PAIRWISE CREDIT IN SEARCH EQUILIBRIUM*

PETER DIAMOND

Pairwise extension of credit is introduced into the barter-search economy previously analyzed by the author. The penalty for failure to repay a debt is modeled as the end of trading opportunities. Since credit availability makes access to trade more valuable, there may be multiple equilibrium credit limits. Since the credit limit affects the implicit interest rate and the stock of inventories, it is necessary to check the net impact of the credit limit on the incentive to repay. In a calculated example, with lumpy credit availability, multiple equilibria are very common with a greater credit limit associated with a lower implicit interest rate. With smooth credit availability no multiple equilibria were found. Surprisingly, credit can break the no-production equilibrium.

The factors determining the extent of liquidity in an economy can be divided into two groups. One group relates to the characteristics of the investment opportunities in the economy, including the costs of observation and verification of the characteristics of the investments and the extent of uncertainty about investment returns. The second group has to do with the organization of the availability of credit. The interesting recent literature on the characteristics of credit markets and their properties has focused in particular on the presence of multiple equilibria in such markets. These papers have concentrated either on the workings of conventional financial markets or on the workings of financial intermediaries. The Chatterjee [1988] and Pagano [1989] papers considered fixed costs of entering markets as one of the determinants of the thickness of the markets and found multiple equilibria. Similarly, multiple equilibria play a central role in the recent bank run literature following the Douglas Diamond and Dybvig paper[1] [1983]. In that literature agents can move very quickly to withdraw funds while the production technology is illiquid. This paper examines whether there are similar multiple equilibria in a model where the underlying technology for bringing people together has everyone moving slowly; thus basing illiquidity in credit and trading limitations rather than in the production technology.

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1. For a partial survey of this literature, see my [1987a] paper.

We consider two situations; first where credit is lumpy, then where it is smooth. The former leads frequently to multiple equilibria. The latter does not appear to. This contrast highlights the importance of lumpiness or fixed costs in the possibility of multiple equilibria. Fixed costs associated with credit transactions or the arrangement of lines of credit are realistic phenomena. This suggests that multiple equilibria might be reintroduced into the smooth model from a further extension which added costs of arranging credit as a substitute for arbitrarily lumpy debt positions.

The paper starts with a simplification of my [1982] barter model of search equilibrium and then introduces lumpy credit to examine its effects. For mathematical tractability there is no money. Two questions are asked. The model is set up in a world where not only is there no money because, let us say, nobody has thought of the idea, but also there is no credit because no one has thought of that idea. If someone thinks of the idea of credit, will credit be introduced into this economy? Credit appears in the model to finance lumpy purchases. The central focus is on the conditions one must look for with pairwise trade for credit not to be introduced into an economy like this. That is, the continuous time steady state equilibrium without any credit remains an equilibrium with no actual credit taking place within the rules of credit introduction assumed.

Then we consider an economy where credit is readily available with the same sorts of information and penalty rules that go with the first economy. We derive the parameters for which there is a steady state equilibrium with readily available credit. The step of bringing these two models together is to ask whether there is an overlap in parameter space between those parameters that allow an equilibrium with no credit and those parameters that allow an equilibrium with readily available credit. We shall find that there are many such parameters. This implies that this economy may have multiple steady state equilibria: one equilibrium where the ready availability of credit is a self-fulfilling description of the state of the economy and another where the absence of credit is a self-fulfilling description of the economy. This implies that in a model not restricted to steady states one could construct all sorts of rational expectations dynamic paths [Diamond and Fudenberg, 1989]. Some of these may resemble things that happen in economies where credit availability changes rather rapidly, and this has a major feedback on the production level in the economy. The underlying presumption here is that some aspects of the fluctuations in the economy come from credit feedback mechanisms.

Lumpiness plays an important role in finding multiple equilibria, as is spelled out below. With no fixed setup costs for arranging credit, credit is always introduced to the economy (except at the knife-edge where equilibrium with production is just sustainable). Interestingly, for some parameters credit is also introduced in the equilibrium with no production.

Before getting into the details, let me give a flavor of what makes this work. Essential for credit is a belief in repayment by the borrower. The penalties for failure to repay affect the incentive to repay. We assume that failure to repay keeps the borrower out of the trading network. Therefore, anything making trade more valuable increases the incentive to repay. Ceteris paribus, generally available credit has this effect. Thus, a greater credit limit tends to justify itself by the induced increases in the willingness to repay. However, a greater credit limits affects prices and quantities in the economy. These also affect the incentive to repay. Thus, it is necessary to examine the net effect of a greater credit limit on the incentive to repay in order to evaluate the possibility of multiple equilibria.

When supplying credit, one is tying up one's purchasing power for some length of time. One's interest in tying up funds depends in part on the different scenarios one sees happening, in particular the possibility that one might need or want purchasing power before the debt is repaid. If that is true, the creditor has the option of borrowing. The willingness of someone to provide credit thus depends in part on the prospects of getting credit himself at some point in the future. So if credit is perceived as hard to get, then lenders are relatively unwilling to provide credit. Similarly, if credit is easy to get, lenders are relatively willing to provide credit, making it easy to get. This positive feedback mechanism affects the endogenous terms of credit. This feedback loop arises naturally in the economy that is modeled here. It is not what I believe to be the most important feedback loop, although I think the important loops have some of the same character to them. There is a great deal of short term debt that is regularly rolled over. One's willingness to lend to someone who is regularly rolling over short-term debt depends more on one's belief of their continued ability to roll over the short-term debt than on one's own ability to borrow. I think that is the important feedback loop when we get rather rapid cutoffs of credit to firms that were previously capable of borrowing considerable sums. This paper represents an early step in the development of models of the micro foundations of the endogenous and varying availability of credit and the link of availability with demand and production.

I. Basic Model Without Credit

In order to have a model with both continuous time and discrete transactions, one needs to have a complicated purchase and storage technology or a preference structure that is different from the standard integral of discounted utility of instantaneous consumption. The alternative preferences I work with have the consumption good in an indivisible unit, which is consumed from time to time. Denote by y the utility that comes whenever one of these units is consumed. This is an instantaneous utility from a discrete consumption at an instant of time. That is a mathematically convenient approximation to the fact that it does take a while to consume goods. But we also do not go around consuming (or purchasing) nondurable goods continuously through the day. Similarly, production of consumer goods takes time but is modeled as an instantaneous process. (Modeling the length of time to complete production as a Poisson process permits a straightforward generalization of this class of models.) After production, the good is carried in inventory until it can be traded. Denote by c the labor disutility of instantly producing one unit of this good. All opportunities involve the same cost. Instantaneous utility thus satisfies

$$U = y - c. \tag{1}$$

For viability of the economy, we assume that $0 < c < y$. Over time there is a sequence of dates, t_i, at which one will have opportunities either to acquire a unit to consume or to produce a unit for trade. The preference of the individual (identical for all agents) are representable as the expected discounted sum of the utilities associated with this random stream of discrete events as given in equation (2):

$$V = \sum_{i=1}^{\infty} e^{-rt_i} U_{t_i}. \tag{2}$$

The focus of this analysis is on trade, so it will not do to have this economy collapse into autarchy, with people producing and promptly consuming what they produce themselves. Therefore, we add some restrictions. The first restriction is that individuals never consume what they produce themselves. This can be thought of as a physical impossibility or an element of preferences—people just do not like the good that they themselves produce. On producing a unit, agents look for someone else who also has one unit with whom to barter. The other restriction that will keep the model simple is that the inventory carrying costs are such that one never carries more than one unit of good available for trade. Thus, an individual in this economy is in one of two positions. Either he has no goods in inventory and is unable to trade, or has one unit of good in inventory and is available to trade. In the former case the agent is looking for an opportunity to produce. I spread opportunities out smoothly in time by assuming a Poisson process with arrival rate a for the opportunity to give up the labor disutility c and add one unit to inventory. This process goes on continuously: there is no cost in being available to produce; there is merely a cost in actually carrying out production. Of course once one has an opportunity to produce, one still has a choice. One does not have to produce. If one has a unit in inventory, one does not produce because one cannot carry the good in inventory. Without a unit in inventory one looks ahead to the length of time it will take to trade a unit if produced. The utility y obtained when the good is traded one-for-one and consumed will happen some time in the future and will be discounted by the utility discount rate r. Therefore, it will only be worthwhile to produce for trade if the process of carrying out a trade is fast enough relative to the utility discount rate and to the gap between the utility of consumption and the disutility of production.

Denote by e the fraction of people with inventory for sale. If every opportunity is carried out, and that will be the first assumption, then e is growing as all the people without goods for sale, the fraction $1 - e$, carry out all of their opportunities. (Thus, we normalize the implicit continuum of the population to one.) In addition, people will be meeting each other. They will carry out a trade whenever they have the opportunity. In a barter economy with no money and no credit, such a trade can be carried out only when both of the people meeting have inventory to trade. We are not concerned here with the double coincidence of their liking each other's goods. That is assumed to happen automatically. But we are concerned with a double coincidence in timing. Two people must come together at a time when they both have goods in inventory.

They do not have the ability—the communications technology—to keep track of lots of potential trading partners and so instantly trade on completing production. The underlying idea here is that for many goods consumers are not searching for the good, they are searching for the good in the right size, color, and design. So retailers stock large quantities of goods that are held for consumers who do a great deal of shopping, not because it is hard to find out who is a supplier but because it may take some time to find one that has available precisely what is wanted.

We assume that this meeting process takes the simplest possible stochastic form of random meetings between individuals. These meetings are going on all of the time. Any individual experiences a Poisson arrival of people at rate b. This is again a Poisson process with an exogenous technological parameter. But some of the people met have no inventory and cannot be traded with. Some of the people met have inventory and can be traded with. So the rate at which goods can be traded is be, an endogenous variable depending on the stock of inventories in the economy. An economy with a high level of production will have strong incentives to produce for inventory because it is easy to meet trading partners. Equation (3) is the differential equation for the behavior of inventories over time assuming that all production opportunities are carried out (below we give a sufficient condition for this behavior to be consistent):

$$\dot{e} = a(1 - e) - be^2. \tag{3}$$

That is, each of the fraction e with inventories faces the probability be of having a successful trade meeting and being freed to seek a new opportunity. Each of the $1 - e$ without inventories has the flow probability a of learning of an opportunity. With all opportunities taken, the employment rate converges to e_0, the solution to $\dot{e} = 0$ in (3):

$$2be_0 = (a^2 + 4ab)^{1/2} - a. \tag{4}$$

Note that e_0 is homogeneous of degree zero in (a,b). Note also that as b/a varies from 0 to $+\infty$ so does be_0/a. Equation (4) describes the steady state equilibrium at which we shall evaluate the possibility of credit.

In this steady state equilibrium we can calculate the expected discounted value of lifetime utility for those with and without inventory (W_e and W_u, respectively), assuming that production opportunities are worth carrying out. (If they are not, W_u is zero.) For each value, the utility rate of discount times value equals the expected dividend plus the expected capital gain.

$$rW_e = be(y - W_e + W_u); \tag{5}$$

$$rW_u = a(W_e - W_u - c). \tag{6}$$

Those with inventory wait for the utility from consumption plus a change in status to being without inventory. Those without inventory wait for the disutility of labor plus a change in status. Note that the value equations are homogeneous of degree one in (c,y) and homogeneous of degree zero in (a,be,r) and so in (a,b,r) given (4).

All projects will be taken if the capital gain from production, $W_e - W_u$, exceeds the cost of a project. To have an equilibrium at e_0, the economy must be productive enough to satisfy this condition. This condition is called the breakeven constraint and denoted by (B_0). Subtracting (6) from (5), we can write this breakeven conditions as

$$(B_0): \quad c \leq W_e - W_u = (bey + ac)/(r + a + be) = c_0^*(e). \tag{7}$$

We write the cost of a project that is just worth taking as $c_0^*(e)$. For later use we note that

$$rW_u = a(c_0^* - c) = (a(be(y - c) - rc))/(r + a + be) \tag{8}$$

where $W_u > 0$ is equivalent to $c < W_e - W_u$. Solving (7), we see that willingness to produce for sale can be written as

$$(B_0): c/y \leq be/(r + be). \tag{9}$$

In Figure I we plot the breakeven conditions relating c/y to b/a for given values of r/a, where e in (9) is set equal to e_0, given in (4) and dependent on b/a. We have an equilibrium below the curve B_0. That is, projects are worth undertaking if the arrival rate of trade opportunities is sufficiently large relative to the ratio of cost to value of a good. There are five parameters in this economy but with two normalizations there are really only three. There is the utility of consumption and the disutility of labor. All we are really interested in is their relative size c/y, which is on the vertical axis. There are three flow rates per unit of time: the utility discount rate, the arrival rate of production opportunities, and the arrival rate of trading partners. Since we are free to measure time any way we want, again we have a normalization. We divide through by a so b/a is on the

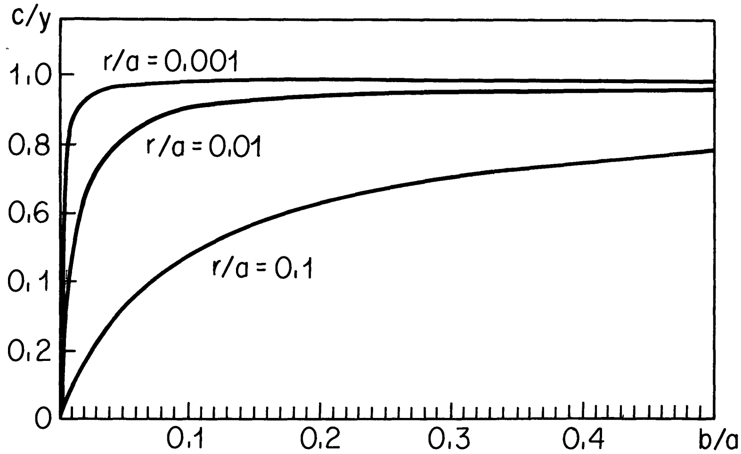

FIGURE I
Breakeven Condition

horizontal axis, and I have drawn the curves for three different utility discount rates.

That completes the picture of the economy. It is simpler than in my [1982] paper by having all projects cost the same. Of course, there is another uniform equilibrium in this economy that we ignore for now. If nobody ever produces anything, then it is obviously not worthwhile to produce for barter. Even with no production for trade, there is a possibility of introducing credit. We return to this issue below.

II. A Single Credit Transaction

We now wish to consider the introduction of credit to this barter economy, preserving the details of the search-trade technology and the simplicity of uniform inventory holdings. To do this, we introduce two assumptions. The first is that repayment of a loan involves no transactions cost and represents consumable output. (It would be straightforward to add a transaction cost in labor units paid by either the borrower or the lender.) That is, individuals have sufficient memory to costlessly find each other to complete the (delayed) barter transaction, but this memory (or perhaps taste for variety) does not permit a new transaction at the same time, nor the opening of a regular channel of trade. Nor do two individuals without inventory enter into contracts for two future deliveries.[2]

The second assumption (for now) is that credit terms are smoothly varied by changing the probabilities in a lottery but always involve repayment of a single unit. This simplifies keeping track of the state of the economy since all debtors will owe a single unit. Let us consider a pair of individuals who have come together in this no-credit steady state equilibrium. One of them has a unit of the good to trade, and the other one does not. The proposed trade begins with realization of a random variable. With probability p the inventory on hand is delivered for immediate consumption. Independent of the outcome of the random variable, the debtor promises that at his next opportunity to produce he will carry out production and deliver that good to the creditor. Thus, a probability of current delivery below one corresponds with a positive implicit interest rate. As we confirm below, there is always a probability of delivery, p, $0 < p \leq 1$, such that this trade is (ex ante) mutually advantageous provided that the borrower always repays. However, unless the borrower is known to be totally honest, the lender must check whether it is in the borrower's interest to repay. We turn to this concern after examining the existence of a mutually advantageous probability p. (A more complicated argument would consider the subjective probability of total honesty.)

Assuming repayment, the lender compares the probabilistic dynamic programing cost of giving up her unit of inventory with the utility gain from her own consumption adjusted for the expected waiting time. The trade is advantageous to the lender if

$$p(W_e - W_u) < ay/(r + a). \tag{10}$$

The condition is that the probability of delivering the good times the value of a unit of inventory be less than the expected value of delayed payment. Delayed payment is a Poisson process with arrival rate a and a payoff y discounted at rate r. The borrower needs to compare the certain cost of being in debt to the probabilistic gain of current consumption. The trade is advantageous to the borrower if

$$W_u - (a/(r + a))(W_u - c) \leq py. \tag{11}$$

2. I suspect that costs of completing transactions could be used to justify the value of one delayed payment but not two. A need to inspect goods, plus symmetry in evaluations, is an alternative route to justification.

If he enters the trade, the debtor switches from the status of waiting for production (with value W_u) to waiting for the opportunity to repay his debt (at cost c) which will then restore him to the status of waiting for production. Combining (10) and (11), there is a mutually advantageous trade if there is a value of p, $0 < p \leq 1$, satisfying

$$\left(\frac{r}{r+a}\right)\frac{W_u}{y} + \left(\frac{a}{r+a}\right)\frac{c}{y} \leq p \leq \left(\frac{a}{r+a}\right)\frac{y}{W_e - W_u}. \tag{12}$$

Using (7) and (8), we have

$$\left(\frac{r}{r+a}\right)\frac{W_u}{y} + \left(\frac{a}{r+a}\right)\frac{c}{y} = \left(\frac{a}{r+a}\right)\frac{W_e - W_u}{y}. \tag{13}$$

Since the value of a unit of inventory is less than its value from instant trade, we have $W_e - W_u < y$, implying that one can always find an interval of values of p satisfying.

$$\left(\frac{a}{r+a}\right)\left(\frac{W_e - W_u}{y}\right) \leq p \leq \left(\frac{a}{r+a}\right)\left(\frac{y}{W_e - W_u}\right). \tag{14}$$

III. Nonpayment Penalities

The lender must ask whether the borrower has an incentive to repay this loan if made. The answer depends on the structure of penalties available for enforcing contracts. Before turning to the particular example of penalty modeled here, I digress on alternative costs of bankruptcy. In the Arrow-Debreu model, individuals have lifetime budget constraints. Since the patterns of earnings and spendings are unconstrained, such a model is likely to result in a pattern with considerable amounts of credit being extended to individual consumers. It is an assumption of the model that all agents are completely honest and so budget constraints are balanced over lifetimes. In contrast, most consumer borrowing in the United States takes place against collateral, particularly houses and cars. The presence of widespread uncollateralized lending in the form of education loans promptly created a problem of the use of the bankruptcy institution to end the debt obligation. While there have been times when there were penalties such as debtors prison for those unable to pay off their debts, in the United States today not only are the direct costs of going through bankruptcy small, but individuals are allowed to keep noticeable amounts of their wealth to start them on their post-bankruptcy economic life. Where there may be a noticeable penalty associated with individual bankruptcy is where the occurrence of bankruptcy is a signal that will limit the borrowing, and so the trading opportunities, of individuals after bankruptcy. D. Diamond [1986] has examined such a model. In such a model the cost of bankruptcy is endogenous, depending on the trading opportunities that are being forgone.

The issue with corporate bankruptcy is somewhat more complex because the financial position of corporations is often more complicated, and because there is a question to be resolved as to whether the corporation should continue as an operating economic agent or should be liquidated. The process of bankruptcy is fairly slow [White, 1984], and the need to clear nonroutine transactions with trustees again implies a decrease in the range of possible trades.

Existing legal institutions for bankruptcy are unlikely to be optimal from the point of view of all debtor-creditor pairs [White, 1989]. With nonoptimal legal rules, the opportunities for contracting alternative penalties are limited by the set of rules that the legal system will enforce. This paper is not meant to explore in realistic detail the structure of bankruptcy penalties. Rather, the purpose is to use a structure of the cost of bankruptcy with reasonable properties in order to examine the implications of that structure for the possibility of multiple equilibria. For this purpose, the following structure is assumed.

We assume that it is observable to everybody whenever a production opportunity is carried out and that the legal system is available to enforce delivery to the lender if one is carried out. Thus, a loan is a form of equity contract, being a claim on the next unit of output with no specification of an exact date of repayment. But we assume that no one can observe whether there is in fact an opportunity which is not taken. So if a borrower chooses not to pay back, he does that by ceasing production. In other words, not repaying a loan implies dropping out of the economy, going to the autarchic state that I have implicitly modeled as the origin. Thus, the penalty is endogenous, depending on the value of continued trade. Having the cost of the penalty depend on the value of trade seems realistic. Naturally, I hope that the nature of the results depends on this fact, rather than on the precise structure of the penalty. These rules do not conform with modern bankruptcy law, but have the advantages of simplicity and of easy construction of a consistent equilibrium.

Debtors will repay if it is worthwhile to pay the cost of production to get access to later production opportunities. That there be inadequate incentive to repay I call the credit limit constraint. We shall have an equilibrium with a zero credit limit if the value of being in the economy with zero inventory W_u is less than the cost c of repaying a debt:

$$(CL_0)\colon W_u < c. \tag{15}$$

From the equation for W_u, (8), the credit limit condition can be written as

$$(CL_0)\colon c_0^* < ((r + a)/a)\, c. \tag{16}$$

Combining (16) with the breakeven constraint (7), we have an equilibrium if the parameters satisfy the inequalities,

$$c \le c_0^* < ((r + a)/a)\, c. \tag{17}$$

Substituting from (8), we write the credit limit condition as

$$(CL_0)\colon \frac{c}{y} > \frac{abe}{r(r + be) + abe + 2ar}. \tag{18}$$

Again, the condition is evaluated at e_0 satisfying (4). For $r/a = 0.1$, the shaded area between the two curves in Figure II contains parameter values $(c/y, b/a)$ for which we have an equilibrium without credit: the credit limit condition is satisfied above the curve

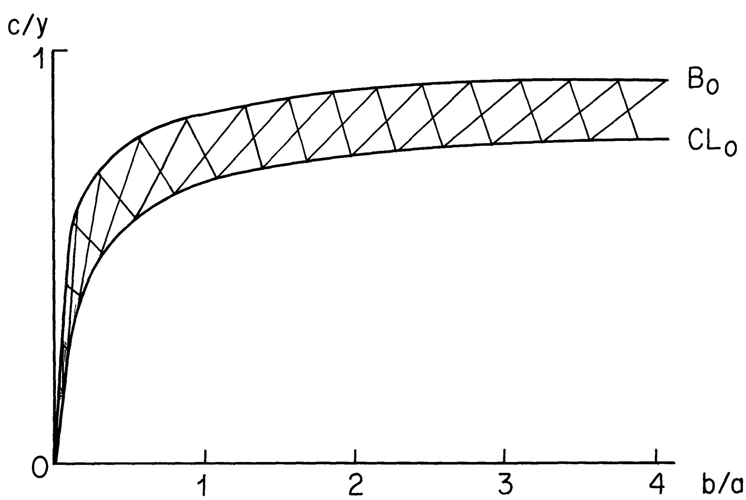

FIGURE II
No-credit Equilibrium

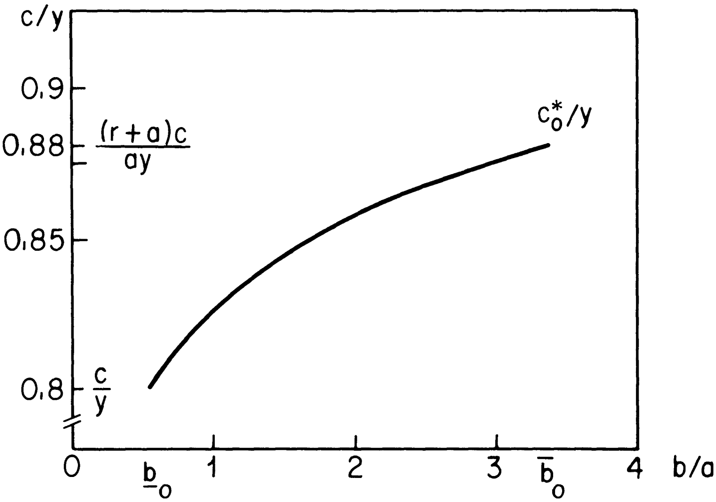

FIGURE III
No-credit Equilibrium

labeled CL_0; the breakeven condition is satisfied below the curve labeled B_0. With a potential moral hazard problem, credit will appear at the no-credit equilibrium as soon as someone has the idea if (CL_0) is not satisfied. This will hold for c/y sufficiently small. Conversely, the economy can be trapped in a no-credit equilibrium if this condition is satisfied.

In Figure III we examine equilibrium for c/y equal to 0.8 and r/a equal to 0.1 by plotting c_0^*/y, c/y, and $(r + a)c/(ay)$ against b/a. There is an equilibrium without credit for b/a in the interval $(\underline{b}_0,\bar{b}^0)$.

IV. BASIC MODEL WITH CREDIT

We turn next to the situation where credit is always available. Assume that if you have no goods in inventory and if you are not in debt, then somebody with inventory is willing to lend to you, willing to provide you (stochastically) consumption in return for future delivery of goods. However, we shall not consider being willing to lend to someone because she is a creditor of someone else. Now we shall look for parameters so that the (endogenous) credit limit is one. Thus, there are three possible positions an individual can be in. (1) He can have a unit of good available for trade. He may or may not also be a creditor, but since that is just future consumption, it does not affect his trading abilities. As before, e is the fraction of the population in this position. (2) He may not have a unit available to trade and also not be a debtor. We denote by u the fraction of the population in that position. Or, (3) he may be a debtor. The fraction of the population in that position is denoted by d. These people cannot borrow any more; they are up against their credit limit.

We now consider dynamics where credit is given by those with inventory to finance all potential transactions with nondebtors but no transactions with debtors. The fraction with inventory, e, drops by any contact with someone with inventory and drops with the probability p from a contact with a nondebtor. The number with inventory rises from any production by a nondebtor. The latter lowers the fraction of nondebtors without inventory. This fraction also rises whenever two agents with inventory trade and whenever a debtor produces. There is an expected change of $(p - 1)$ in the number of nondebtors without inventory from a trade involving credit. The number of debtors, d, falls from production and rises from the acceptance of credit. Thus, we have the differential equations,

$$\begin{aligned} \dot{e} &= -be(e + up) + au, \\ \dot{u} &= -au + be^2 + ad + beu(p - 1), \\ \dot{d} &= -ad + beu. \end{aligned} \tag{19}$$

It is convenient to eliminate d from these equations, giving us the pair of differential equations:

$$\begin{aligned} \dot{e} &= au - be^2 - beup, \\ \dot{u} &= -au + be^2 + a(1 - e - u) + beu(p - 1). \end{aligned} \tag{20}$$

Since $\dot{e} + \dot{u} = a(1 - e - u) - beu$, any intersection of $\dot{e} = 0$ and $\dot{u} = 0$ with $e > 0$ and $u > 0$ must have $e + u < 1$.

In $e - u$ space we examine the phase diagram for (20) inside the triangle $e \geq 0, u \geq 0, e + u \leq 1$. Setting $\dot{e} = 0$, we have

$$u = be^2/(a - bep). \tag{21}$$

The relevant portion of this curve rises from the origin and is asymptotic to the line $e = a/(pb)$. This shown in Figure IV for $b = a$ and $p = 1$.

Setting $\dot{u} = 0$ and solving for u, we have

$$u = (be^2 - ae + a)/(2a + be(1 - p)). \tag{22}$$

For $e = 0$ and $e = (a/b) = (1 - p)/2$, we have $u = 1/2$. The curve is

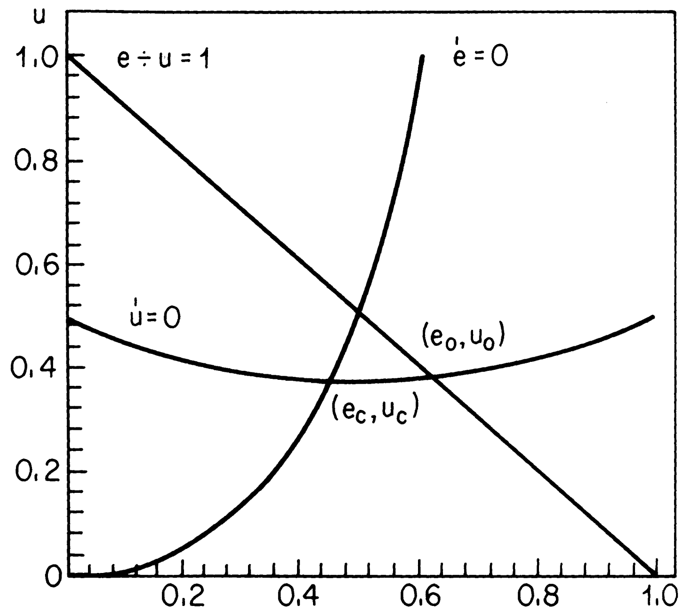

FIGURE IV
Equilibrium with Credit

decreasing as it leaves the vertical axis and has a unique turning point with e positive. The curve is also shown in Figure IV for $b = a$ and $p = 1$.

Equating (21) and (22), we have a cubic equation for e. Defining $a' = a/b$, we can write the cubic equation as

$$e^3 + a'(1-p)e^2 + (a'^2 + a'p)e - a'^2 = 0. \tag{23}$$

Define

$$\begin{aligned} x &= (3(a'^2 + a'p) - a'^2(1-p)^2)/3, \\ z &= (2a'^3(1-p)^3 - 9a'(1-p)(a'^2 + a'p) - 27a'^2)/27, \\ A &= \left(\frac{-z}{2} + \left(\frac{z^2}{4} + \frac{x^3}{27}\right)^{1/2}\right)^{1/3}, \\ B &= \left(\frac{-z}{2} - \left(\frac{z^2}{4} + \frac{x^3}{27}\right)^{1/2}\right)^{1/3}. \end{aligned} \tag{24}$$

Then, we write the equilibrium level e_c as

$$e_c = A + B - a'(1-p)/3. \tag{24}$$

Thus, there is a unique intersection of (21) and (22). It is straightforward to check the stability of this equilibrium. Implicitly differentiating (23), we see that e_c decreases with p; that is, the greater the probability of delivery in a stochastic credit transaction, the lower the steady state stock of inventory.

Eliminating p from (21) and (22), we have

$$beu = a(1 - e - u). \tag{25}$$

Thus, u_c is decreasing in e_c and so is increasing in p.

Note from (3) that (e_0,u_0) lies at the intersection of $\dot{e} = 0$ drawn for $p = 0$ and $e + u = 1$ in $e - u$ space. Thus, $e_0 > e_c$ for all p. We can have either sign for $u_0 - u_c$.

Next we examine wealths under the assumption that all projects are carried out and credit is extended up to a credit limit of one. As above, the utility discount rate times the value of being in a position equals the expected flow of utility dividends and capital gains:

$$rW_e = be\,(y - W_e + W_u) + bu\,(ay/(r + a) - pW_e + pW_u), \tag{27}$$

$$rW_u = be\,(yp - W_u + W_d) + a(W_e - W_u - c), \tag{28}$$

$$rW_d = a(W_u - W_d - c). \tag{29}$$

Following the notation used above, we write the cost of projects just worth taking as c_i^*:

$$\begin{aligned} c_u^*(e,u) &= W_e - W_u, \\ c_d^*(e,u) &= W_u - W_d. \end{aligned} \tag{30}$$

Subtracting (28) from (27) and (29) from (28), we have

$$(r + a + be + pbu)c_u^* = bec_d^* + ac + bu\,ay/(r + a) + bey\,(1 - p), \tag{31}$$

$$(r + a + be)c_d^* = bepy + ac_u^*. \tag{32}$$

Solving from (31) and (32), we have

$$\left(r + a + be + pbu - \frac{abe}{r + a + be}\right)c_u^* = \frac{b^2e^2yp}{r + a + be} + ac + bu\frac{ay}{r + a} + bey(1 - p), \tag{33}$$

$$\begin{aligned}((r + a + be + pbu)(r + a + be) - abe)c_d^* &= (r + a + be + pbu)bepy + a^2c \\ &\quad + (bua^2y/(r + a)) + abey(1 - p). \end{aligned} \tag{34}$$

V. Terms of Credit

In order to determine the terms of credit (the value of p), we use the Nash bargaining solution for a single credit transaction, assuming that all other credit transactions occur with delivery probability p; i.e., that position values satisfy (27) to (29). We want a fixed point in p so that the condition for the Nash bargaining solution is satisfied. Without this credit transaction the pair of agents who have met have values (W_e, W_u). With a credit transaction with delivery probability p, their values become

$$(W_e + ay/(r + a) - p(W_e - W_u), W_d + py). \tag{35}$$

Using the willingnesses to produce, (30), the gains from trade can be written as

$$(ay/(r + a) - pc_u^*, py - c_d^*). \tag{36}$$

The Nash bargaining solution satisfies the maximization problem,

$$\max\,(py - c_d^*)(ay/(r + a) - pc_u^*). \tag{37}$$

Calculating the first-order condition, we have

$$y(ay/(r + a) - pc_u^*) = (c_u^*)\,(py - c_d^*). \tag{38}$$

Solving for p, we have

$$p = \frac{c_u^* c_d^* + ay^2/(r + a)}{2yc_u^*}. \tag{39}$$

For there to be a mutually advantageous trade, the gains to trade to both parties must be nonnegative. Thus, p must satisfy

$$c_d^*/y \le p \le ay/((r + a)c_u^*). \tag{40}$$

The Nash bargaining solution value of p is the mean of the two limits in (40). Thus, we have a mutually advantageous trade provided that

$$c_u^* c_d^* \le ay^2/(r + a). \tag{41}$$

There is no guarantee that the solution to (39) is less than one. However, the equations are valid only when $p \le 1$. Thus, we restrict attention to parameter values having this property. I have not examined whether there are alternative equilibria corresponding to a negative interest rate and a lottery on repayment of the debt rather than a lottery on delivery of goods on hand.

VI. General Credit Availability

For an equilibrium with a credit limit of one and a mutually agreeable probability of delivery p, $0 < p < 1$, we need to check three conditions. We must check a breakeven condition: it is worth producing for inventory. We must check the moral hazard condition: it is worth repaying debt if you succeed in borrowing. And we must check the credit limit condition: lending to someone beyond the credit limit would violate a moral hazard constraint. In this section we examine the three conditions. To distinguish these conditions from those above, we drop the subscript and write then as (B), (MH), and (CL). We have an equilibrium with credit if the three conditions are met:

$$\text{(B): } c_u^* \geq c, \qquad \text{(MH): } c_d^* \geq c, \qquad \text{(CL): } W_d < c. \tag{42}$$

Using (29) and (30), one can write the credit limit constraint as

$$\text{(CL): } c_d^* < ((r + a)/a)\, c. \tag{43}$$

The conditions are summarized in Table I.

Using (33), we can write the breakeven constraint as

$$\text{(B): } \left(r + be + pbu - \frac{abe}{r + a + be}\right)\frac{c}{y} \leq \frac{b^2e^2p}{r + a + be} + \frac{bua}{r + a} + be(1 - p). \tag{44}$$

TABLE I

Credit Limit	0	1
Endogenous variables	e_0	e_c, u_c
	c_0^*	c_u^*, c_d^*
		p
Breakeven constraint	$c_0^* \geq c$	$c_u^* \geq c$
Moral hazard condition		$c_d^* \geq c$
Credit limit condition	$W_u < c$	$W_d < c$
or		
	$c_0^* < \left(\frac{r + a}{a}\right)c$	$c_d^* < \left(\frac{r + a}{a}\right)c$

Using (34), we can write the moral hazard constraint as

$$(45)\quad \text{(MH)}: ((r + be + pbu)(r + a + be) + ar)(c/y) \leq (r + be + pbu)bep + (bua^2/(r + a)) + abe.$$

Using (34), we can write the credit limit condition as

$$(46)\quad \text{(CL)}: \left((r + be + pbu)(r + a + be)\left(\frac{r + a}{a}\right) + r^2 + 2ar\right)\frac{c}{y} \geq (r + be + pbu)\, bep + \frac{bua^2}{r + a} + abe.$$

We evaluate all three conditions at e_c, satisfying (25) and u_c satisfying (26).

In Figure V we calculate the values of c_u^*/y and c_d^*/y as functions of b/a for $r/a = 0.1$ and $c/y = 0.8$. We have an equilibrium in the range $[\underline{b}_c, \bar{b}_c]$. Comparing Figures III and V, we have two equilibria for a sizeable fraction of the values of b/a for which we have an equilibrium with either a zero or one credit limit.

In Figure VI we show p as a function of b/a over the range of values for which we have an equilibrium with $c/y = 0.8$ and $r/a = 0.1$. Also shown in Figure VI is the value of p, denoted p_0, which would satisfy the Nash bargaining solution if a single credit transaction were to take place at the no-credit equilibrium because

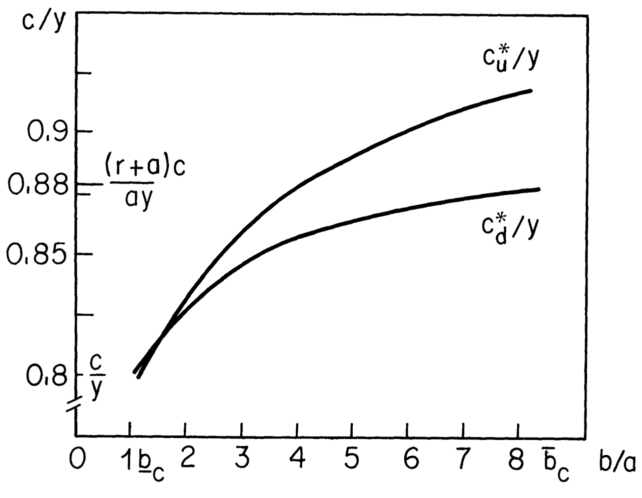

FIGURE V
Equilibrium with Credit

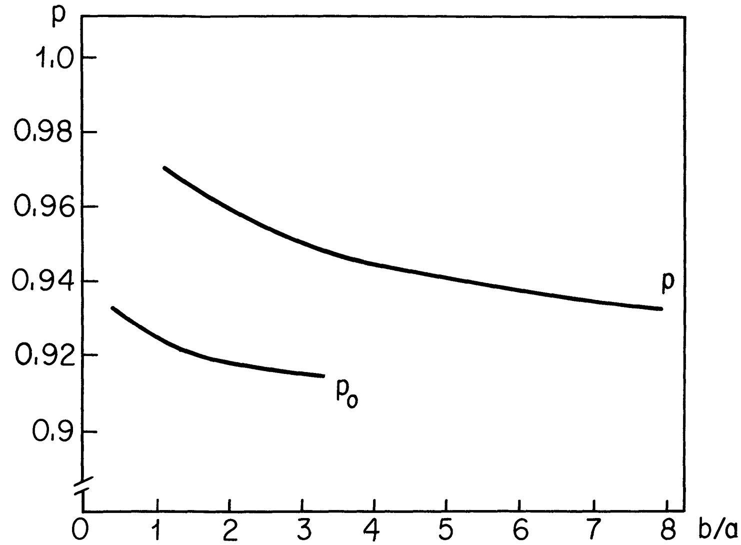

FIGURE VI
Probability of Delivery

of total honesty and so irrelevance of the credit limit constraint for a single transaction:

$$p_0 = ((y^2 + c_0^{*2})/2yc_0^*)(a/(a + r)). \quad (47)$$

A further difference between the two equilibria is in the stock of inventory available for trade. In Figure VII we show e_0 and e_c as functions of b/a for $c/y = 0.8$ and $r/a = 0.1$.

VII. DISCUSSION

For the values $r/a = 0.1$ and $c/y = 0.8$, we have found an equilibrium with a credit limit of zero over a range of values of b/a running from 0.5 to 3.4. For all of these values of b/a in excess of 1.2, there is another equilibrium with a credit limit of one. Thus, multiple equilibria are not merely possible in this model but are in one sense a common phenomenon. Given the discrete nature of allowable credit limits in the model as formulated, multiple equilibria imply two different credit limits with the property that individuals are willing to pay back debts up to their credit limit and are unwilling to pay back debts one unit greater than that credit limit. In a model with a continuous credit limit, multiple equilibria would

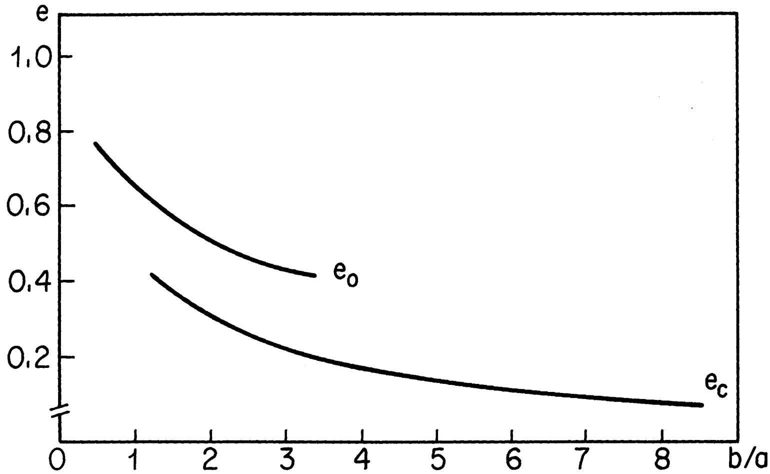

FIGURE VII
Stock of Inventory

be represented by multiple solutions to the condition that at the credit limit the individual is just willing to pay back. The lumpiness of the credit limit makes multiple equilibria easier to find than they would be in a continuous model. In other words, the magnitude of the feedback effects that raise the willingness to pay when the credit limit rises can be small and still have multiple equilibria in a lumpy model. To see this, let us formulate the problem symbolically.[3] Assume that one had a model with a continuous credit limit L. Assume that the equilibrium in the economy is unique for each L and is described as $x(L)$ for some vector x. In a steady state of this economy, we can write individual expected discounted lifetime utility as a function of x, L and the individual debt (or inventory) position D, $W(x,L,D)$. Continuing to use the level of lifetime utility in autarchy as the origin, the endogenous credit limit in a continuous model satisfies $W(x(L),L,L) = 0$. With L restricted to the integers a credit limit satisfies $W(x(L),L,L) > 0 > W(x(L),L,L + 1)$. We can have a second equilibrium at $L + 1$ with $W(x(L + 1),L + 1, L + 1)$ less than $W(x(L),L, L)$ and still positive. With L continuous, two equilibria at L and L' must have the same values of W. Thus, the

3. I am indebted to Kevin Murphy for a valuable discussion of the connection between lumpiness and multiple equilibria.

continuous model requires stronger feedbacks to achieve multiple equilibria.

It is interesting to contrast the two equilibria that exist for some values of b/a. We see that p is three to four percentage points larger than p_0, the credit terms that would hold in a zero credit limit equilibrium if a single transaction were not concerned with the moral hazard constraint. Thus, in conformity with the usual image of easy credit, a higher credit limit is associated with a lower implicit interest rate. This difference in probabilities represents a sizeable difference in implicit interest rates and adds to the incentive to pay back one's debt in order to stay in the trading network. On the other hand, the stock of inventories drops by approximately 0.2, representing a large decrease in the rate of meeting individuals with goods to trade. This decreases the advantage of being part of the trading network. With lower inventories there are more individuals available to produce. Thus, the greater credit limit is associated with a higher level of production. In sum, there are three effects combining to change the net incentive to pay back one's debt: the direct impact of the change in the credit limit, the improvement in the terms of credit, and the deterioration in the speed of transactions. The net feedback must be positive in order to sustain the second credit limit. That is, with a general credit limit of zero and inventory level e_0, an individual with a debt of one would not pay back. Yet when the credit limit is one and inventories are e_c, an individual with a debt of one will pay back. Net, the value of trade increases by enough to justify paying the debt to stay in the trading economy.

VIII. A Single Credit Transaction with Smoothed Credit Limits

We now introduce a second lottery to smooth the credit limit. As in Section II let us consider a pair of individuals who have come together in a no-credit steady state equilibrium. One of them has a unit of the good to trade, and the other one does not. As above, the proposed trade begins with realization of a random variable. With probability p the inventory on hand is delivered for immediate consumption. Independent of the outcome of the random variable, the debtor promises that at his next opportunity to produce he will carry out production and with independent probability q will deliver that good to the creditor. That is, we have expanded the allowable set of trades by permitting a delivery probability that can be less than one.

Paralleling the analysis of Section II, assuming repayment, trade is advantageous to the lender if

$$(qay/(r + a)) - p(W_e - W_u) \geq 0. \tag{48}$$

The condition is that the probability of delivering the good now times the value of a unit of inventory be less than the expected value of delayed payment. The borrower needs to compare the cost of being in debt with the probabilistic gain of current consumption. Again assuming repayment, the trade is advantageous to the borrower if

$$py - (qa/(r + a))(W_e - W_u) \geq 0. \tag{49}$$

Note that when the left-hand sides of (48) and (49) are positive, they are both increasing in the size of trade, p, holding constant the ratio q/p. Thus, p will be chosen as large as possible subject to p being no larger than one, q being no larger than one, and q being small enough to make repayment attractive.

Combining (48) and (49), there is a mutually advantageous trade if there is a value of p/q satisfying

$$\left(\frac{a}{r + a}\right)\left(\frac{W_e - W_u}{y}\right) \leq \frac{p}{q} \leq \left(\frac{a}{r + a}\right)\left(\frac{y}{W_e - W_u}\right). \tag{50}$$

Since $y \geq W_e - W_u$ when production is worthwhile (cf. (7)), there is always an interval of values of p/q that can satisfy this condition.

A debtor will repay if it is worthwhile to pay the cost of production to remain in the economy. Thus, the debtor will repay if q is sufficiently small that it is worth paying c for the lottery of being in position W_u with probability q and position W_e with probability $(1 - q)$. Provided that both p and q are less than one, the maximal promise to repay which is credible satisfies

$$qW_u + (1 - q)W_e = c \tag{51}$$

or

$$q = (W_e - c)/(W_e - W_u). \tag{52}$$

If the breakeven condition (7) is satisfied with a strict inequality, there always exists a value of q that permits the introduction of credit. Thus, the lumpiness of credit assumed above is essential for preventing the appearance of credit with the penalties for nonpayment as modeled here. We note that if the economy is on the knife-edge of just satisfying the breakeven condition (7) so that $W_u = 0$ and $W_e = c$, then $q = 0$ in (52), and (48) requires that $p = 0$ as well.

In order to have $q < 1$, we needed $W_u < c$. We restrict analysis below to parameters that yield a solution to (51) with both p and q less than one at equilibrium with credit. Otherwise we would need to examine further lending to a debtor.

It is natural to think of the implicit interest rate for this credit transaction. The claim on future stochastic delivery of the consumer good trades at a "price" p in terms of the current consumer good. Given the stationary character of the Poisson process determining the date of repayment, the price does not change over time. Thus, the implicit interest rate on this transaction times the "price" is equal to the flow probability of a repayment, a, times the expected return on repayment, which is the probability of delivery, q, less the loss in value of the asset, p, which becomes zero on repayment. Thus, we have

$$i/a = (q - p)/p. \tag{53}$$

IX. Basic Model with Smooth Credit

We turn now to equilibrium with credit. We restrict analysis to parameters for which the (endogenous) credit limit q is less than or equal to one. As before, there are three possible positions an individual can be in: with inventory, without inventory, or in debt. We now consider dynamics where credit is given by those with inventory to finance all potential transactions with nondebtors but no transactions with debtors. Rewriting (19) to have repayment by the fraction q of debtors, we have

$$\begin{aligned} \dot{e} &= -be(e + up) + au + ad(1 - q), \\ \dot{u} &= -au + be^2 + adq + beu(p - 1), \\ \dot{d} &= -ad + beu. \end{aligned} \tag{54}$$

Eliminating d from these equations, we have

$$\begin{aligned} \dot{e} &= au - be^2 - beup + a(1 - e - u)(1 - q), \\ \dot{u} &= -au + be^2 + a(1 - e - u)q + beu(p - 1). \end{aligned} \tag{55}$$

Since $\dot{e} + \dot{u} = a(1 - e - u) - beu$, any intersection of $\dot{e} = 0$ and $\dot{u} = 0$ with $e > 0$ and $u > 0$ must have $e + u < 1$.

Solving for $\dot{e} = \dot{u} = 0$, we have the same cubic equation for e as above, (23), except that p must be replaced by $s = p + q - 1$.

Thus, we have a unique solution to the cubic, with e_c decreasing in s. We note that the root occurs where

$$aq > bep. \tag{56}$$

Eliminating $aq - bep$ from $\dot{e} = 0$ and $\dot{u} = 0$, we again have

$$(a + be)u = a(1 - e). \tag{57}$$

Thus, u_c is decreasing in e_c and so increasing in p and q.

Next we examine wealths under the assumption that all projects are carried out and credit is extended up to a credit limit of $q < 1$.

$$rW_e = be\,(y - W_e + W_u) + bu\,((aqy/(r + a)) - pW_e + pW_u). \tag{58}$$

$$rW_u = be\,(yp - W_u + W_d) + a\,(W_e - W_u - c), \tag{59}$$

$$rW_d = a\,(qW_u + (1 - q)W_e - W_d - c). \tag{60}$$

We write the cost of a project just worth taking to shift status from position u to position e as

$$c^* = W_e - W_u. \tag{61}$$

As above, q is set as large as possible consistent with a willingness to pay back. Thus, $W_d = 0$. Subtracting (59) from (58), we have

$$(r + a + be + pbu)c^* = be\,W_u + ac + bu\,(aqy/(r + a)) + bey(1 - p), \tag{62}$$

$$(r + be)W_u = bepy + ac^* - ac. \tag{63}$$

Solving from (62) and (63), we have

$$\left(r + a + be + pbu - \frac{abe}{r + be}\right)c^* = \frac{r(ac - bepy)}{r + be} + bu\,\frac{aqy}{r + a} + bey. \tag{64}$$

X. Terms of Credit

With q set as large as possible consistent with a willingness to pay back, we have $W_d = 0$ or

$$q = (W_u + c^* - c)/c^*. \tag{65}$$

Positive production ($c^* > c$) implies that $q > 0$. The Nash bargaining solution for the terms of credit now solves the maximization problem,

$$\max\,(py - W_u)\,(aqy/(r + a) - pc^*). \tag{66}$$

Calculating the first-order condition and solving for p, we have

$$p = \frac{c^* W_u + aqy^2/(r + a)}{2yc^*}. \tag{67}$$

We note that $q > 0$ implies that $p > 0$. For there to be a mutually advantageous trade, the gains to trade to both parties must be nonnegative. Thus, p must satisfy

$$W_u/y \leq p \leq aqy/((r + a)c^*). \tag{68}$$

The Nash bargaining solution value of p is the mean of the two limits in (68). Thus, we have a mutually advantageous trade provided that

$$c^* W_u \leq aqy^2/(r + a), \tag{69}$$

or

$$c^{*2} W_u \leq (a/(r + a))y^2 (W_u + c^* - c).$$

XI. Equilibrium with Smooth Credit

For an equilibrium with a credit limit of q, $0 < q < 1$, and a probability of delivery p, $0 < p < 1$, we need to have a solution to the six equations (23, with s substituted for p), (57), (63), (64), (65), and (67), yield probability values for p and q between zero and one and a willingness to produce, c^*, satisfying $c^* > c$. To distinguish this breakeven condition from that above, we write it as (B_s). Since (B_s) implies that both p and q are nonnegative, we have an equilibrium with credit if the three conditions are met:

$$p \leq 1,\ q \leq 1,\ (B_s)\text{: } c^* \geq c. \tag{70}$$

Using (64), the breakeven constraint can be written as

$$(B_s)\text{: } \left(r + be + pbu - \frac{abeq}{r + a + be}\right)\frac{c}{y} \leq \frac{b^2e^2p}{r + a + be} + \frac{buaq}{r + a} + be(1 - p). \tag{71}$$

We evaluate this condition at e_c, satisfying (23) and u_c satisfying (57).

To examine the curve on which the breakeven condition is just satisfied, we note from (65) and (63) that when $c^* = c$, we have

$$q = W_u/c = bepy/(r + be). \tag{72}$$

With $c^* = c$, substituting for q from (65) in (67), we have

$$p = \left[c + \left(\frac{a}{r + a}\right)\frac{y^2}{c}\right]\frac{W_u}{2yc}. \tag{73}$$

Substituting for p/W_u from (63), we have be/a as a function of c/y and r/a:

$$\frac{r + be}{be} = \frac{1}{2}\left(1 + \left(\frac{a}{r + a}\right)\frac{y^2}{c^2}\right). \tag{74}$$

Using (72) to eliminate q, (23) can be solved for p as a function of b/a, be/a, and c/y. Similarly, using (57) to eliminate u and (72) to eliminate q, (64) evaluated at $c^* = c$ can be solved for p as a function of the same variables. Equating these expressions for p, and recognizing that be/a is a function of c/y and r/a in (74) we get an expression for $c^* = c$ which is quadratic in b/a. Using the normalizations $y = a = 1$, this expression is (75):

$$\begin{aligned} &\left[\frac{be}{1 + r} - c^2(r + be)\right]b^2 + \left[(be)^2\left(\frac{rbe - 1}{1 + r}\right) - berc\,(1 + be) + bec^2\,(r + be - rbe - r^2)\right]b \\ &+ \left[b^2e^2\,(r + be)^2c^2 + (b^3e^3 + b^2e^2)\,(r + be)\,c + b^3e^3\left(\frac{1 - rbe}{1 + r}\right)\right] = 0. \end{aligned} \tag{75}$$

In Figure VIII, for $r/a = 0.1$, we show (75) along with the no-credit breakeven curve (B_0) which is an additional locus on which $c^* = c$ (with $p = q = 0$).

The equations are valid only when $q \leq 1$. Thus, we also plot the locus $q = 1$, which is the locus $W_u = c$. The constraint $p \leq 1$ was not binding. The relevant parts of the $c^* = c$ locus are to the right of B_0 and above $W_u =$ c. Thus, the shaded region in Figure IX shows parameters that yield an equilibrium credit limit between zero and one.

With six equations in six unknowns (e, u, p, q, c^*, W_u), the search for multiple equilibria was by calculated example, not analysis of the equations. No multiple equilibria were found.

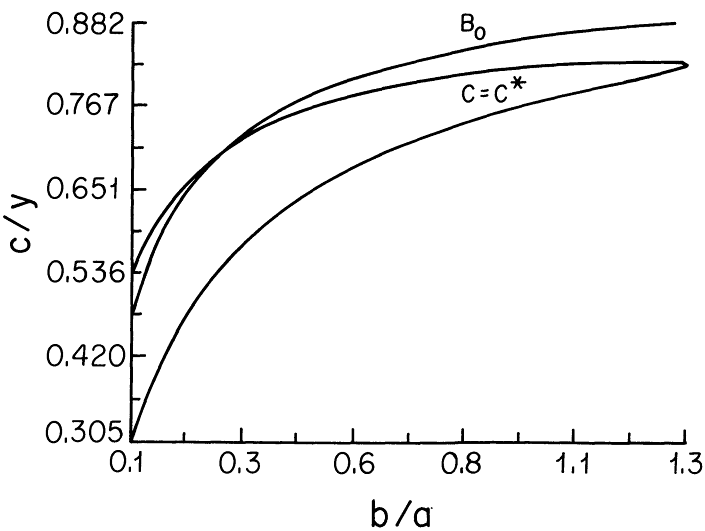

FIGURE VIII
Breakeven Condition

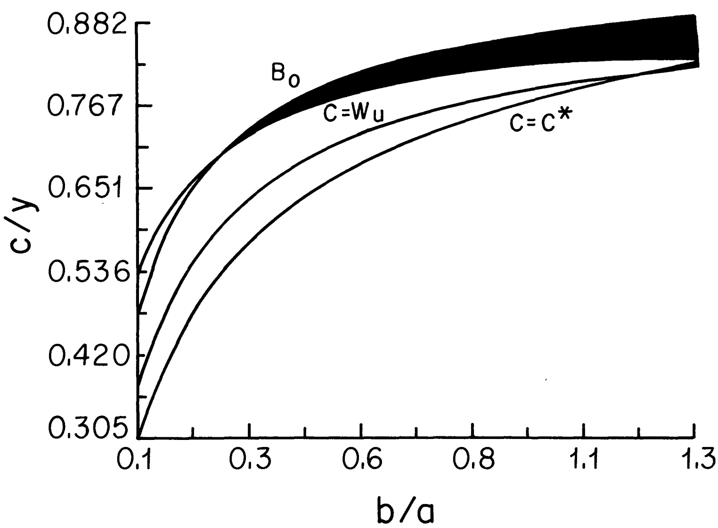

FIGURE IX
Equilibrium with Smooth Credit

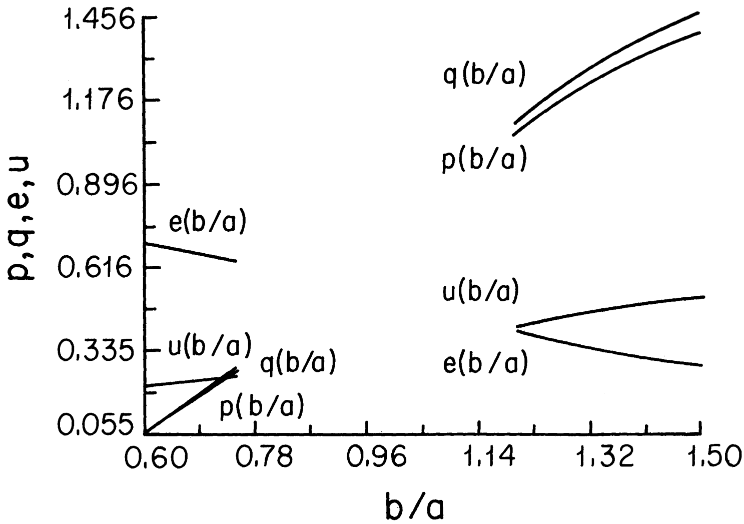

FIGURE X
Equilibrium with Smooth Credit: $c/y = 0.8, r/a = 0.1$

XII. COMPARATIVE STATICS

In Figure X we show e, u, p, and q as functions of b/a for $r/a = 0.1$ and $c/y = 0.8$. The figure shows the values only for the parameter values for which there exists an equilibrium. In Figure XI we show the implicit interest rate, i (given in (53)), as a function of b/a for the same values. Figures XII and XIII show the same variables as functions of c/y for $r/a = 0.1$ and $b/a = 1.27$. Again, the curves are drawn only for values for which there exists an equilibrium. In these calculations the implicit interest rate is positive.[4] The monotonicity properties shown in the figures are present on figures drawn on a number of other parameter values.

A more efficient trade technology (increase in b/a) directly reduces the stock of inventory. In turn, this increases the rate of production in the economy, $a(1 - e)$. More efficient trade technology enhances the value of being in the trade network, tending to raise q and so p. These increases, in turn, also contribute to the

4. Assuming equal gains from trade rather than the Nash bargaining solution, I was able to prove that $i > 0$. With the Nash bargaining solution, the ratio of marginal utilities of probability changes, c^*/y, enters the formula, complicating analysis.

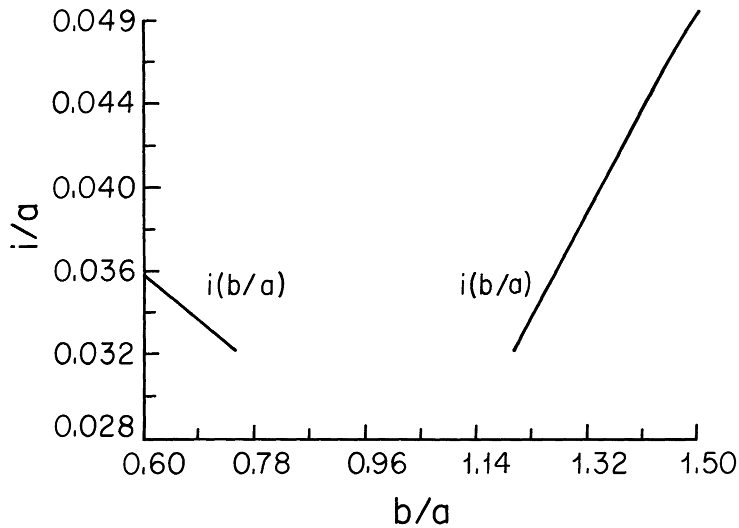

FIGURE XI
Implicit Interest Rate: $c/y = 0.8$, $r/a = 0.1$

decline in inventories. The figures suggest that the indirect effects of e and u on p do not offset these direct effects.

XIII. No-Trade Equilibrium

Above, we saw that credit would be introduced to the no-credit equilibrium with positive production, except at the knife-edge where the breakeven condition is just satisfied. In this section we examine the same question for the barter equilibrium with no production and no trade. If everyone believes that future credit transactions will not occur, then they will not occur, since there is no cost associated with being excluded from future trading opportunities. If agents believe that future credit transactions will occur if individually rational (given belief in their occurrence), then they may occur. To explore this possibility, we first derive a condition such that naive extrapolation of the meeting probabilities justifies production for a credit transaction, i.e., destroys the no-production equilibrium. However, this naive extrapolation may have a Ponzi character to it. (In fact, this calculation may seem worthwhile even when $c > y$.) We then add the condition that after a single credit transaction the economy converges back to the no-production equilibrium, implying that the myopic forecast is correct. This

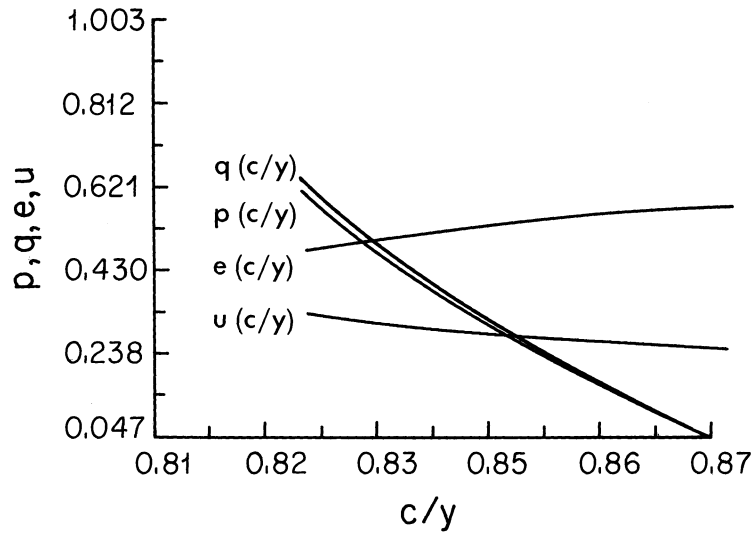

FIGURE XII
Equilibrium with Smooth Credit: $b/a = 1.27$, $r/a = 0.1$

results in a more stringent condition that is sufficient for the introduction of credit.

Consider an equilibrium with no inventories and no production. A single individual considers bearing the cost c to produce a unit. We denote by V the dynamic programming value of this unit to the producer. With arrival rate b the individual experiences the arrival of an individual to whom to propose a credit transaction. As above, a credit transaction is described by a pair of probabilities (p,q) of delivery of goods immediately and after future production. Both parties to the credit transaction believe that in the event of nondelivery of the good, that good can be the basis of a future credit transaction (which arrives at rate b) on the same terms as this one. In order for this credit transaction to occur, three conditions must be satisfied. First, the initial production must seem worthwhile:

$$V \geq c. \tag{76}$$

Second, later production to satisfy the debt must seem worthwhile. The debtor will have a unit of the good as a basis for a future credit transaction with probability $(1 - q)$. Thus, the latter production constraint is

$$(1 - q)\, V \geq c. \tag{77}$$

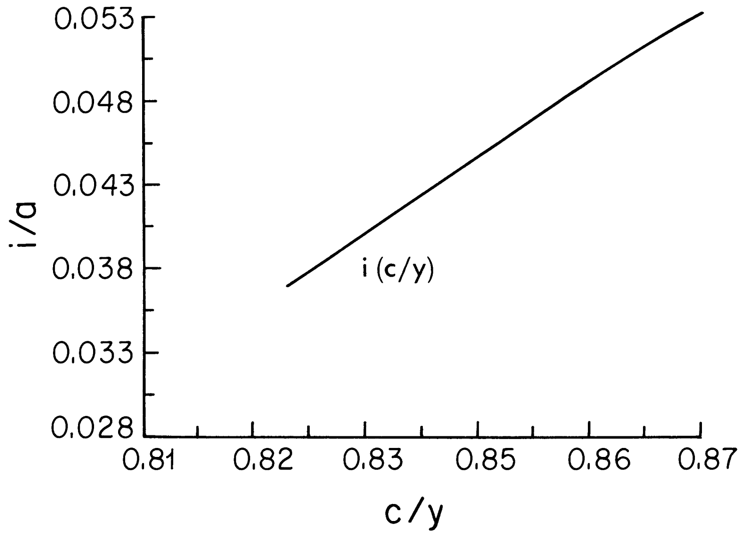

FIGURE XIII
Implicit Interest Rate: $b/a = 1.27, r/a = 0.1$

Since (77) is more stringent than (76), we can ignore (76). Third, the credit transaction must seem worthwhile to the debtor. With production for repayment worthwhile the credit transaction is worthwhile with any nonnegative value of p. This peculiar result follows from the lack of alternative activities with production for barter not profitable and the assumption that the debtor cannot take the idea of credit and produce to become a creditor rather than becoming a debtor. We shall review that assumption below.

To examine the condition such that a pair of probabilities (p,q) can be found which satisfy (77), we need to derive the value of a unit of inventory. The dynamic programming equation for V is that the utility discount rate times V is equal to the arrival rate of a credit partner times the value of a credit transaction to the lender. The value of the credit transaction is the expected value of later consumption $(a/(r + a))qy$ less the expected cost of delivering the good, pV. Thus, we have

$$rV = b((a/(r + a))\, qy - pV). \tag{78}$$

Solving for V, we have

$$V = \frac{abqy}{(r + a)\,(r + bp)}. \tag{79}$$

We can now write the condition for the introduction of credit from (77) as

$$(1 - q)q > \frac{(r + a)(r + bp)c}{aby}. \tag{80}$$

The product $q(1 - q)$ varies between 0 and 0.25 as q varies between 0 and 1. Thus we can find a satisfactory value of q provided that the right-hand side of (80) is less than 0.25. The smallest value of this expression is achieved by setting $p = 0$. That is, the most favorable case for the possibility of credit comes from marketing the idea of credit; there is no need to actually deliver any goods to initiate the credit contrast. This strongly suggests the potential Ponzi nature of the introduction of credit. Note that with p less than one and r small relative to b, (80) can be satisfied with $c > y$.

If goods are never delivered at the initiation of a credit transaction ($p = 0$), then the stock of inventory in the economy will grow, never returning to the zero stock initial position. This makes the naive forecast of an arrival rate b of potential debtors wrong. Thus, we add a second condition that the stock of inventory returns to zero at the probabilities (p,q). This ensures that the naive forecast is correct.

Denote the number of individuals with goods to trade by e and the number of debtors by d, in the neighborhood of zero these numbers satisfy

$$\begin{aligned} \dot{e} &= a(1 - q)d - bpe, \\ \dot{d} &= be - ad. \end{aligned} \tag{81}$$

For the origin to be locally stable, we need

$$p > (1 - q). \tag{82}$$

For the introduction of credit without Ponzi expectations, we need to satisfy (80) and (82). That is, we need to find a value of q, $1/2 < q < 1$, such that

$$q(1 - q) \geq \frac{(r + a)(r + b - bq)c}{aby}. \tag{83}$$

The left-hand side of (83) is quadratic in q, while the right-hand side is linear. To solve for the parameter values for which we can find values of q satisfying (83), we solve for parameter values so that the two curves are tangent. Calculating the values for which this condition holds, the no-production equilibrium is not sustainable in the presence of credit possibilities with rules as modeled here when

$$(84)\qquad \frac{r+b}{b} < \left[1 + 2\left(\frac{r+a}{a}\right)\left(\frac{c}{y}\right) + \left(\frac{r+a}{a}\right)^2\left(\frac{c}{y}\right)^2\right]\left[4\left(\frac{r+a}{a}\right)\left(\frac{c}{y}\right)\right]^{-1}.$$

When r is small relative to both a and b, (84) can be satisfied for nearly all c/y less than one.

In assuming that the offer of credit is accepted if profitable, we have preserved the assumption that autarchy is the only alternative. However, the introduction of the idea of credit introduces a second possibility unless the initiator of the idea has contractually bound the would-be debtor not to use the idea except in contract with the initiator. This is a common sort of contract with intellectual property. Without such a contract credit is accepted only if it is more valuable than waiting to produce for a future credit contract. Assuming initiation of only one credit contract is contemplated, this requires that

$$(85)\qquad p > (a/(r+a))^2\,(b/(r+bp))\,q^2.$$

The absence of financial intermediaries in this model severely limits the lessons that might be drawn from it. Financial intermediaries are natural institutions since purchasing power is fungible (i.e., variety per se is not desired by borrowers), and it is cost reducing to have repeated dealings with the same lender. While financial intermediaries will internalize some of the externalities that appear in the model, they will not eliminate all of them if one preserves transactions and information limitations that are realistic. In addition, much credit is extended without intermediation. Thus, the paper points to the value of continued exploration of models of credit that recognize that smoothly functioning credit markets are only part of the story of credit provision.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

REFERENCES

Chatterjee, Satyajit, "Participation Externality as a Source of Coordination Failure in a Competitive Model with Centralized Markets," Working Paper, University of Iowa, June 1988.

Diamond, Douglas, "Reputation Acquisition in Debt Markets," Center for Research in Security Prices, Working Paper 134, University of Chicago, 1986.

——, and Philip Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy,* XCI (1983), 401–19.

Diamond, Peter, "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy,* XC (1982) 881–94.

——, "Equilibrium Without an Auctioneer," in *Advances in Economic Theory,* Truman Bewley, ed. (Cambridge: Cambridge University Press, 1987a).

———, "Credit in Search Equilibrium," in *Financial Constraints, Expectations, and Macroeconomics,* Meir Kohn and Sho-Chieh Tsiang, eds. (Oxford: Oxford University Press, 1987b).

———, and Drew Fudenberg, "Rational Expectations Business Cycles in Search Equilibrium," *Journal of Political Economy,* XCVII (1989) 606–19.

Pagano, Marco, "Endogenous Market Thinness and Stock Price Volatility," *Review of Economic Studies,* LVI (1989), 269–87.

White, Michelle, "Bankruptcy Liquidation and Reorganization," in *Handbook of Modern Finance,* Dennis Logue, ed. (Boston, MA: Warren, Gorham and Lamont, 1984).

———, "The Corporate Bankruptcy Decision," *Jounal of Economic Perspectives,* III (1989), 129–52.