Matching and Money

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In Corbae, Temzelides, and Wright (2001) (hereafter, CTW) we proposed a new version of the framework that uses bilateral matching to model the exchange process, and in particular to model the use of money as a medium of exchange. Our version does not have agents meeting exogenously and at random, but rather has agents meeting endogenously. That is, agents are matched at each date subject to a stability condition that requires, roughly, that no agents prefer to be paired with each other or to be unmatched, rather than to be paired with the partners they get along the equilibrium path. While similar in spirit to the cooperative matching concept introduced by David Gale and Lloyd Shapley (1962), we had to generalize their framework to dynamic models because we are interested in monetary economics. Here we present a version of the solution concept in CTW, specialized in some ways but also generalized to include extrinsic uncertainty (sunspots).

We then discuss some applications of endogenous matching models to issues that have previously been addressed using random matching, including the existence of sunspot equilibria and the efficiency of inside versus outside money. One of our main goals is to show how endogenous matching is a useful alternative to random matching. This may be interesting to those who think that bilateral trade is a reasonable friction upon which to build a theoretical foundation for monetary economics but perhaps think that random matching is an extreme and unrealistic simplification. Another goal is to provide examples where it makes a difference for substantive results how we model the matching process, and also examples where it does not.

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I. Endogenous Matching

We present a version of the equilibrium concept in CTW specialized to the applications studied below. Time is discrete and unbounded. At each date there are K indivisible and nonstorable goods. The set of agents $\mathcal{A} = [0, 1]$ is equally divided into K types, where each type-i agent produces good i and consumes good $i + 1$ (modulo K). For all agents, production costs C, consumption yields utility U, and the discount factor is $\beta < 1$. In addition to type, agents are also indexed by their (outside) money holdings, $m \in \{0, 1\}$. The amount of money, and also the total number of agents with money, is $M \in (0, 1)$. In any equilibrium considered here it will be the case that a fraction M of each type holds money at every date. An agent's type and money holding are publicly observable, but his trading history is not; that is, agents are *anonymous.*

The aggregate state of the economy is represented by two objects. First, it includes the distribution of money holdings across agents, say γ_t. Second, it includes a random variable s_t representing extrinsic uncertainty—as it is called in the literature, a *sunspot.* At each date, conditional on the state, agents make two types of decisions. First they decide who to match with (if an agent is unmatched, we say he matches with himself). Second, if matched, agents have to decide whether to trade. Given goods are indivisible and $m \in \{0, 1\}$, given there is no direct barter (assuming $K > 2$), and given agents are anonymous, the only trades that can occur here are when a type-i agent with money is paired with a type-$(i + 1)$ agent without money, and the former gives up his money for his consumption good.

This describes the environment we will study below, although to present the equilibrium concept it is useful to proceed somewhat more generally. Thus, for any set $\mathcal{A}$ with any preferences and technologies, matching at t can always be described by a partition θ_t of $\mathcal{A}$ into subsets of size 1 or 2, called *coalitions*. A matching rule is a function $\theta_t(\gamma_t, s_t)$ that partitions agents into coalitions depending on the state of the economy. A trading rule $\tau_t(\theta_t, \gamma_t, s_t)$ lists the trading decisions of each agent given the current partition and state. If trading histories were observable, as in CTW, we would also have to include them in the state variable, in which case matching and trading could also be conditioned on agents' past behavior; this is not the case here.

The instantaneous utility of agent i is $w_t^i(\theta_t, \tau_t)$. Notice that w_t^i may depend on matching and trading but not directly on the state (since money is intrinsically useless and sunspots represent extrinsic uncertainty). Lifetime utility is described recursively by

$$v_t^i(\gamma_t, s_t) = w_t^i[\theta_t(\gamma_t, s_t), \tau_t(\theta_t(\gamma_t, s_t), \gamma_t, s_t)] + \beta v_{t+1}^i(\gamma_{t+1}, s_{t+1})$$

where γ_{t+1} is determined from γ_t, given θ_t and τ_t, in the obvious way, and s_{t+1} evolves according to some exogenous process. Agent i's individual state is contained in γ_t (i.e., γ_t lists the money holdings of each i). An *equilibrium* consists of matching and trading rules such that, for every t and (γ_t, s_t): no coalition can do better by matching in some way other than as prescribed by the equilibrium; and given matching, no coalition can do better by trading in some way other than as prescribed.

To make this precise we need to describe what kind of deviations are allowed and to say what it is agents take as given when they contemplate a deviation from the equilibrium. First, it is feasible for any agent to be unmatched at any t rather than following the equilibrium. Second, any two agents can deviate by matching with each other at t and, if the equilibrium had prescribed them other partners, abandoning the other partners. When agents deviate they take as given that all other agents continue to match and trade as prescribed by the equilibrium, except for any agents they abandon. An equilibrium is simply a matching and trading pattern from which no coalition wants to deviate.[1]

II. Monetary Equilibrium

There is always a nonmonetary (no-trade) equilibrium. Consider monetary equilibria. Ignoring temporarily the sunspot s, under an incentive condition given below, we claim that there is an equilibrium where, for all t, every agent on the short side of the market (those with money if $M < 1/2$ and those without money if $M > 1/2$) finds someone with whom to trade money for goods or vice versa. To see this, observe that, whenever a type-i agent with money trades with a type-$(i + 1)$ without money, neither strictly prefers to be with anyone else, nor will either prefer to be unmatched or prefer not to trade given the parameter condition below. Of course, some agents on the long side of the market are left unmatched, which they do not like, but no one prefers being with them over the equilibrium pattern.

Assume that agents on the short side pick a partner at random.[2] Then the relevant probabilities of a trade each period for agents with and without money are

$$a_1^e = \min\left\{1, \frac{1-M}{M}\right\}$$

$$a_0^e = \min\left\{1, \frac{M}{1-M}\right\}$$

respectively (the superscript e is for endogenous matching). Let V_m be the value function for an agent with money $m \in \{0, 1\}$, where the

[1] Details are in CTW. We emphasize here that we allow bilateral and not just unilateral deviations. Also, as in cooperative equilibrium theory, we do not need to take a stand on the *process* by which agents match or trade, only the outcome. Also, note that deviations here only have future implications if they change individual money holdings, but with observable trading histories, as in CTW, reputations are also relevant. Finally, a technical detail is that we formally allow only finite deviations, but this does not matter for anything done here (see CTW for an example where it can matter).

[2] For instance, one can always find the right type (a taxi) but not a particular individual (a particular driver).

dependence on the money holdings of everyone else, given by γ, is implicit but it is understood that a fraction M of each type always have $m = 1$. Then Bellman's equations are

$$V_0 = \beta\{a_0^e(-C + V_1) + (1 - a_0^e)V_0\}$$

$$V_1 = \beta\{a_1^e(U + V_0) + (1 - a_1^e)V_1\}.$$

The binding incentive condition for no one to deviate is $V_1 - C \geq V_0$ or, letting $\beta = 1/(1 + \rho)$ and rearranging,

$$a_1^e(U - C) \geq \rho C.$$

For comparison, consider random matching where α is the probability of meeting anyone at t and every meeting is a random draw from $\mathcal{A}$. Let $a_0^r = \alpha M/K$ and $a_1^r = \alpha(1 - M)/K$ (the superscript r is for random matching). Replacing a_m^e with a_m^r in both Bellman's equations and the incentive condition yields a standard random matching model of monetary exchange with indivisible goods (e.g., Nobuhiro Kiyotaki and Wright, 1993). Hence, we see that the random- and endogenous-matching models are qualitatively the same, although the incentive condition is stricter in the random-matching version, because $a_1^e > a_1^r$. Intuitively, one can spend money faster with endogenous matching, and this makes money more desirable.

Additionally, if $M = 1/2$, in the endogenous-matching model monetary exchange achieves the efficient outcome conditional on bilateral trade, where each agent consumes every second period. This is as good as we could do if we had a public record of all meetings and transactions and used punishment threats to sustain cooperative exchange (see CTW). By contrast, with random matching, money can never do as well as complete record-keeping. Intuitively, money is an imperfect substitute for record-keeping with random matching because there can occur meetings where one wants to trade but has no cash; this does not happen with endogenous matching.

III. Sunspot Equilibria

We now examine how the possibility of sunspot equilibria depends on whether matching is endogenous or random. One perhaps might think that, because there is less intrinsic uncertainty in the endogenous-matching framework, equilibria would be less susceptible to sunspots; we will see that this is not the case. This is interesting for its own sake, and also because it provides another example of how endogenous matching is a useful alternative to random matching.

As in Wright (1994), assume $s_t \in \{1, 2\}$ with $\Pr(s_{t+1} = 2|s_t = 1) = H_1$ and $\Pr(s_{t+1} = 1|s_t = 2) = H_2$, and let V_m^s be the value function for an agent with money m in state s. We assume $z \equiv a_1(U - C) - \rho C \geq 0$, as required for the existence of a monetary equilibrium without sunspots. The goal is to construct an equilibrium where money trades for goods in state 2 but not state 1. When $s = 1$, there is no trade, and hence there is no point to matching at all; there is nothing to do but wait for the state to switch to $s = 2$. Hence, for $m = 1$ or 0, we have

$$V_m^1 = \beta\{H_1 V_m^2 + (1 - H_1)V_m^1\}.$$

In state 2, there is trade, and

$$V_0^2 = \beta\{a_0[-C + H_2 V_1^1 + (1 - H_2)V_1^2] + (1 - a_0)[H_2 V_0^1 + (1 - H_2)V_0^2]\}$$

$$V_1^2 = \beta\{a_1[U + H_2 V_0^1 + (1 - H_2)V_0^2] + (1 - a_1)[H_2 V_1^1 + (1 - H_2)V_1^2]\}.$$

These equations apply in both the random- and endogenous-matching models if we assign the appropriate superscript to a_m. The incentive condition that makes agents trade goods for money in state 2 is

$$-C + H_2 V_1^1 + (1 - H_2)V_1^2 \geq H_2 V_0^1 + (1 - H_2)V_0^2$$

and the condition that makes them not do the same trade in state 1 is

$$-C + H_1 V_1^2 + (1 - H_1)V_1^1 \leq H_1 V_0^2 + (1 - H_1)V_0^1.$$

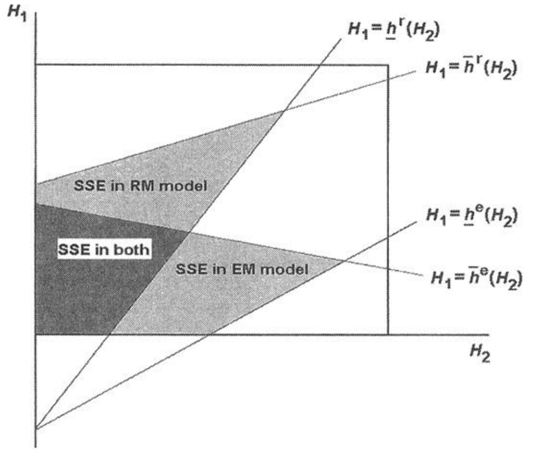

FIGURE 1. EXISTENCE REGIONS FOR SUNSPOT EQUILIBRIA (SSE)

Algebra implies that these two conditions hold if and only if

$$H_1 \geq -\rho + \frac{[a_1(U - C) + C]\rho}{z} H_2 = \underline{h}(H_2)$$

$$H_1 \leq \frac{(\rho + a_0 + a_1)\rho C}{z + \rho(a_0 C + a_1 U)} + \frac{(1 - a_0 - a_1)\rho C}{z + \rho(a_0 C + a_1 U)} H_2 = \bar{h}(H_2).$$

The equilibrium under construction exists in (H_2, H_1) space in the regions shown in Figure 1, for both endogenous and random matching. It exists in the endogenous-matching model when H_2 is high and H_1 is low, and it exists in the random-matching model under the opposite conditions. The reason is that money works better with endogenous matching. Intuitively, when matching is purposeful, money is more valuable, so we need a lower H_1 to make agents willing to forgo trade in state 1; and we can have a lower H_1 and still have agents willing to trade in state 2. Still, contrary to what one might have thought, it is not the case that sunspot equilibria are more likely to exist in one model or the other.

IV. Inside and Outside Money

Sometimes endogenizing the meeting process changes the quantitative results but not the basic point. Here we present a very different example. In a random-matching model, Ricardo Cavalcanti and Neil Wallace (1999) argue that inside money yields superior allocations to outside money. Inside money consists of notes issued by agents called *banks,* who are the same as other agents except that there is a public record of their trading histories. Hence, their behavior can be monitored, and they can be punished if they do not behave appropriately. These agents may issue *bank notes,* or inside money, and must redeem them (or get punished) whenever someone with a note wants their output. Cavalcanti and Wallace (1999) show such an arrangement is superior to one with only outside money.

The economic intuition is simple: in an outside-money regime bankers can buy from nonbankers only if they have on hand cash from a past sale; in an inside-money regime they can trade whenever they meet nonbankers since they can print money. However, with endogenous matching, bankers without money do not meet nonbankers who produce their consumption goods in equilibrium. Hence, the advantage of inside money according to Cavalcanti and Wallace (1999) requires randomness in the meeting process and is not due to anything that is essential for money to be valued. As we have shown, there is a well-defined role for money in the endogenous-matching model, and indeed outside money can support the efficient outcome in our endogenous-matching model. Hence, the advantage of inside money vanishes.[3]

V. Conclusion

We believe that endogenous matching is a natural and interesting equilibrium concept. Sometimes the results with endogenous matching are similar to random matching, and sometimes not. Also, endogenous matching can be

[3] A simplified version of the Cavalcanti-Wallace model, similar in many details to the basic model in this paper, is contained in Wright (1999) and makes it easy to verify the claims in the text.

much more tractable, which is a big advantage; for example, models in which agents can hold money in some generalized set, say $m \in \{0, 1, \ldots \bar{m}\}$, should be much simpler with endogenous matching. Finally, endogenous matching may be a palatable alternative to random matching for those who are sympathetic to building foundations for monetary theory based on bilateral trade, but who think random meetings are extreme and unrealistic. Of course, both approaches are useful, and for some issues random matching is better. For example, one may find something reasonable in the comparison of inside and outside money that was formalized using random matching. Still, it is good to know which results in these analyses depend on bilateral matching per se, and which depend on randomness.

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