of trade. In particular, conditional on being matched with a good trader, the expected quantity of services supplied by a money trader is less than that supplied by a good trader: $x(1-x)(q_b+q_s) > xq_m + (1-x)q_m^M$. This is a condition we derive later as necessary for all robust monetary equilibria. Here it implies that $(1-x)q_b > q_m$.

The improved terms of trade yields greater surpluses for those in a money trade than in an AGM, $S_m > S_b$ and $S_g > S_s$. Were it possible to isolate an AGM without upsetting the monetary equilibrium, both the buyer and the seller would benefit from the buyer's good being replaced with a unit of money. The barter inefficiency stemming from AGMs is necessary for the monetary equilibrium.

However, the value of money is not monotonically increasing in the degree of inefficiency. When $\sigma = 1$, the barter inefficiency is the greatest. Nevertheless, no monetary equilibrium exists. Good traders are able to extract all the surplus from money traders which implies that money is valueless by Eq. (14). The distribution of the surplus is too extreme. In contrast, when $\sigma \in (\bar{\sigma}, 1)$ the Nash solution shares the surplus such that $S_m > 0$ and $S_b > bu$ and monetary equilibria exist.

5. The optimal quantity of money

As before, we consider the optimal quantity of money that maximizes the expected utility function, $Z = MV_m + (1-M)V_g$. Differentiating the function yields,

$$\frac{dZ}{dM} = Y + M\frac{dV_m}{dM} + (1-M)\frac{dV_g}{dM}.$$

The first term on the right-hand side is the benefit from replacing a good trader with a money trader whereas the other two terms describe the effect of the additional money trader on existing money and good traders.

In the model without services, there was no advantage to holding money relative to a good, $Y = 0$. Money simply displaces goods making both good traders and money traders worse off, $dV_m/dM < 0$ and $dV_g/dM < 0$. Not only does money play no useful role, it also reduces welfare at a dramatic rate, $dZ/dM = -Z^*$.

In contrast, in the model with services, the optimal supply of money may be positive. This is because the agent given money strictly prefers it, $Y > 0$. As in other search models, increasing money also has a welfare decreasing effect from reducing the number of good traders (which is an implication of the inventory assumption made for tractability reasons that an agent cannot hold money and a good at the same time). The welfare decreasing effect from displacing good traders is particularly strong in our model because of the universal double coincidence of wants in goods. Thus, the range of parameters for positive optimal money supply is more severely restricted than in the Kiyotaki and Wright models. Nevertheless, the bargaining terms of trade may be sufficiently beneficial to produce a welfare-improving role for money in the absence of other frictions.

We examine the optimal money supply in the strong monetary equilibrium. Substituting $V_{\mathrm{m}}(E^{\mathrm{s}})$ and $V_{\mathrm{g}}(E^{\mathrm{s}})$ into the expected utility function yields:

$$Z = (1-M)\left\{1-(1-M)(1-x)\left[e\sigma - b\left(\frac{1}{x}+\frac{e(1-\sigma)}{1-e}\right)\right]\right\}u/R.$$

The optimal quantity of money, M^{o}, is derived by maximizing this function subject to the nonnegativity condition $M \geqslant 0$ and the equilibrium constraint $b \leqslant b^*$. The solution depends on critical values for σ and e in terms of the other parameters. Define

$$\sigma_{\mathrm{L}} \equiv \frac{1-e}{e(1-e+b)}\left[\frac{1}{2(1-x)} + b\left(\frac{1}{x}+\frac{e}{1-e}\right)\right],$$

$$\sigma_{\mathrm{H}} \equiv \frac{-B_{\mathrm{H}} - \sqrt{B_{\mathrm{H}}^2 - 4A_{\mathrm{H}}C_{\mathrm{H}}}}{2A_{\mathrm{H}}},$$

$$e_\sigma \equiv \frac{2(1+R)[x+2(1-x)b]}{2x(1-x)+(1+2R)[x+2(1-x)^2 b]},$$

where

$$A_{\mathrm{H}} = -e^2\left(1+\frac{b}{1-e}\right), \quad B_{\mathrm{H}} = e\left[1+\frac{b(1+e)}{1-e}+\frac{b}{x}\right] - \frac{(1-e)(1+R)}{1-x}$$

and $C_{\mathrm{H}} = -b\left(\dfrac{1}{x}+\dfrac{e}{1-e}\right)$

so that $\sigma_{\mathrm{H}} < 1$. Note that $e_\sigma < 1$ if b is sufficiently small,

$$b < \min\left[b^*, \frac{1-2x}{2(1-x)(\frac{1}{x}+1+2R)}\right].$$

Proposition 9. (i) *The optimal quantity of money in the strong equilibrium is*

$$M^{\text{o}} = \begin{cases} 1 - \dfrac{1}{2(1-x)[e\sigma - b(\frac{1}{x} + \frac{e(1-\sigma)}{1-e})]} > 0, \\ \qquad \text{if } \sigma_{\text{L}} < \sigma < \dfrac{1}{e + 2(1+R)(1-e)}; \\ 1 - \dfrac{\sigma(1-e)(1+R)}{(1-e\sigma)(1-x)\ [e\sigma - b(\frac{1}{x} + \frac{e(1-\sigma)}{1-e})]} > 0, \\ \qquad \text{if } \dfrac{1}{e + 2(1+R)(1-e)} \leqslant \sigma < \min[\sigma_{\text{H}}, 1]; \\ 0 \quad \text{otherwise;} \end{cases}$$

(ii) *The interval* $(\sigma_{\text{L}}, \min[\sigma_{\text{H}}, 1])$ *is nonempty if and only if* $e > e_\sigma$.

The optimal quantity of money is positive over the interval $(\sigma_{\text{L}}, \min[\sigma_{\text{H}}, 1])$ provided that services poorly transfer utility $e > e_\sigma > \frac{2}{3}$. The first part of the interval $\sigma \in (\sigma_{\text{L}}, 1/(e + 2(R+1)(1-e)))$ corresponds to when the constraint $b \leqslant b^*$ does not bind. Over this range the optimal money supply is increasing in σ. The remaining portion of the interval $\sigma \in (1/(e + 2(R+1)(1-e))), \min[\sigma_{\text{H}}, 1])$ corresponds to when the constraint $b \leqslant b^*$ binds. $M^{\text{o}} = 0$ corresponds to the nonmonetary equilibrium: $Z = V_{\text{g}}^{\text{n}}(N)$. Though money may improve welfare, it can not achieve the constrained optimum.

Factors that generate a substantial inefficiency from barter lead to a welfare-improving role for money. Money improves welfare only when the seller has substantial bargaining power, $\sigma > \sigma_{\text{L}}$.[12] This implies that the buyer must produce more little-valued services for the seller in AGMs. Thus, the inefficiency from barter is substantial. Increasing the money supply has the effect of replacing AGMs with money matches. Each AGM replaced with a money match reduces the exchange of little-valued services and increases the surplus by $S_{\text{m}}(E^{\text{s}}) + S_{\text{g}}(E^{\text{s}}) - (S_{\text{s}} + S_{\text{b}}) = e\sigma u - b(1-e\sigma)u/(1-e) > 0$. This difference is increasing in σ and explains why the optimal money supply increases with σ when the constraint is not binding. The optimal money supply is also increasing in e, when b is sufficiently small.

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[12] In the particular environment we are using, $\lim_{b\to 0}\sigma_{\text{L}} \to 1/2e(1-x) > 1/2$ from above. Introducing a fixed cost to producing services yields a greater bargaining range $\sigma_{\text{L}} < 1/2$ consistent with positive optimal money supply.

6. Arbitrary terms of trade

The above robust monetary equilibria displayed: asymmetric matches ($0 < x < 1$), services transfer utility imperfectly ($e > 0$), and an improvement in the expected terms of trade for money holders. We now show that these elements are necessary to the existence of any robust monetary equilibrium when Restrictions 2 and 3 are relaxed. We also show that these elements are also necessary for money to play a welfare-improving role.

Consider a random matching economy where the terms of trade may not be determined by bargaining and money may be exchanged for mediocre goods. In particular, let the service payments $q_b \geqslant 0$, $q_s \geqslant 0$, $q_m \geqslant 0$, $q_g \geqslant 0$, $q_m^M \geqslant 0$, and $q_g^M \geqslant 0$ be chosen arbitrarily consistent with feasibility and individual rationality. Otherwise, exchanges are as before.

To develop a restriction on Y we subtract Eq. (2) from Eq. (3) and substitute the definitions of the surpluses, Eqs. (4), (9) and (11). This yields

$$\left(1+\frac{r}{\beta}\right)Y = (1-M)(1-x)xce(q_b+q_s) - (1-eM)[xcq_m + (1-x)cq_m^M] + [1-e(1-M)][xcq_g + (1-x)cq_g^M]. \tag{18}$$

Since S_g, $S_g^M \geqslant 0$, the expressions for S_g and S_g^M imply

$$cq_g \leqslant Y + c(1-e)q_m; \qquad cq_g^M \leqslant Y + c(1-e)q_m^M.$$

Using these inequalities into Eq. (18) yields

$$[1+\beta e(1-M)/r]Y \leqslant (1-M)\beta x(1-x)ce(q_b+q_s)/r - \beta(2-e)(1-M)ce[xq_m + (1-x)q_m^M]/r.$$

This is the inequality we need. Using Eq. (8) the first term on the RHS can be expressed

$$(1-M)\beta x(1-x)ce(q_b+q_s)/r = (1-M)(Z^* - V_g^n),$$

where $Z^* - V_g^n$ is the inefficiency from barter in a nonmonetary economy (for given values of q_b and q_s). Thus, this first term can be interpreted as the inefficiency from barter exchanges in the monetary economy. Clearly, this inefficiency must be positive for a robust monetary equilibrium, $Y > 0$. Hence, $0 < x < 1$, $e > 0$, and $q_b + q_s > 0$ are all necessary for a robust monetary equilibrium. These same elements generated an inefficient nonmonetary economy in Proposition 5.

Further, for $Y > 0$ and $e > 0$ the above inequality implies:

$$x(1-x)(q_b+q_s) > \frac{2-e}{1-x}[xq_m + (1-x)q_m^M] > xq_m + (1-x)q_m^M.$$

That is, a robust monetary equilibrium requires that conditional on being matched with a good trader, the expected quantity of services supplied by a money trader is less than that supplied by a good trader. Clearly this requirement is satisfied only if $0 < x < 1$ and $q_b + q_s > 0$. The following proposition summarizes the results.

Proposition 10. *For a robust monetary equilibrium to exist all of the following are necessary*: (i) $0 < x < 1$, (ii) $e > 0$, *and* (iii) *service payments satisfy* $x(1-x)(q_b + q_s) > xq_m + (1-x)q_m^M$. *These elements generate an inefficiency from barter exchanges.*

In a robust monetary equilibrium agents strictly prefer to hold money as it improves their expected terms of trade. In Section 4 we found that the Nash bargaining solution could induce the improvement in the expected terms of trade required for a robust monetary equilibrium. Though this section examines arbitrary terms of trade, we believe that the Nash solution is the natural mechanism to concentrate on in the current bilateral matching environment. Obvious alternative candidates fail to yield robust monetary equilibria. For example, consider the case in which those who own the more dearly valued item – the seller in AGMs and the good trader in money matches – make take it or leave it offers. This is equivalent to $\sigma = 1$ in our model so no robust monetary equilibrium exists.

The following proposition establishes that the same elements which are necessary for a robust monetary equilibrium are also necessary for a welfare-improving role for money.

Proposition 11. *If the barter terms of trade*, (q_b, q_s), *are the same in a nonmonetary and a monetary equilibrium, then the monetary equilibrium is welfare improving only if*: (i) $0 < x < 1$; (ii) $e > 0$; *and* (iii) *services satisfy* $x(1-x)(q_b + q_s) > xq_m + (1-x)q_m^M$.

Proof. From the expressions of V_m and V_g, Eqs. (2) and (3), and surpluses, Eqs. (4), (9) and (11), we obtain the following relation for the welfare level Z:

$$rZ = r(1-M)Z^* - (1-M)^2\beta x(1-x)ec(q_b + q_s)$$

$$- M(1-M)\beta ec[x(q_m + q_g) + (1-x)(q_m^M + q_g^M)].$$

The welfare level in a nonmonetary equilibrium, denoted V_g^n, is given by Eq. (8)

$$rV_g^n = rZ^* - \beta x(1-x)ec(q_b + q_s).$$

We can establish that $V_g^n > 0$ by substituting q_b and q_s from Eq. (5) and imposing individual rationality. Subtracting the above expressions yields

$$r(Z - V_g^n) = M(1 - M)\beta ec[x(1 - x)(q_b + q_s) - x(q_m + q_g) - (1 - x)(q_m^M + q_g^M)] - MrV_g^n.$$

As $V_g^n > 0$, for any $M > 0$, we have

$$r(Z - V_g^n) < M(1 - M)\beta ec[x(1 - x)(q_b + q_s) - x(q_m + q_g) - (1 - x)(q_m^M + q_g^M)].$$

As q_g, $q_g^M \geq 0$, it is clear that for any $M > 0$, $Z > V_g^n$ implies conditions (i)–(iii). □

The restriction that the barter terms of trade are the same in both non-monetary and monetary equilibria need not be restrictive. For example, Nash bargaining yields the same barter terms of trade in both equilibria.

7. Conclusions

Using the search-theoretic approach to monetary economics, this paper provides insights into two classic questions in monetary economics: Why do people readily accept fiat money in exchange? and When is the use of fiat money welfare improving? We find that people strictly prefer to accept fiat money when money improves their expected terms of trade relative to barter. Money is welfare improving when it improves money traders' expected terms of trade sufficiently. Thus there is both a positive and normative role for money in improving the terms of trade.

This role of money only arises when barter is inefficient so that utility is transferred imperfectly in positive surplus matches. For this to be the case in our model, it is necessary that services transfer utility imperfectly. However, it is not sufficient. In our setup, trading goods but not services in all matches achieves the constrained optimum. Therefore, for barter to be inefficient, the terms of trade must also be inefficient and require the provision of services. This is the natural outcome in asymmetric matches where the surplus is divided between trading partners according to the Nash solution.

For robust or welfare improving monetary equilibria, money traders must receive better expected terms of trade than good traders. The Nash solution induces better terms of trade for money traders because money is more valuable in equilibrium than trading with goods. Money's greater value gives the trader generalized bargaining power, independent of the trading partner's preferences.

As in all monetary models, the value of money depends on beliefs. In our model the Nash solution yields the buyer better terms of trade the more highly valued is money. There are multiple monetary equilibria. Both the value of money and the terms of trade are greater in the strong equilibrium and it Pareto dominates the weak equilibrium.

The value of money also depends on the bargaining specification. Interestingly, under Restriction 2 (the seller's weight in an AGM is the same as a good trader's weight in a money match) only bargaining weights which strictly divide the surplus yield robust monetary equilibria. Thus, when the seller/good trader has all the bargaining power there is no monetary equilibrium even though the barter inefficiency is greatest. On the other hand, if the seller/good trader has little bargaining power, there is no inefficiency and no robust monetary equilibrium exists. Only when the bargaining power is divided relatively evenly will money improve the terms-of-trade.

Though some of our findings are special to our model, we believe that the presence of asymmetry, imperfectly transferable utility, and the role of money in improving the terms of trade are important elements for economies with robust or welfare-improving monetary equilibria. Of course, these elements may manifest themselves in different ways depending on the model.[13] For example, in Kiyotaki and Wright (1993) the absence of the universal double coincidence of wants results in matches where only one agent values the other's good. In these asymmetric good matches, no barter takes place at feasible terms of trade because the buyer cannot transfer sufficient utility to the seller.[14] Money improves the expected terms of trade by facilitating more exchanges which speeds up the rate of acquiring the preferred good. In Williamson and Wright (1994) private information yields informationally asymmetric matches where barter does not take place because agents are not sure of the quality of the other agent's good. Money improves the expected terms of trade by increasing the probability of acquiring high quality goods. As in Kiyotaki and Wright, money improves

[13] Non-search models also display the same elements. For example, Engineer and Bernhardt (1991) examine a turnpike model where good valuation is asymmetric and barter involves the imperfect transfer of utility. Money is robustly valued and welfare improving when it improves users terms of trade by transferring utility more effectively than barter. Engineer and Bernhardt also examine an adverse selection problem. There the asymmetry arises from the exchange of the wrong goods in barter and utility is transferred imperfectly. Again money has value if it has a lower opportunity cost of use than barter.

[14] Kiyotaki and Wright (1993) assume a cost of accepting a good $\varepsilon > 0$ to preclude barter in AGMs. Similarly, we can also preclude barter in AGMs by introducing a sufficiently high cost of accepting the good $\varepsilon > [(1 - e) + b]u/(2 - e)$. This condition makes it impossible for the buyer in AGMs to compensate the seller. Notice that if utility is not transferable, $e = 1$, then the condition is $\varepsilon > bu$. Further, if there is no double coincidence of wants in AGMs, $b = 0$, then the condition is $\varepsilon > 0$. Kiyotaki and Wright's (1993) model can be thought to correspond to our model when $e = 1$, $b = 0$, and $\varepsilon > 0$.

liquidity. A key distinguishing feature of our analysis is that money improves the expected terms of trade not by generating more exchanges but rather by generating better exchanges for money traders.

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