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MONEY AS A MEDIUM OF EXCHANGE WHEN GOODS VARY BY SUPPLY AND DEMAND

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Models of the exchange process based on search theory can be used to analyze the features of objects that make them more or less likely to emerge as money in equilibrium. These models illustrate the trade-off between endogenous acceptability (an equilibrium property) and intrinsic characteristics of goods, such as storability or recognizability. We look at how the relative supply and demand for various goods affect their likelihood of becoming money. Intuitively, goods in high demand and/or low supply are more likely to appear as commodity money, subject to the qualification that which object ends up circulating as a medium of exchange depends at least partly on convention. Welfare properties and fiat money are discussed.

Keywords: Money, Search Theory, Supply and Demand

1. INTRODUCTION

Models of the exchange process based on search theory can be used to analyze the features of objects that make them more or less likely to emerge as money in equilibrium. Features that are important include the intrinsic properties of an object, which determine how good it is as a store of value, and its acceptability, which determines how good it is as a medium of exchange. Acceptability depends on the behavior of individuals in the economy, because the objects that one chooses to accept as money depend at least in part on what others are accepting (that is, acceptability is a property of an equilibrium more than an intrinsic property of an object).

Search models illustrate the trade-off between endogenous acceptability and intrinsic properties. Several versions of the model have been constructed, focusing on different intrinsic properties that have been suggested as important in the traditional literature [see, e.g., Jevons (1875) and Menger (1892) for early discussions of the desirable intrinsic properties of money]. For instance, Kiyotaki and Wright (1989), Aiyagari and Wallace (1991), Burdett et al. (1995), and Cuadras-Morató (in press) look at what may be thought of as various forms of storability, durability, or portability; and Williamson and Wright (1994), Cuadras-Morató (1994), Li (1995), and Trejos (1997) look at what may be thought of as homogeneity or recognizability. We continue this line of research by looking at how the relative supply and demand for various goods affect their likelihood of emerging as media of exchange; that is, we ask how the number of producers and consumers of the different commodities influence whether they become money. Both scarcity (a relatively low supply) and intrinsic utility (a relatively high demand) often have been identified in the traditional literature as crucial factors determining if a good may serve as a commodity money [see, e.g., Jevons (1875), who seems to argue that demand is the most important factor].

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To pursue this, we construct a search model in which agents are specialists in production but generalists in consumption. On the one hand, an agent of a particular type always produces the same commodity, and the distribution of types thereby determines the relative supply of the different goods. On the other hand, agents of every type have tastes that change over time according to some common distribution, and this determines the relative demand for the different commodities. One of the main advantages of this particular modeling strategy is its tractability: even if types differ in the production process, it is shown that, once they enter the exchange process, their trading strategies are all identical. Without this, there would seem to be little hope of solving the model. Moreover, these assumptions seem quite natural, in the sense that here agents are all specialists in production and generalists in consumption, whereas previous commodity money models assumed that agents specialized in both consumption and production in a particular way.

Otherwise, all goods and individuals are perfectly symmetric, and, in particular, all goods are perfectly and costlessly storable, recognizable, and so on. Individuals meet in a bilateral exchange process in which trade is quid pro quo. They always accept a good that they wish to consume, and the interesting question is whether they accept commodities that they currently are not interested in consuming. A high probability of wanting to consume a good encourages an individual to accept it even if he does not want to consume it now, not only because he is more likely to want to consume it next period, but also because, even if he doesn't, trading partners may be more likely to want it next period. Hence, goods in high demand seem more likely to end up serving as money. At the same time, if the number of producers of a good is large, holding demand constant, individuals may be less likely to accept it now because it is easy to acquire when needed. Hence, goods in scarce supply also seem more likely to end up as money. But, again, what agents accept as money also depends on what they think others are likely to accept. In addition to these fairly intuitive results, the present model makes some predictions that differ quite dramatically from previous models, including the fact that it is not at all easy to find equilibria where fiat money is valued. This is despite the apparently minor changes in the environment, and for reasons that we spell out in detail below.

The model of Jones (1976) considers similar issues. However, there are two drawbacks to that model from our current perspective. First, individuals in Jones' model choose and commit to a sequence of trades before the exchange process starts, rather than using optimal sequential strategies to determine when to trade as a function of who you meet and what you both have. Second, he generally does not impose rational expectations concerning the distribution of goods held by other traders in equilibrium [further analyses of Jones' model by Iwai (1988) and Oh (1989) go only part way to addressing these problems]. Another related model is described by Wright (1995). However, that model adopts the very special assumption that each type i specializes in the consumption of good i and production of good $i + 1$ (modulo the number of types), which obviously precludes the independent investigation of supply and demand effects. The attempt to generalize that model to study these effects introduced several complications, and led to the rather different model presented here.

In terms of results, the first thing that we show is that all types use the same trading strategies in equilibrium. This is important not only because it reduces the set of candidate equilibria rather dramatically, but also because it means that, if an object is used as money, it is used as money universally (i.e., all agents use the same objects). The next thing that we show is that goods are more likely to be accepted as money if and only if holding them implies a higher probability of consumption next period. This is very convenient in terms of solving the model, although note that the relevant probability is still endogenous because the likelihood that you consume next period of course depends on the trading strategies of others. We then show that high demand for a good is a necessary condition for it to serve as money, in the following sense: If the demand for a good is sufficiently small, then there exists no equilibrium where it is used as a medium of exchange (this result is interesting because, as we stated above, it also can be interpreted as saying that fiat money is unlikely to emerge in a decentralized economy such as the one we present here). Indeed, a sufficiently high demand always guarantees that a good will emerge as money. We also show that a high supply of a good does not preclude it from being used as money, nor do low values of supply guarantee that it will be used as money. Given supply and demand parameters, multiple equilibria with different media of exchange are possible for the reason to which we alluded earlier: The use of money is at least partly a convention. We show one agent is always better off than another agent if we are in an equilibrium in which the former's production good is used as money and the latter's is not. We also show that equilibria generally are not efficient; examples demonstrate that several equilibria may coexist, with one being dominated by some other.

2. MODEL

The economy is populated by a [0,1] continuum of infinite-lived agents. There are three types of agents and three consumption goods (three is the minimum number that makes things interesting). These goods are costlessly storable and indivisible. We assume that type i can produce good i, and only good i, at a cost in terms of

disutility denoted by D. Let σ_i denote the fraction of the population that is type $i (0 < \sigma_i < 1)$, $\Sigma_i \sigma_i = 1$, and let $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$. Then, $\boldsymbol{\sigma}$ describes the relative supplies of the goods.[1]

Although agents are specialists in production, they are generalists in consumption. In this model, we assume that at every date $t = 0, 1, 2, \ldots$, each agent gets a taste shock (independent across agents and across time) that determines the good he desires that period. If an agent desires good i, he gets utility U from consuming it and no utility from consuming any other good. In what follows, we write $u = U - D$ for the net utility from consumption and production. Let δ_i be the probability that any agent desires good $i (0 < \delta_i < 1)$, $\Sigma_i \delta_i = 1$, and let $\boldsymbol{\delta} = (\delta_1, \delta_2, \delta_3)$. Then, $\boldsymbol{\delta}$ describes the relative demands for the goods.[2]

If we endow everyone with a single unit of their production good at the initial date, and assume thereafter that agents can produce i.f.f. they consume, then every agent will always be holding one unit of one of the goods. If an agent realizes a desire for the good he is holding, he consumes it, produces a new unit of his production good (assuming that D is not too large), and waits for the next period. If he realizes a desire for something other than what he is holding, he enters a trading process, or a *market*, wherein he is randomly matched with another agent who also demands a good other than what he is holding. If both agree to trade, they swap inventories one-for-one (because the goods are indivisible); otherwise, they part company and wait for the next period.

If you meet someone in the trading process who has the good that you desire and wants to trade, then obviously you trade. The interesting question is whether you should agree to a trade for a good that you do not currently desire. To analyze this, let V_{ij} denote the value function for a type-i individual, at the end of a period, holding a good j other than the one currently desired for consumption. One can interpret V_{ij} as the value for individual i of good j as an asset. Because taste shocks are independent over time, V_{ij} does not depend on the good that is desired in the current period. Also, because we consider only stationary equilibria, V_{ij} does not depend on time.

The strategic problem faced by an agent of type i can be summarized in the following way: He wants to trade good j for good k i.f.f. $V_{ij} < V_{ik}$, that is, i.f.f. the value of k as an asset is larger than the value of good j as an asset.[3] Hence, once we rank the three value functions, we know which trades agent i wants to make. For example, if $V_{11} < V_{12} < V_{13}$, then, assuming he did not want to consume the good he was currently holding, type 1 would trade good 1 for good 2, good 1 for good 3, and good 2 for good 3. He would not make any other trades, except of course to acquire a good that he currently desires for immediate consumption.

We can describe behavior of agents of a particular type by a strategy vector, denoted $\mathbf{s}^i = (\ldots s^i_{jk} \ldots)$, where $s^i_{jk} = 1$ if type i wants to trade good j for good k and $s^i_{jk} = 0$ otherwise. In fact, because $s^i_{jk} = 1 \Leftrightarrow V_{ik} > V_{ij}$, the vector $\mathbf{s}^i$ is completely summarized by any three independent elements, say $\hat{\mathbf{s}}^i = (s^i_{12}, s^i_{23}, s^i_{31})$. Given $\hat{\mathbf{s}}^i$, we know how to rank the value functions, which gives us all of the s^i_{jk}. Provided that we restrict our analysis to equilibria in which agents of the same type use the same strategies, we can summarize the behavior of all agents by $S = (\hat{s}^1, \hat{s}^2, \hat{s}^3)$, which describes the strategies of each type.

Let p_{ij} denote the measure of type-i agents with good j at the start of a period, $\Sigma_j p_{ij} = \sigma_i$, and let $\boldsymbol{p} = (\ldots p_{ij} \ldots)$. The measure of agents with good j is $P_j = \Sigma_i p_{ij}$, and the measure of agents with good j who go to the market is $(1 - \delta_j)P_j$, because every agent with good j has probability δ_j of being able to consume without trading. Then, the total number of agents in the market is $N = \Sigma_j (1 - \delta_j) P_j$, and the fraction of agents in the market who have good i and want good $j \neq i$ is $\pi_{ij} = P_i \delta_j / N$. Because meetings are random, π_{ij} is the probability of meeting someone who has good i and wants good j. Given S, the distribution $\boldsymbol{p}$ evolves according to some law of motion $\boldsymbol{p}' = f(\boldsymbol{p}; S)$; for now, we simply note that a steady-state distribution is a solution to $\boldsymbol{p} = f(\boldsymbol{p}; S)$ (see the Appendix for details). If we know $\boldsymbol{p}$, we can determine the steady-state value of $\boldsymbol{\pi} = (\ldots \pi_{ij} \ldots)$ as a function of S (given $\boldsymbol{\sigma}$ and $\boldsymbol{\delta}$). Agents have rational expectations regarding these meeting probabilities.

We now derive the value function for type 1 with an inventory of good 1, looking forward to the next period, for given strategies of others as described by S. This derivation is somewhat tedious because of the many possible things that can happen to an individual in a period. Propositions 1 and 2, below, simplify the analysis considerably, but it is necessary to consider the general situation before proving these results. To proceed, we partition the different things that can happen to an individual by considering each of three events in turn.

Event 1. With probability δ_1, he desires good 1. In this case, he does not need to go to the market but simply consumes his inventory and produces a new unit of good 1, for a payoff of

$$W_1 = u + V_{11}.$$

Event 2. With probability δ_2, he desires good 2. Then he goes to the market, where several things can happen. With probability $\pi_{12} + \pi_{13}$, he meets a partner who also has good 1 and they cannot trade, which yields payoff V_{11}. With probability π_{21}, he meets a partner with good 2 who wants good 1 and they trade, which yields payoff $u + V_{11}$. With probability $(\delta_3/N)p_{12}$ he meets a partner of type 1 who holds good 2 but wants good 3, and they trade i.f.f. this partner is willing to exchange good 2 for good 1, which yields an expected payoff of $s_{21}^1(u + V_{11}) + (1 - s_{21}^1)V_{11}$. A similar argument can be applied to the case in which he meets agents of type 2 and type 3 holding good 2 and wanting to consume good 3. With probability π_{31}, he meets a partner with good 3 who wants good 1 and they trade i.f.f. our agent is willing to exchange good 1 for good 3, which yields expected payoff $s_{13}^1 V_{13} + (1 - s_{13}^1)V_{11}$. With probability $(\delta_2/N)p_{23}$, he meets a partner of type 2 holding good 3 who wants good 2, and they trade i.f.f. $s_{13}^1 = 1$ and $s_{31}^2 = 1$. A similar argument applies when our agent meets an agent of type 3.[4] This yields payoff $s_{13}^1(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3)V_{13} + [(1 - s_{13}^1)(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3) + p_{13}(1 - s_{31}^1) + p_{23}(1 - s_{31}^2) + p_{33}(1 - s_{31}^3)]V_{11}$.

Putting all of this together, the expected payoff in Event 2 is

$$\begin{aligned} W_2 = {} & (\pi_{12} + \pi_{13})V_{11} + \pi_{21}(u + V_{11}) \\ & + (\delta_3/N)\{(p_{12}s_{21}^1 + p_{22}s_{21}^2 + p_{32}s_{21}^3)(u + V_{11}) \\ & + [p_{12}(1 - s_{21}^1) + p_{22}(1 - s_{21}^2) + p_{32}(1 - s_{21}^3)]V_{11}\} \\ & + \pi_{31}[s_{13}^1 V_{13} + (1 - s_{13}^1)V_{11}] \\ & + (\delta_2/N)\{s_{13}^1(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3)V_{13} \\ & + [(1 - s_{13}^1)(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3) \\ & + p_{13}(1 - s_{31}^1) + p_{23}(1 - s_{31}^2) + p_{33}(1 - s_{31}^3)]V_{11}\}. \end{aligned}$$

Event 3. With probability δ_3, he desires good 3. An analysis similar to the preceding case implies that the expected payoff in Event 3 is

$$\begin{aligned} W_3 = {} & (\pi_{12} + \pi_{13})V_{11} + \pi_{31}(u + V_{11}) \\ & + (\delta_2/N)\{(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3)(u + V_{11}) \\ & + [p_{13}(1 - s_{31}^1) + p_{23}(1 - s_{31}^2) + p_{33}(1 - s_{31}^3)]V_{11}\} \\ & + \pi_{21}[s_{12}^1 V_{12} + (1 - s_{12}^1)V_{11}] \\ & + (\delta_3/N)\{s_{12}^1(p_{12}s_{21}^1 + p_{22}s_{21}^2 + p_{32}s_{21}^3)V_{12} \\ & + [(1 - s_{12}^1)(p_{12}s_{21}^1 + p_{22}s_{21}^2 + p_{32}s_{21}^3) \\ & + p_{12}(1 - s_{21}^1) + p_{22}(1 - s_{21}^2) + p_{32}(1 - s_{21}^3)]V_{11}\}. \end{aligned}$$

The value function V_{11} satisfies

$$V_{11} = \frac{1}{1+r} \sum_j \delta_j W_j,$$

where $r > 0$ is the rate of time preference. Substituting the W_j formulas into V_{11} and simplifying, we arrive at

$$\begin{aligned} rV_{11} = {} & \gamma_1 u + \delta_2 s_{13}^1[\pi_{31} + (\delta_2/N)(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3)](V_{13} - V_{11}) \\ & + \delta_3 s_{12}^1[\pi_{21} + (\delta_3/N)(p_{12}s_{21}^1 + p_{22}s_{21}^2 + p_{32}s_{21}^3)](V_{12} - V_{11}), \end{aligned} \tag{1}$$

where

$$\begin{aligned} \gamma_1 = {} & \delta_1 + \delta_2[\pi_{21} + (\delta_3/N)(p_{12}s_{21}^1 + p_{22}s_{21}^2 + p_{32}s_{21}^3)] \\ & + \delta_3[\pi_{31} + (\delta_2/N)(p_{13}s_{31}^1 + p_{23}s_{31}^2 + p_{33}s_{31}^3)] \end{aligned} \tag{2}$$

is the probability of consumption next period conditional on holding good 1, either because good 1 is desired or because something else is desired and acquired in exchange for good 1.

By similar reasoning (or, more efficiently, by appropriately modulating the subscripts), one can derive

$$rV_{12} = \gamma_2(u + V_{11} - V_{12}) + \delta_1 s^1_{23}\left[\pi_{32} + (\delta_1/N)\left(p_{13}s^1_{32} + p_{23}s^2_{32} + p_{33}s^3_{32}\right)\right] \times (V_{13} - V_{12}) + \delta_3 s^1_{21}\left[\pi_{12} + (\delta_3/N)\left(p_{11}s^1_{12} + p_{21}s^2_{12} + p_{31}s^3_{12}\right)\right](V_{11} - V_{12}), \quad (3)$$

where

$$\gamma_2 = \delta_2 + \delta_3\left[\pi_{32} + (\delta_1/N)\left(p_{13}s^1_{32} + p_{23}s^2_{32} + p_{33}s^3_{32}\right)\right] + \delta_1\left[\pi_{12} + (\delta_3/N)\left(p_{11}s^1_{12} + p_{21}s^2_{12} + p_{31}s^3_{12}\right)\right] \quad (4)$$

and

$$rV_{13} = \gamma_3(u + V_{11} - V_{13}) + \delta_2 s^1_{31}\left[\pi_{13} + (\delta_2/N)\left(p_{11}s^1_{13} + p_{21}s^2_{13} + p_{31}s^3_{13}\right)\right] \times (V_{11} - V_{13}) + \delta_1 s^1_{32}\left[\pi_{23} + (\delta_1/N)\left(p_{12}s^1_{23} + p_{22}s^2_{23} + p_{32}s^3_{23}\right)\right](V_{12} - V_{13}), \quad (5)$$

where

$$\gamma_3 = \delta_3 + \delta_1\left[\pi_{13} + (\delta_2/N)\left(p_{11}s^1_{13} + p_{21}s^2_{13} + p_{31}s^3_{13}\right)\right] + \delta_2\left[\pi_{23} + (\delta_1/N)\left(p_{12}s^1_{23} + p_{22}s^2_{23} + p_{32}s^3_{23}\right)\right]. \quad (6)$$

Using the same reasoning, we can derive similar expressions for the value functions for types 2 and 3. Having done all of this, we are in position to prove that agents of different types necessarily rank the value functions in the same order, and hence use the same strategies.

PROPOSITION 1. *For any two types h and i and two goods j and k,* $V_{hj} > V_{hk} \Leftrightarrow V_{ij} > V_{ik}$.

Proof. After solving the system of linear equations (1), (3), and (5) for the V_{1j} (and the similar system for agents of type 2 and type 3), we get the following result:

$$V_{i1} - V_{i2} > 0 \quad \text{i.f.f.}$$

$$(\gamma_1 - \gamma_2)\left(r + \gamma_3 + \delta_2 s^i_{31}\epsilon_1 + \delta_1 s^i_{32}\epsilon_2\right) + (\gamma_1 - \gamma_3)\delta_1 s^i_{23}\epsilon_3 + (\gamma_3 - \gamma_2)\delta_2 s^i_{13}\epsilon_4 > 0, \quad (7)$$

$$V_{i2} - V_{i3} > 0 \quad \text{i.f.f.}$$

$$(\gamma_2 - \gamma_3)\left(r + \gamma_1 + \delta_3 s^i_{12}\epsilon_5 + \delta_2 s^i_{13}\epsilon_4\right) + (\gamma_2 - \gamma_1)\delta_2 s^i_{31}\epsilon_1 + (\gamma_1 - \gamma_3)\delta_3 s^i_{21}\epsilon_6 > 0, \quad (8)$$

$$V_{i1} - V_{i3} > 0 \quad \text{i.f.f.}$$

$$(\gamma_1 - \gamma_3)\left(r + \gamma_2 + \delta_3 s^i_{21}\epsilon_6 + \delta_1 s^i_{23}\epsilon_3\right) + (\gamma_2 - \gamma_3)\delta_3 s^i_{12}\epsilon_5 + (\gamma_1 - \gamma_2)\delta_1 s^i_{32}\epsilon_2 > 0, \quad (9)$$

where

$$\epsilon_1 = \pi_{13} + (\delta_2/N)\left(p_{11}s^1_{13} + p_{21}s^2_{13} + p_{31}s^3_{13}\right) > 0,$$
$$\epsilon_2 = \pi_{23} + (\delta_1/N)\left(p_{12}s^1_{23} + p_{22}s^2_{23} + p_{32}s^3_{23}\right) > 0,$$
$$\epsilon_3 = \pi_{32} + (\delta_1/N)\left(p_{13}s^1_{32} + p_{23}s^2_{32} + p_{33}s^3_{32}\right) > 0,$$
$$\epsilon_4 = \pi_{31} + (\delta_2/N)\left(p_{13}s^1_{31} + p_{23}s^2_{31} + p_{33}s^3_{31}\right) > 0,$$
$$\epsilon_5 = \pi_{21} + (\delta_3/N)\left(p_{12}s^1_{21} + p_{22}s^2_{21} + p_{32}s^3_{21}\right) > 0,$$
$$\epsilon_6 = \pi_{12} + (\delta_3/N)\left(p_{11}s^1_{12} + p_{21}s^2_{12} + p_{31}s^3_{12}\right) > 0.$$

The argument now proceeds by contradiction. Suppose that two agents rank the value functions differently and hence use different strategies. For instance, suppose $\mathbf{s}^1 = (0, 0, 1)$ and $\mathbf{s}^2 = (1, 0, 0)$. Then, $V_{11} > V_{12}$, and, by (7), $(\gamma_1 - \gamma_2)(r + \gamma_3 + \delta_2\epsilon_1 + \delta_1\epsilon_2) > 0$, which implies that $\gamma_1 > \gamma_2$. Also, $V_{22} > V_{21}$, and, again by (7), $(\gamma_1 - \gamma_2)(r + \gamma_3 + \delta_1\epsilon_2) + (\gamma_3 - \gamma_2)\delta_2\epsilon_4 < 0$, which implies that $\gamma_2 > \gamma_3$ (given that $\gamma_1 > \gamma_2$). Also, $V_{21} < V_{23}$, and, by (9), $(\gamma_1 - \gamma_3)(r + \gamma_2) + (\gamma_2 - \gamma_3)\delta_3\epsilon_5 + (\gamma_1 - \gamma_2)\delta_1\epsilon_2 < 0$, which implies $\gamma_3 > \gamma_1$. This contradicts $\gamma_1 > \gamma_2$ and $\gamma_2 > \gamma_3$. Consequently, we cannot have $\mathbf{s}^1 = (0, 0, 1)$ and $\mathbf{s}^2 = (1, 0, 0)$. Following an identical procedure, one can derive similar contradictions whenever $\mathbf{s}^2 \neq (0, 0, 1)$. One can do the same thing whenever $\mathbf{s}^3 \neq (0, 0, 1)$. Hence, if $\mathbf{s}^1 = (0, 0, 1)$, then $\mathbf{s}^2 = \mathbf{s}^3 = (0, 0, 1)$. A similar argument applies to any $\mathbf{s}^1$. This completes the proof. ■

Proposition 1 simplifies things a lot, because we only need to analyze situations in which agents rank the goods in the same order, and we can summarize S by one vector. The following result further simplifies the analysis.

PROPOSITION 2. $V_{ij} > V_{ik} \Leftrightarrow \gamma_j > \gamma_k$.

Proof. Because all types use the same strategies by Proposition 1, we can substitute $s_{jk} = s^i_{jk}$ in (7)–(9). Then, using $s_{jk} \cdot s_{kj} = 0$, we get the following results for agents of type 1:

$$V_{11} > V_{12} \quad \text{i.f.f.}$$

$$(\gamma_1 - \gamma_2)(r + \gamma_3 + \delta_2 s_{31}\pi_{13} + \delta_1 s_{32}\pi_{23}) + (\gamma_3 - \gamma_2)\delta_2 s_{13}\pi_{31} + (\gamma_1 - \gamma_3)\delta_1 s_{23}\pi_{32} > 0, \quad (10)$$

$$V_{12} > V_{13} \quad \text{i.f.f.}$$

$$(\gamma_2 - \gamma_3)(r + \gamma_1 + \delta_3 s_{12}\pi_{21} + \delta_2 s_{13}\pi_{31}) + (\gamma_1 - \gamma_3)\delta_3 s_{21}\pi_{12} + (\gamma_2 - \gamma_1)\delta_2 s_{31}\pi_{13} > 0, \quad (11)$$

$$V_{13} > V_{11} \quad \text{i.f.f.}$$

$$(\gamma_3 - \gamma_1)(r + \gamma_2 + \delta_1 s_{23}\pi_{32} + \delta_3 s_{21}\pi_{12}) + (\gamma_2 - \gamma_1)\delta_1 s_{32}\pi_{23} + (\gamma_3 - \gamma_2)\delta_3 s_{12}\pi_{21} > 0. \quad (12)$$

Given (10)–(12), choose any ranking $V_{ih} > V_{ij} > V_{ik}$, and substitute the implied $\boldsymbol{s}$. Then, check that $\gamma_h > \gamma_j$ and $\gamma_j > \gamma_k$ are necessary and sufficient conditions for the V's to be ranked in the assumed way (these computations involve simple algebra which is available upon request). ■

One can define an equilibrium (symmetric, steady-state, pure strategy) in the usual way here as a vector of strategies, $\boldsymbol{s}$, such that every s_j maximizes the payoff of type-j agents, as described by the value functions V_{ji}, taking as given the strategies of others. However, based on the above analysis summarized in Propositions 1 and 2, the best response conditions in this model are completely summarized by the values of γ and, therefore, we can more conveniently define an equilibrium as follows:

DEFINITION. *A steady-state, pure-strategy equilibrium is a vector* $\boldsymbol{s}$ *that satisfies* $s_{ij} = 1$ i.f.f. $\gamma_j > \gamma_i$, *where the* γ*'s are defined by* (2), (4), *and* (6).

In terms of intuition, Proposition 2 says that the only value to holding an asset is its use in facilitating future consumption. This is because ultimately all that agents care about is consumption. Proposition 1 says that, irrespective of their production types, all agents rank the goods the same way in terms of their asset value. This is because all agents draw their taste shocks from the same distribution, and so, conditional on holding the same good, they all have the same chance of consuming next period. Of course, once they consume, they will be in different positions if they produce different goods, but this does not affect their trading strategies.

In terms of monetary economics, we can say the following. If $\gamma_i > \gamma_j > \gamma_k$, then good i is the best asset and hence is always accepted in trade even if the recipient does not want to consume it; good j is the second-best asset and hence is accepted in some trades where the recipient does not want to consume it, but not others; and good k is the worst asset and hence is never accepted in trade unless the recipient wants to consume it. According to standard definitions [see, e.g., Kiyotaki and Wright (1989) and the references contained therein], this means that good i serves as a universally accepted commodity money, good j serves as a partially accepted commodity money, and good k never serves as money.[5]

We now consider some special cases where supply or demand parameters take on certain extreme values (e.g., $\sigma_i = 1$ or $\delta_j = 0$) which allow us to say quite a bit analytically. Although the model is not very interesting at these extreme values (e.g., at $\sigma_i = 1$ there is only one type), by continuity we know that for parameter values not too different from these extremes the results will be qualitatively similar. The following result says that when demand for a particular good is very small the only possible equilibria are those in which it is the worst asset, and therefore it can never be used as money.

PROPOSITION 3. *When demand for good i is very small, it must be the worst asset, and either of the other two goods will be the best asset depending on which has the greater demand relative to supply (in a sense to be made precise in the proof).*

Proof. We prove this result for the case of a very low demand for good 3; exactly the same argument applies to any other good. Consider the limiting case where $\delta_3 = 0$. Then, it is easy to show $P_j = p_{jj} = \sigma_j$ for all j (all types exclusively hold their production good, because, e.g., given $\delta_3 = 0$, type 1 either demands good 1, in which case he consumes and produces a new unit of good 1, or demands good 2, in which case the only trade he ever makes is for immediate consumption). This implies $N = 1 - \sigma_1\delta_1 - \sigma_2\delta_2$, $\pi_{12} = \sigma_1\delta_2/N$, $\pi_{13} = 0$, $\pi_{21} = \sigma_2\delta_1/N$, and $\pi_{23} = 0$ (we do not need π_{3j}). From these, one can solve for

$$\gamma_1 = \frac{\delta_1(1-\sigma_1\delta_1)}{1-\sigma_1\delta_1-\sigma_2\delta_2}, \qquad \gamma_2 = \frac{\delta_2(1-\sigma_2\delta_2)}{1-\sigma_1\delta_1-\sigma_2\delta_2},$$

$$\gamma_3 = \frac{\delta_1\delta_2(\sigma_1 s_{13}+\sigma_2 s_{23})}{1-\sigma_1\delta_1-\sigma_2\delta_2}.$$

It is now easy to see that equilibria where good 3 ranks as the best or second-best asset do not exist. Consider, for instance, $\boldsymbol{s} = (0, 1, 1)$, which requires $\gamma_1 > \gamma_3 > \gamma_2$. Using $\delta_1 = 1 - \delta_2$ one can show that $\gamma_3 - \gamma_2 > 0$ i.f.f. $\sigma_2 > 1$, which is a contradiction. Similar contradictions can be derived for all equilibria which imply that good 3 would be ranked as best or second-best asset, and so, the only possible equilibria have $\gamma_3 = 0$, and good 3 is the worst asset.

Given this, we now proceed to rank goods 1 and 2. Observe that $\gamma_1 - \gamma_2$ is proportional to $H(\delta_1) = (\sigma_1 - \sigma_2)\delta_1^2 - 2(1-\sigma_2)\delta_1 + 1 - \sigma_2$. Because $H(0) > 0 > H(1)$ and $H'(\delta_1) < 0$ for all $\delta_1 \in (0, 1)$, there is a unique δ_1^* such that $\gamma_2 > \gamma_1$ i.f.f. $\delta_1 < \delta_1^*$, where

$$\delta_1^* = \begin{cases} \frac{1}{2} & \text{if} \quad \sigma_1 = \sigma_2 \\ \dfrac{1}{\sigma_1-\sigma_2}\left[1-\sigma_2-\sqrt{(1-\sigma_1)(1-\sigma_2)}\right] & \text{if} \quad \sigma_1 > \sigma_2 \\ \dfrac{1}{\sigma_1-\sigma_2}\left[1-\sigma_2+\sqrt{(1-\sigma_1)(1-\sigma_2)}\right] & \text{if} \quad \sigma_1 < \sigma_2. \end{cases}$$

We conclude that when $\delta_3 = 0$, good 3 is the worst asset, and either good 1 will be the best asset i.f.f. it has a greater demand relative to supply than good 2 in the sense that $\delta_1 > \delta_1^*$; in particular, if good 1 and good 2 are supplied equally ($\sigma_1 = \sigma_2$), then good 1 is the best asset i.f.f. $\delta_1 > \delta_2$. This establishes the desired results for the limiting case $\delta_3 = 0$. By continuity, the equilibrium will be qualitatively similar if δ_3 is strictly positive but not too big. ■

The above argument actually implies that any of the possible strategy profiles can be an equilibrium for the appropriate choice of $\boldsymbol{\delta}$, regardless of $\boldsymbol{\sigma}$. Consider, e.g., the case $V_{i1} > V_{i2} > V_{i3}$. If we make δ_3 sufficiently small and δ_1 sufficiently big, then $V_{i1} > V_{i2} > V_{i3}$ in equilibrium. Moreover, it implies the following: As demand for good 1 gets sufficiently big ($\delta_1 \to 1$, which requires δ_2 and $\delta_3 \to 0$), good 1 will necessarily be the best asset. Hence, high enough demand always guarantees that good 1 will emerge as money in equilibrium.

The result stated in Proposition 3 is also interesting because it can be interpreted as saying that fiat money (a medium of exchange with no intrinsic value) is unlikely to emerge in a decentralized economy such as the one we describe. This suggests a need for centralized institutions to support the circulation of fiat money, which is what seems to have happened historically. This is an important difference with other models of commodity money [e.g., Kiyotaki and Wright (1989)] in which an intrinsically useless commodity takes on value because of its universal acceptability. In a version of that model, traders were always willing to accept fiat money, an object that was both universally accepted and easily storable. In fact, one could show in these models that fiat money does not need to have the best intrinsic properties in order to have value, because higher acceptability may well overcome a high storage cost [in particular, Aiyagari and Wallace (1992) construct an equilibrium in which fiat money is valued without having the lowest storage cost]. In the present model, this is no longer the case. Here, we always have a good j with universal acceptability (the one with the highest γ_j), and so, fiat money cannot be better in those grounds; but good j also has potential consumption value whereas fiat money does not. Hence, if we assume equal storage costs (and other fundamental properties), fiat money will not be accepted by anyone with good j and, consequently, not by anyone at all, so there cannot be an equilibrium in which fiat money has positive value. Of course, if fiat money had superior storability (or some other superior intrinsic property), presumably it could be valued. But this is a key difference between those previous models and our model: For a fiat object to be valued in our model, it would need to have the best intrinsic properties.

The next result implies that we cannot rule out the existence of equilibria in which goods in very high supply are used as money, or guarantee that goods in low supply are used as money. We proceed by characterizing the set of equilibria in the extreme case of $\sigma_1 = 1$, which might suggest that good 1 is the worst asset because it is so plentiful.

Given $\sigma_1 = 1$, it is easy to see that $p_{11} = 1$ and $p_{ij} = 0$ unless $i = j = 1$. Therefore, $P_1 = 1$, $P_2 = P_3 = 0$, $N = 1 - \delta_1$, $\pi_{12} = \delta_2/(1 - \delta_1)$, $\pi_{13} = \delta_3/(1 - \delta_1)$. Now, the general formulas for γ_j reduce to

$$\gamma_1 = \delta_1 = 1 - \delta_2 - \delta_3, \qquad \gamma_2 = \frac{\delta_2 + \delta_3(1 - \delta_2 - \delta_3)s_{12}}{\delta_2 + \delta_3},$$
$$\gamma_3 = \frac{\delta_3 + \delta_2(1 - \delta_2 - \delta_3)s_{13}}{\delta_2 + \delta_3}.$$

This means that $\gamma_i - \gamma_j$ is proportional to D_{ij}, where

$$D_{21} = (\delta_2 + \delta_3)^2 - \delta_3 + \delta_3(1 - \delta_2 - \delta_3)s_{12},$$
$$D_{31} = (\delta_2 + \delta_3)^2 - \delta_2 + \delta_2(1 - \delta_2 - \delta_3)s_{13},$$
$$D_{32} = \delta_3 - \delta_2 + \delta_2(1 - \delta_2 - \delta_3)s_{13} - \delta_3(1 - \delta_2 - \delta_3)s_{12}.$$

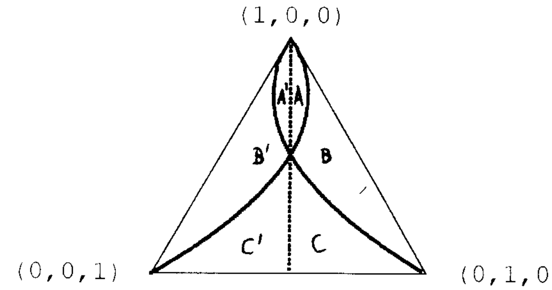

FIGURE 1. Equilibria when $\boldsymbol{\sigma} = (1, 0, 0)$.

We now look for equilibria of various types. First, suppose good 1 is the best asset. For one such case, let $V_{i1} > V_{i2} > V_{i3}$, which means $s_{12} = s_{13} = 0$ and requires $D_{21} < 0$, $D_{31} < 0$, and $D_{32} < 0$. Inserting $s_{12} = s_{13} = 0$ into D_{ij} yields

$$D_{21} = (\delta_2 + \delta_3)^2 - \delta_3 < 0 \quad \text{i.f.f.} \quad \delta_2 < \sqrt{\delta_3} - \delta_3,$$
$$D_{31} = (\delta_2 + \delta_3)^2 - \delta_2 < 0 \quad \text{i.f.f.} \quad \delta_3 < \sqrt{\delta_2} - \delta_2,$$
$$D_{32} = \delta_3 - \delta_2 < 0 \quad \text{i.f.f.} \quad \delta_2 > \delta_3.$$

These conditions hold, and hence $V_{i1} > V_{i2} > V_{i3}$ is an equilibrium, in the region labeled A in Figure 1. The other case in which good 1 is the best asset is $V_{i1} > V_{i3} > V_{i2}$, and, by symmetry, it is an equilibrium in the region labeled A'.

Now suppose good 1 is the second-best asset. One case is $V_{i2} > V_{i1} > V_{i3}$, which means $s_{12} = 1$ and $s_{13} = 0$ and requires $D_{21} > 0$, $D_{31} < 0$, and $D_{32} > 0$. Inserting $s_{12} = 1$ and $s_{13} = 0$ yields:

$$D_{21} = \delta_2(\delta_2 + \delta_3) > 0 \quad \text{for all} \quad \boldsymbol{\delta},$$
$$D_{31} = (\delta_2 + \delta_3)^2 - \delta_2 < 0 \quad \text{i.f.f.} \quad \delta_3 < \sqrt{\delta_2} - \delta_2,$$
$$D_{32} = -\delta_2 + \delta_2\delta_3 + \delta_3^2 < 0 \quad \text{i.f.f.} \quad \delta_2 > \delta_3^2/(1 - \delta_3).$$

It can be shown easily that $D_{31} < 0$ implies $D_{32} < 0$, and therefore this equilibrium exists i.f.f. $\delta_3 < (\delta_2)^{1/2} - \delta_2$, which holds in the union of regions A, A', and B in Figure 1. Symmetrically, the other equilibrium where good 1 is second best, $V_{i3} > V_{i1} > V_{i2}$, exists in the union of regions A', A, and B' in Figure 1.

Finally, suppose good 1 is the worst asset. One case is $V_{i2} > V_{i3} > V_{i1}$, which means $s_{12} = 1$ and $s_{13} = 1$ and requires $D_{21} > 0$, $D_{31} > 0$, and $D_{32} < 0$. Inserting $s_{12} = s_{13} = 1$ yields

$$D_{21} = \delta_2(\delta_2 + \delta_3) > 0 \quad \text{for all} \quad \boldsymbol{\delta},$$
$$D_{31} = \delta_3(\delta_2 + \delta_3) > 0 \quad \text{for all} \quad \boldsymbol{\delta},$$
$$D_{32} = \delta_3^2 - \delta_2^2 < 0 \quad \text{i.f.f.} \quad \delta_2 > \delta_3.$$

Hence, this equilibrium exists in the union of regions A, B, and C in Figure 1. By symmetry, the other equilibrium where good 1 is the worst asset, $V_{i3} > V_{i2} > V_{i1}$, exists in the symmetric region, the union of regions A', B', and C'. In particular, notice that for all $\boldsymbol{\delta}$ there is an equilibrium for which good 1 is the worst asset, and either good 2 or good 3 is the best asset depending only on $\delta_2 - \delta_3$. Although we have verified this for the limiting case in which $\sigma_1 = 1$, by continuity, Figure 1 looks similar, and the same economics apply if σ_1 is strictly less than one but sufficiently large.

The economics of large σ_1 can be explained as follows: If δ_1 is very high, which occurs near the origin in Figure 1, then good 1 can be the best, second-best, or third-best asset; if δ_1 is less high, then good 1 can be the second- or the third-best, but not the best, asset; and, finally, if δ_1 is low, then good 1 must be the worst asset. Given high supply, high demand is a necessary but not sufficient condition for good 1 to be used as money, in the sense that there always exists an equilibrium in which good 1 is the worst asset. Hence, high supply makes it less likely that good 1 will be money; it could be money, but demand for that good has to be high and also beliefs have to be right (in the sense that whenever there is an equilibrium in which good 1 is used as money, there is another equilibrium in which it is not). We summarize certain relevant aspects of the above analysis as follows:

PROPOSITION 4. *Even if the supply of good i is very high (low), there is always a value of $\boldsymbol{\delta}$ for which it is (is not) used as money.*

The results contained in Propositions 3 and 4 reflect an asymmetry between demand and supply in the way that they affect our results about what commodities will emerge as money. As we have seen, very high supply for good i does not work exactly the same way as very low demand. The reason for this asymmetry lies in the fact that, in this model, demand for final consumption is what ultimately drives the desirability and hence the moneyness of objects. Of course, supply and speculative beliefs are also factors that play a role in the determination of money in equilibrium, but they cannot overcome the effect of big-enough or small-enough demand.

The last result of this section concerns welfare. It says that, in equilibrium, the producers of the most accepted good are better off than the producers of the second most accepted good, and the latter are better off than the producers of the least accepted good. Hence, in a given equilibrium, producing a good that is used as money gives a higher payoff.

PROPOSITION 5. *If $\gamma_i > \gamma_j$, then $V_{im} > V_{jm}$ for all m; that is, type-i agents are better off than type-j agents (holding the same good) in any equilibrium in which good i is ranked above good j.*

Proof. The proof involves simple but tedious algebra. We need to solve a system of nine linear equations, composed of (1), (3), and (5) for type 1 and something similar for types 2 and 3, in which we insert for s_{ij} the values implied by the γ's. Given the solution, it is easy to check that if $\gamma_i > \gamma_j > \gamma_k$, then $V_{im} > V_{jm} > V_{km}$ for $m = 1, 2, 3$ (details are available upon request). ■

Multiple equilibria with different media of exchange exist for a large set of supply and demand parameters. In general, we cannot Pareto rank those equilibria,[6] but we can construct examples in which some of the equilibria are inefficient. Consider the case in which $\boldsymbol{\delta} = (0.18, 0.10, 0.72)$ and $\boldsymbol{\sigma} = (0.1/9, 0.1/9, 8.8/9)$. It can be shown that the following rankings of assets are equilibria: (a) $V_{i1} > V_{i2} > V_{i3}$, (b) $V_{i2} > V_{i3} > V_{i1}$, (c) $V_{i3} > V_{i1} > V_{i2}$, and (d) $V_{i1} > V_{i3} > V_{i2}$. It is easy to prove that (c) is inefficient because all agents are better off in equilibrium (a). The intuition for this is the following: An equilibrium in which a good in very high supply performs the role of main medium of exchange is Pareto dominated by another equilibrium in which this same good is the worst asset. This shows that there exist equilibria in which an ill-suited object is used as money (e.g., one in very high supply), but this is inefficient and everyone would be better off if they could coordinate on a different equilibrium with different money.

3. NUMERICAL ANALYSIS

In the preceding section, we presented some analytic results displaying how the set of equilibria behaved for certain extreme values of parameters (high or low demand for a good, and high supply of a good). Here, we consider a range of other parameter values. The method that we use for finding equilibria is as follows: First, choose a ranking for good 1, 2, and 3. Next compute $\boldsymbol{\pi}$ numerically using the formulas in the Appendix. Then, compute γ and check whether it is consistent with the ranking with which we started. Whether it is will depend on the supply and demand parameters $\boldsymbol{\sigma}$ and $\boldsymbol{\delta}$ (because γ depends on $\boldsymbol{\delta}$ directly, and on $\boldsymbol{\pi}$, which itself depends on $\boldsymbol{\sigma}$ and $\boldsymbol{\delta}$). In fact, because of the symmetry of the model, for most of our analysis we need only focus on a single case. To see this, note that if $\hat{s}$ is an equilibrium for parameters $\boldsymbol{\sigma}$ and $\boldsymbol{\delta}$ and ρ is a permutation operator, then $\rho(\hat{s})$ is an equilibrium for parameters $\rho(\boldsymbol{\sigma})$ and $\rho(\boldsymbol{\delta})$. For example, fix $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ and $\boldsymbol{\delta} = (\delta_1, \delta_2, \delta_3)$ and suppose that $\hat{s} = (0, 0, 1)$ implies $\gamma_1 > \gamma_2 > \gamma_3$, which means that $\hat{s} = (0, 0, 1)$ is an equilibrium. Then, for the permutation $\rho(\boldsymbol{\sigma}) = (\sigma_3, \sigma_2, \sigma_1)$ and $\rho(\boldsymbol{\delta}) = (\delta_3, \delta_2, \delta_1)$, it is clear that $\rho(\hat{s}) = (1, 1, 0)$ implies $\gamma_3 > \gamma_2 > \gamma_1$, because this is merely a change in notation, and therefore $\hat{s} = (1, 1, 0)$ is an equilibrium. Hence, if we find the set of $(\boldsymbol{\sigma}, \boldsymbol{\delta})$ parameters such that $\hat{s} = (0, 0, 1)$ is an equilibrium, we know the set of parameters such that any other s is an equilibrium simply by permuting the subscripts.

We focus on the case $\hat{s} = (0, 0, 1)$, which means $\gamma_1 > \gamma_2 > \gamma_3$. For this particular case, the steady-state distribution of inventories is described as follows (see the Appendix for more details): First, the producers of good 1 never trade it (except for a good they consume), and so, $p_{11} = \sigma_1$ and $p_{12} = p_{13} = 0$. Second, the producers of good 2 trade it for good 1 but not for good 3, and $p_{21} = (\sigma_2/\sigma_3)p_{31}$, $p_{22} = \sigma_2 - p_{21}$, and $p_{23} = 0$, where p_{31} is defined next. The values of p_{31} and p_{32} are found by solving

$$P_1(\sigma_3 - p_{31})\delta_2\delta_3 = [N\delta_1 + P_2\delta_2(1 - \delta_2) + P_3\delta_3(1 - \delta_3)]p_{31},$$

$$P_2 P_3 \delta_1 \delta_3 = [N\delta_2 + P_1\delta_2(1 - \delta_1) + P_3\delta_3(1 - \delta_3)]p_{32},$$

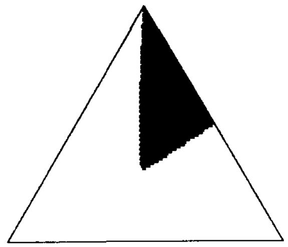

FIGURE 2. $\sigma = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

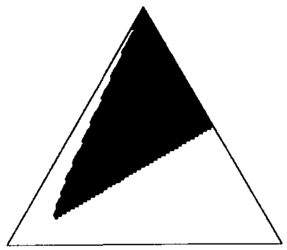

FIGURE 3. $\sigma = (0.1/9, 0.1/9, 8.8/9)$.

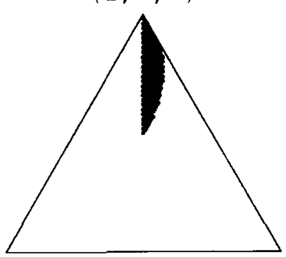

FIGURE 4. $\sigma = (0.98, 0.01, 0.01)$.

where

$$P_1 = \sigma_1 + [(\sigma_2 + \sigma_3)/\sigma_3] p_{31}, \qquad P_2 = \sigma_2 - (\sigma_2/\sigma_3) p_{31} + p_{32},$$

$$P_3 = \sigma_3 - p_{31} - p_{32}.$$

For what values of (σ, δ) is $\hat{s} = (0, 0, 1)$ an equilibrium? To answer this question, for all points on a grid in the set $\Theta = \{(\sigma, \delta) : \Sigma_i \sigma_i = \Sigma_i \delta_i = 1\}$, we compute γ_i and check when $\gamma_1 > \gamma_2 > \gamma_3$. Notice that this does not require specifying values for the preference parameters u and r.

Figures 2, 3, and 4 show the set of δ for which, given values of σ, the equilibrium where $\gamma_1 > \gamma_2 > \gamma_3$ exists. Recall that in this equilibrium, good 1 is a

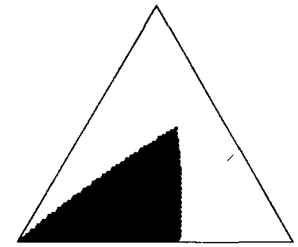

FIGURE 5. $\delta = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

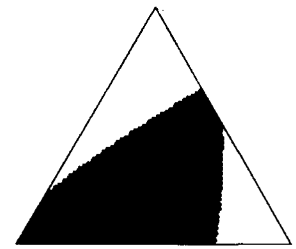

FIGURE 6. $\delta = (3.42/9, 3/9, 2.58/9)$.

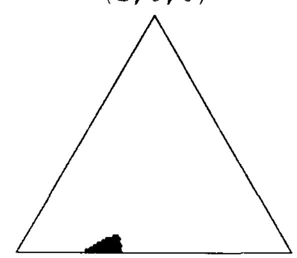

FIGURE 7. $\delta = (2/9, 3/9, 4/9)$.

generally accepted medium of exchange and good 2 is a partially accepted medium of exchange. Each of Figures 2, 3, and 4 differ in the relative supply of goods, σ. Notice that when supply of goods 1 and 2 is markedly scarcer than that of good 3, this equilibrium exists for a much larger region (Figure 3). When the supply of good 1 is large (Figure 4), the region where this equilibrium exists is small, and corresponds very closely to region A in Figure 1.

Figures 5, 6, and 7 display values of the parameters σ for which the same equilibrium exists, given different values for δ. For high enough demand for goods 1 and 2, this equilibrium exists for a large region (Figures 5 and 6), whereas for

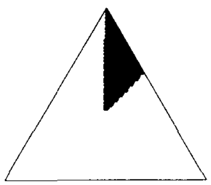

FIGURE 8. Equilibrium $\gamma_1 > \gamma_2 > \gamma_3$ when $\sigma = (0.70, 0.15, 0.15)$.

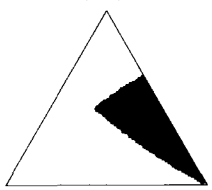

FIGURE 9. Equilibrium $\gamma_2 > \gamma_1 > \gamma_3$ when $\sigma = (0.70, 0.15, 0.15)$.

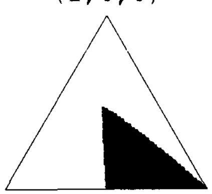

FIGURE 10. Equilibrium $\gamma_1 > \gamma_2 > \gamma_3$ when $\sigma = (0.70, 0.15, 0.15)$.

very low demand for goods 1 and 2, this equilibrium exists only for a very small set of values for σ (Figure 7).

We also can use numerical methods to look at the changes that occur in Figure 1 when we move away from the extreme assumption $\sigma_1 = 1$ used in constructing that diagram analytically. In Figures 8, 9, and 10, we fix $\sigma = (0.70, 0.15, 0.15)$, and show the set of δ for which the following equilibria exist: respectively, $\gamma_1 > \gamma_2 > \gamma_3$, $\gamma_2 > \gamma_1 > \gamma_3$, and $\gamma_2 > \gamma_3 > \gamma_1$. These figures show regions in the parameter space that should be compared with the corresponding regions showed in Figure 1 (respectively, A, the union of A, A', and B, and the union of A, B, and C). What these figures indicate is that when the supply of good 1 goes down, there are more values of δ such that good 1 serves as a general medium of exchange (Figure 8) and fewer values such that good 1 is the least valued asset (Figure 10).

To conclude the numerical analysis, we compare different equilibria and also different economies as to their degree of monetization. To do this, we propose the

following measure, m, for the equilibrium with $\gamma_1 > \gamma_2 > \gamma_3$ (simply modulating the subscripts gives a similar expression for the other equilibria):

$$m = p_{33}\delta_3(P_2\delta_1 + P_1\delta_2) + (p_{22} + p_{32})\delta_2\delta_3 P_1 = P_3\delta_3(P_2\delta_1 + P_1\delta_2) + P_2\delta_2\delta_3 P_1.$$

In the specified equilibrium, type-1 agents never make any trades for a good they do not consume immediately. Type-2 agents sometimes accept good 1 as a medium of exchange when they are holding good 2, whereas type-3 agents accept good 1 and good 2 as money, provided they are holding good 3. What m measures is the frequency with which goods are accepted to be used as money and not for immediate consumption.

One conclusion from the experiments we ran is that economies in which agents are relatively equally distributed in terms of both supply and demand parameters are more highly monetized than economies in which the distribution is polarized. An example serves to illustrate the general idea. An economy with a high level of monetization $m = 0.0390728$ is given by $\delta = (0.31, 0.34, 0.35)$ and $\sigma = (0.31, 0.34, 0.35)$. An economy with a low degree of monetization $m = 0.00000293$ is given by $\delta = (0.98, 0.01, 0.01)$ and $\sigma = (0.98, 0.01, 0.01)$. In the latter case, agents very often produce goods that they consume, yielding little trade. In the former case, production and consumption are not closely linked, implying the need for trade and the use of media of exchange in equilibrium. This result is reminiscent of the observation dating back at least to Adam Smith that specialization leads to a greater role for money [see Kiyotaki and Wright (1993) and Siandra (1991)].

4. CONCLUSION

This paper provides a model that allows one to study the influence of relative supply and demand on the equilibrium choice of commodity money. The model tends to confirm the intuition that goods with relatively high demand and/or low supply are more likely to be used as commodity money. Nevertheless, a good with relatively low demand may be used as money if agents believe that it will be acceptable, as long as demand is not too low. In fact, we show that if the demand for a good is low enough, then there exists no equilibrium in which it is used as a medium of exchange; a sufficiently high demand always guarantees that a good will emerge as money. Related to this, the model predicts that fiat money cannot get off the ground unless it has some superior fundamental property such as low storage cost, superior recognizability, etc. because there is always a real commodity money that can have universal acceptability plus potential consumption value. On the supply side, we could not make analogous statements, in the sense that high supply of a good does not preclude, and low supply does not guarantee, its use as medium of exchange. We show that there are multiple equilibria for large regions of the space of parameters and some of them are not efficient.

One thing we do not do here is to provide an existence proof. Rather, our goal was to see when each equilibrium of a particular type (i.e., with particular commodity monies) exists in the parameter space. To establish existence of equilibria goes well beyond the scope of the present analysis. In particular, because it is well known from the study of related models, one presumably would need to extend the model to allow mixed strategies (or, equivalently, strategies that are asymmetric in the sense that different agents of the same type use different strategies), as in Aiyagari and Wallace (1991).

Another extension of the analysis would be to consider a version of the model in which each good j yields utility from consumption equal to U_j. Then, if U_1, for example, is low relative to other U_j's, there may be an equilibrium in which agents with a taste for good 1 who acquire it may prefer not to consume it but keep it to use as money, and so, it will never be consumed. (Note that to get a steady-state equilibrium with this property we would have to assume that good 1 depreciates or disappears with some probability; otherwise it will pile up until everyone holds it and then agents might as well eat it after all.) Hence, good 1 potentially could be used as a commodity money that is never consumed. This extension seems both feasible and interesting, because one could ask more generally how a low or high U_j impacts on the possibility of good j serving as money.

Perhaps the main contribution has been to show that a very natural model, which on the surface seems exceedingly complicated, can actually be analyzed in a fairly clean and simple way and, moreover, makes predictions that differ from previous models and adds new insights into the general issue of the nature of money.

NOTES

1. Actually, σ_i is a measure of the *potential* supply of good i, for although it equals the number of agents who can produce it, this is generally different from the number who actually produce the good each period, or from the number of agents with it in inventory.

2. Related to note 1, δ_i is actually a measure of the *potential* demand for good i, for although it equals the number of agents who would like to consume it each period, this generally is different from the number who actually consume it, or from the number who accept it in trade.

3. We assume that agents do not trade if they are indifferent; however, as long as we restrict attention to pure strategy equilibria, this is innocuous because $V_{ij} = V_{ik}$ only on a set of measure zero in parameter space.

4. Obviously, no trade will take place when agent of type 1 holding good 1 and wishing to consume good 2 meets another agent of type 1 holding good 3 and wishing also to consume good 2, because s^1_{13} and s^1_{31} can never both be equal to one.

5. Note that in this model all commodity money eventually is consumed by someone. That is, even if an agent accepts a good and then trades it away later, in the long run somebody will eat it with probability one. Historically, many commodity monies were rarely consumed but continued to be traded indefinitely (e.g., gold coins) whereas others were regularly consumed (e.g., cigarettes in prisoner-of-war camps). Note also that some objects, such as cigarettes, totally depreciate when used in consumption, whereas others, such as gold, can be turned into consumption goods such as jewelry and then turned back into coins, at least if one is willing to pay some cost, say, by taking the metal to the mint.

6. We define the payoff function of type-i agents as

$$U_i = \sum_i \frac{p_{ij}}{\sigma_i} V_{ij}.$$

Note that, by Proposition 5, $\gamma_i > \gamma_j \Rightarrow U_i > U_j$.

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APPENDIX

This Appendix shows how to compute the steady-state distribution $\boldsymbol{p}$ for the case of $\boldsymbol{s} = (0, 0, 1)$. We must solve the following system of equations, which are derived by setting the inflow to a state equal to the out flow. For type 1,

$$p_{12}[N\delta_2 + P_1(\delta_1\delta_2 + \delta_3\delta_2) + P_3(\delta_3\delta_2 + \delta_3\delta_1)] = P_2 p_{13}\delta_3\delta_1, \quad \textbf{(A.1)}$$

$$p_{13}[(\delta_1\delta_3 + \delta_2\delta_3)P_1 + (\delta_2\delta_3 + \delta_1\delta_3)P_2 + N\delta_3] = 0. \quad \textbf{(A.2)}$$

For type 2,

$$p_{21}[N\delta_1 + (\delta_2\delta_1 + \delta_2\delta_3)P_2 + (\delta_3\delta_1 + \delta_3\delta_2)P_3] = P_1(\sigma_2 - p_{21})\delta_3\delta_2, \quad \textbf{(A.3)}$$

$$p_{23}[(\delta_1\delta_3 + \delta_2\delta_3)P_1 + (\delta_2\delta_3 + \delta_1\delta_3)P_2 + N\delta_3] = 0. \quad \textbf{(A.4)}$$

For type 3,

$$p_{31}[N\delta_1 + (\delta_2\delta_1 + \delta_2\delta_3)P_2 + (\delta_3\delta_1 + \delta_3\delta_2)P_3] = P_1(\sigma_3 - p_{31})\delta_3\delta_2, \tag{A.5}$$

$$p_{32}[N\delta_2 + P_1(\delta_1\delta_2 + \delta_3\delta_2) + P_3(\delta_3\delta_2 + \delta_3\delta_1)] = P_2(\sigma_3 - p_{31} - p_{32})\delta_3\delta_1. \tag{A.6}$$

From (A.2) it is obvious that $p_{13} = 0$. Then, (A.1) implies $p_{12} = 0$. From (A.4), $p_{23} = 0$. From (A.3) and (A.5), $p_{21} = (\sigma_2/\sigma_3)p_{31}$. Consequently, $p_{11} = \sigma_1$ and $p_{22} = \sigma_2[1 - (p_{31}/\sigma_3)]$. We can now use this to get the following expressions for P_i's.

$$P_1 = \sigma_1 + \left(1 + \frac{\sigma_2}{\sigma_3}\right)p_{31}, \qquad P_2 = \sigma_2\left(1 - \frac{p_{31}}{\sigma_3}\right) + p_{32}, \qquad P_3 = \sigma_3 - p_{31} - p_{32}.$$

Substituting into (A.5) and simplifying, we arrive at

$$P_{31}\left\{\delta_1 - \delta_1^2\left[\sigma_1 + \left(1 + \frac{\sigma_2}{\sigma_3}\right)p_{31}\right] + \delta_2\delta_3\left[\sigma_2\left(1 - \frac{p_{31}}{\sigma_3}\right) + \sigma_3 - p_{31}\right]\right\}$$
$$= \left[\sigma_1 + \left(1 + \frac{\sigma_2}{\sigma_3}\right)P_{31}\right](\sigma_3 - p_{31})\delta_2\delta_3,$$

which is a nonlinear equation with only one unknown, p_{31}, and which we can solve numerically for any $\boldsymbol{\delta}$ and $\boldsymbol{\sigma}$. Given p_{31} and $(\boldsymbol{\delta}, \boldsymbol{\sigma})$, we now can solve (A.6) for p_{32}.

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VALUING THE FUTURES-MARKET PERFORMANCE GUARANTEE

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Futures-market exchanges guarantee the performance of all contracts to encourage active trading between anonymous partners. Traders buy from or sell to the clearinghouse. The exchanges rely primarily on a margin system—traders post a performance bond that they forfeit if they default—to manage their risk exposure. The market value of the performance guarantee is the amount a trader would have to pay to insure his performance if there were no exchange guarantee. An underpriced guarantee subsidizes high-risk traders and ultimately undermines the credibility of the exchange's commitment to perform. An adequate margin policy fairly prices the exchange performance guarantee within an (economically insignificant) ε neighborhood. I show that the value of a call (put) option with a strike price equal to the futures price minus (plus) the margin is an upper bound to the market value of the exchange's performance guarantee. The probability that the futures price change exceeds the margin is the probability that the option expires "in the money." I estimate the value of the option on the December S&P 500 futures contract during the market crash in October 1987 to illustrate the technique. For the first half of the month the value of the performance guarantee was fairly priced. On the day of the crash it was underpriced by as much as 10% of the value of the futures contract. Futures-market margin committees moved quickly; by the end of the month the value of the performance guarantee was fairly priced.

Keywords: Performance Guarantee, Futures Market, Contingent Claim Pricing

1. INTRODUCTION

Traders in the futures market can buy or sell contracts by posting a margin that is a fraction of the price. The futures-market clearinghouses protect traders against default by guaranteeing the performance of all contracts. If a buyer or seller defaults, the clearinghouse performs. The clearinghouse guarantee is a credit guarantee similar to the deposit insurance guarantee. The futures-market performance guarantee, however, is a private guarantee backed only by the credit of the exchange and claims that the exchange can impose on its members. Federal deposit insurance is backed by the U.S. government.

At the beginning of October 1987, the clearing margin on the nearby S&P 500 futures contract was slightly less than 3% of the contract price. On October 19, 1987, U.S. futures and equity markets suffered the largest one-day price decline in

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