Buyers and Sellers: Should I Stay or Should I Go?

By KENNETH BURDETT, MELVYN COLES, NOBUHIRO KIYOTAKI, AND RANDALL WRIGHT*

This paper explores some aspects of the exchange process that should be of interest to economists who use search theory, and especially to those who use search as a foundation for monetary economics. We are mainly concerned with a question that seems basic but has not been analyzed previously: is there some way to determine endogenously which agents are willing to invest resources in the process of active search for trading partners, and which agents prefer to wait passively for trading partners to come to them? In particular, given a group of buyers with money and a group of sellers with goods, is there any reason to expect that buyers will search for sellers, rather than the other way around?

One obvious factor is the relative cost of transporting goods and money; but we are interested in examining whether there is anything about the process of monetary exchange per se, and not merely exogenous search costs, that makes buyers more willing than sellers to bear the costs of seeking out trading partners. We investigate this within a generalized version of the search-theoretic model of fiat money in Kiyotaki and Wright (1991, 1993). We find that there may exist equilibria in which buyers search while sellers do not, even if the search cost is greater for the former. However, there can also exist other equilibria with different properties. Perhaps the key finding is that the situation is not symmetric: factors that determine whether to search are fundamentally different for buyers and sellers.

One property of an equilibrium in which only buyers search is that money appears on one side of every transaction, because if sellers do not search they do not meet and cannot barter. This is consistent Robert W. Clower's (1967) observation that money buys goods and goods buy money, but goods do not buy goods, although here it is derived endogenously rather than assumed. Some search-based monetary models simply rule out barter, including Peter A. Diamond (1984), Douglas M. Gale (1986), Diamond and Joel Yellin (1990), Alessandra Casella and Johnathon S. Feinstein (1990), Kiminori Matsuyama et al. (1993), Victor E. Li (1995), and Alberto Trejos and Wright (1995). It is difficult to motivate this absence of barter in these papers. In a model that is similar to the one used here, except search is costless, S. Rao Aiyagari and Neil Wallace (1992) prove that all equilibria involve some barter. A contribution of this paper is to show that barter may disappear once the decision to search is endogenized.

I. The Model

The economy is similar to the one in Kiyotaki and Wright (1993). There is a [0,1]

[†]*Discussants:* Carl Davidson, Michigan State University; Victor Li, Pennsylvania State University; William Roberds, Federal Reserve Bank of Atlanta; Bruce D. Smith, Cornell University. A fourth paper, "Matching Human Capital and Displacement," by V. V. Chari and Hugo Hopenhayn was also presented at this session but is not being published here.

* Burdett and Coles: University of Essex, Colchester CO4 3SQ, United Kingdom; Kiyotaki: University of Minnesota, Minneapolis, MN 55455; Wright: University of Pennsylvania, Philadelphia, PA 19104. We thank the National Science Foundation for research support, and Alberto Trejos and Victor Li for their insightful comments.

continuum of infinite-lived agents. There are k types of agents in equal proportions and k types of consumption goods, $k \geq 3$, where type i produces good i. In addition to these "real" commodities, there is another object called fiat money. No one can produce money. Initially, a fraction m of the agents are each endowed with one unit of money, while $1-m$ are each endowed with one of the consumption goods.

To focus on the exchange process, rather than on the determination of exchange rates, we assume that goods are indivisible and that everyone endowed with money is endowed with exactly one unit of real balances. This means that every trade is a one-for-one swap. Hence, if we endow agents with either one real commodity or one unit of real balances at the initial date, then everyone will continue to have either one real commodity or one unit of real balances at all future dates.

Although agents are specialists in production, they are generalists in consumption. After consuming one good they realize a desire or taste for another good (possibly the same one) drawn at random. A consumer with a taste for good j gets utility $u>0$ from consuming it and no utility from anything else until after j is consumed and a new taste shock is realized. Also, immediately after consuming, agents produce their production goods at a cost normalized to zero. They cannot produce except after consuming. If an agent has a taste for his production good, he consumes it and produces again, repeating the process until he realizes a taste for something else.

When he desires something he does not produce, the agent enters a trading process characterized by bilateral random matching. Each trader with a good is called a "seller," and each trader with money is called a "buyer." Buyers want to trade for goods, while sellers may either trade for money or barter goods directly. Trade requires mutual consent. When trade occurs, there is a transaction cost in terms of disutility $\varepsilon \in (0,u)$ paid by receivers of goods, but *not* by receivers of money (this simplifies the presentation but is otherwise unessential; see Kiyotaki and Wright, 1993).

We now describe the search technology. There are two ways to meet someone: you can either actively search for partners or you can wait for them to come to you. We call those engaged in active search "movers" and those not so engaged "stayers." A mover has a better chance of meeting someone, because he can meet both stayers and other movers, while a stayer can only meet movers and not other stayers. Hence, stayers meet potential trading partners at a rate proportional to the number of movers, while movers meet potential trading partners at a rate that is independent of the number of movers and stayers.

One way to picture this is to imagine a finite number of physical locations and individuals. The probability of a meeting for a mover who samples a location at random is the total number of agents divided by the number of locations. The probability of a meeting for a stayer is the probability that a mover comes to him, which equals the number of movers divided by the number of locations. It may also be helpful to imagine traders as particles colliding in space. A stationary particle can only collide with a moving particle, but a moving particle can collide with either a stationary particle or another moving particle.

If we normalize to 1 the arrival rate of meetings for movers, the arrival rate of meetings for stayers equals the fraction of movers. Each meeting is a random draw from the set of agents. Finally, let $c_1 \geq 0$ and $c_{\mathrm{m}} \geq 0$ denote the disutility of moving per unit time for sellers and buyers (subscript 1 for one good and subscript m for money), and let $r>0$ denote the rate of time preference. This completes the description of the physical environment.

II. Equilibrium

Agents choose strategies to decide whether to move or stay and when to trade. Some properties of strategies are immediate. First, an agent always accepts a good he currently desires for consumption. Second, since we will only consider symmetric equilibria in which no commodity is more acceptable than any other, an agent never accepts a commodity he does not consume (due to the transaction cost ε; see Kiyotaki and Wright [1993] for details).

Hence, the probability that a random seller is willing to accept a given good is $1/k$. The probability that you are willing to accept his good given that he is willing to accept yours is $1/(k-1)$, since you must desire one of the $k-1$ goods other than the one he desires. Here we will only consider equilibria in which everyone accepts money with probability 1 (although other equilibria exist; see Burdett et al., 1993). Thus, since a random trader has money with probability m and a good with probability $1-m$, the rate at which sellers barter is $(1-m)/k(k-1)$, the rate at which sellers acquire money is m/k, and the rate at which buyers acquire goods is $(1-m)/k$.

Let V_1 and V_m denote the value functions for sellers and buyers who move, and let S_1 and S_m be the value functions for sellers and buyers who stay. Let n_1 be the fraction of sellers who move, n_m the fraction of buyers who move, and $n=(n_1,n_\mathrm{m})$. As a final piece of notation, let Φ denote any number in the open interval $(0,1)$; for example, $n=(\Phi,\Phi)$ means that n_1 and n_m are both between 0 and 1, although not necessarily that $n_1=n_\mathrm{m}$.

The value functions satisfy versions of the standard dynamic-programming equations from search theory:

$$(1)\qquad rV_1=\frac{1-m}{k(k-1)}[U+\max(V_1,S_1)-V_1]+\frac{m}{k}[\max(V_\mathrm{m},S_\mathrm{m})-V_1]-c_1$$

$$(2)\qquad rS_1=n_1\frac{1-m}{k(k-1)}[U+\max(V_1,S_1)-S_1]+n_\mathrm{m}\frac{m}{k}[\max(V_\mathrm{m},S_\mathrm{m})-S_1]$$

$$(3)\qquad rV_\mathrm{m}=\frac{1-m}{k}[U+\max(V_1,S_1)-V_\mathrm{m}]-c_\mathrm{m}$$

$$(4)\qquad rS_\mathrm{m}=n_1\frac{1-m}{k}[U+\max(V_1,S_1)-S_\mathrm{m}]$$

where $U\equiv uk/(k-1)-\varepsilon$.[1] Notice how the arrival rates of trading partners are smaller for stayers than for movers: the rate at which a stayer meets a type-j trader is n_j times the rate at which a mover meets a type-j trader.

An equilibrium is a list $(n,V_1,S_1,V_\mathrm{m},S_\mathrm{m})$, satisfying $\max(V_\mathrm{m},S_\mathrm{m})\geq\max(V_1,S_1)$ (which guarantees that money is acceptable) and the following conditions for $j=1$ and $j=\mathrm{m}$: $V_j>S_j$ implies $n_j=1$; $V_j<S_j$ implies $n_j=0$; and $n_j=\Phi$ implies $V_j=S_j$. An immediate result is that $n=(1,1)$ cannot be an equilibrium. To see this, observe that if $n=(1,1)$ then (1)–(4) imply $V_j=-c_j+S_j<S_j$ for $j=1$ and m. The intuition is that, if everyone is moving, the arrival rates are the same for movers and stayers, and so no one would be willing to pay the moving cost.

To see what may be an equilibrium, consider first the case in which both n_1 and n_m are in $\{0,1\}$. Also, to reduce notation, normalize $U=1$ from now on with no loss in generality.

PROPOSITION 1: *The set of equilibria with $n_j\in\{0,1\}$ is as follows:*

(a) $n=(0,0)$ is an equilibrium if and only if

$$c_1\geq\frac{1-m}{k(k-1)}\qquad c_\mathrm{m}\geq\frac{1-m}{k}$$

[1]These can be derived in the usual way (see e.g., Kiyotaki and Wright, 1993). The one tricky bit is to compute the expected utility of an agent who has just acquired his consumption good. Let this be denoted V_0. The agent consumes and draws a new taste shock, which with probability $1/k$ yields V_0 again and with probability $(k-1)/k$ forces him to enter the exchange process. Hence,

$$V_0=u+\frac{1}{k}V_0+\frac{k-1}{k}\max(V_1,S_1)$$

or

$$V_0=\frac{k}{k-1}u+\max(V_1,S_1).$$

Then $V_0-\varepsilon=U+\max(V_1,S_1)$ is the gain from trading for one's consumption good, which is used in deriving (1)–(4).

(b) $n=(0,1)$ *is an equilibrium if and only if*

$$c_1 \geq \frac{1-m}{k(k-1)} \qquad c_m \leq \frac{1-m}{k}$$

(c) $n=(1,0)$ *is an equilibrium if and only if*

$$c_1 \leq \bar{c}_1 \equiv \frac{m(1-m)(k-2)}{k(k-1)(rk+1-m)}.$$

PROOF:

Consider case (b). We must show $S_1 \geq V_1$ (so that sellers stay), $V_m \geq S_m$ (so that buyers move), and $V_m \geq S_1$ (so that money is acceptable). If we insert the candidate strategies $n=(0,1)$ into (1)–(4), we have

$$rV_1 = \frac{1-m}{k(k-1)}(1+S_1-V_1) + \frac{m}{k}(V_m - V_1) - c_1$$

$$rS_1 = \frac{m}{k}(V_m - S_1)$$

$$rV_m = \frac{1-m}{k}(1+S_1-V_m) - c_m$$

$$rS_m = 0.$$

It is routine to check that $S_1 \geq V_1$ if and only if $c_1 \geq (1-m)/k(k-1)$, and both $V_m \geq S_1$ and $V_m \geq S_m$ if and only if $c_m \leq (1-m)/k$. This verifies (b). The proofs of (a) and (c) are similar, and hence they are omitted.

Consider the equilibrium $n=(0,1)$. In this (or any) equilibrium, sellers weigh the cost c_1 against the benefit from moving, which in this case is that if they move they can barter, while if they stay they cannot. The rate at which they barter is $(1-m)/k(k-1)$. Buyers weigh their moving cost c_m against the benefit, which for them is that if they move they consume at rate $(1-m)/k$ while if they stay they do not consume at all (since no sellers move in this equilibrium). The benefit from moving is greater for a buyer than for a seller. Moreover, as k becomes large, the benefit for a buyer goes to 0 at rate k while the benefit for a seller goes to 0 at rate k^2.

In the other nondegenerate equilibrium in Proposition 1, given by $n=(1,0)$, all sellers are moving, and so the only benefit from moving is meeting buyers. Buyers have nothing to gain from meeting other buyers, so they do not move. Sellers do have an incentive to move (the incentive to acquire money), which exceeds the cost as long as $c_1 \leq \bar{c}_1$. As k gets large $\bar{c}_1$ goes to 0 at rate k^2. Hence, if we imagine c_j becoming small at the same time that k gets large, this equilibrium is less likely to exist than the equilibrium discussed above (see below for a more detailed argument).

For some parameter values, multiple equilibria exist. In particular, the economy may end up in equilibrium $n=(0,1)$ or the degenerate equilibrium $n=(0,0)$ with no trade. It is also possible that none of the equilibria in Proposition 1 exists, and so we also want to consider equilibria with $n_j = \Phi$. The general case is analyzed in Burdett et al. (1993); here we concentrate on the special case where $c_1 = c_m = c > 0$.

PROPOSITION 2: *Assume* $c_1 = c_m = c$. *Then equilibrium exists for all* $c > 0$. *The set of possibilities consists of the pure-strategy equilibria (a)–(c) in Proposition 1, which exist under the stated conditions with* $c_1 = c_m$, *plus:*

(d) $n=(\Phi,1)$ *is an equilibrium if and only if*

$$c < \frac{1-m}{k(k-1)}$$

(e) $n=(\Phi,\Phi)$ *is an equilibrium if and only if* $c \leq \bar{c}_2$;

(f) $n=(\Phi,0)$ *is an equilibrium if and only if* $\bar{c}_1 < c \leq \bar{c}_2$

where $\bar{c}_1$ *is defined above and*

$$\bar{c}_2 \equiv \frac{m(1-m)[rk+m+(1-m)/(k-1)]}{k(rk+1)(r+m)}.$$

PROOF:

Consider case (d). We must show that $S_1 = V_1$ (so that sellers are indifferent between staying and moving), $V_m \geq S_m$ (so that buyers move), and $V_m \geq S_1$ (so that money is acceptable). Inserting the candidate strategies into (1)–(4), we have

$$rV_1 = -c_1 + \frac{1-m}{k(k-1)} + \frac{m}{k}(V_m - V_1)$$

$$rS_1 = n_1\frac{1-m}{k(k-1)} + \frac{m}{k}(V_m - S_1)$$

$$rV_m = -c_m + \frac{1-m}{k}(1 + V_1 - V_m)$$

$$rS_m = n_1\frac{1-m}{k}(1 + V_1 - S_m).$$

Algebra implies $V_m \geq S_1$ and $V_m \geq S_m$ for all parameter values. Setting $V_1 = S_1$, we can solve for $n_1 = 1 - ck(k-1)/(1-m)$. We require $0 < n_1 < 1$, which is true if and only if $c < (1-m)/k(k-1)$. This verifies (d). The proofs of (e) and (f) are similar. No other equilibria exist, except possibly on a set of measure zero in parameter space.

We now argue that the subset of parameter space supporting equilibrium $n = (0,1)$ is larger than the subset supporting other equilibria when k is large. Let $\hat{c} = \max\{\bar{c}_1, \bar{c}_2\}$, and define the intervals $I_m = [\hat{c}, (1-m)/k]$ and $I_1 = [0, \hat{c}]$. If $c \in I_m$ the unique equilibrium is $n = (0,1)$; all of the equilibria with some sellers moving require $c \in I_1$. Since $\hat{c}$ is of order $(1-m)/k^2$, the range of I_m is of order k times larger than the range of I_1. For large k, therefore, we conclude that the more likely equilibrium is the one in which no sellers move, and therefore there is no barter. Intuitively, the number of commodities cannot be too large if barter is going to be sufficiently viable for sellers to pay the transportation cost.

III. Conclusion

Perhaps the key insight to emerge from the analysis is that the decision to search is fundamentally different for buyers and sellers. This is most clear in the equilibrium with $n_m = 1$ and $n_1 = 0$, where all buyers search and no sellers search. For this to be an equilibrium, the cost c_m must be below a certain threshold, and the cost c_1 must be above a certain threshold. Since the second threshold exceeds the first, agents with money are more willing to bear transportation costs than are agents with goods. This is because fiat currency is a universally acceptable medium of exchange, and so it is easier for a buyer with money to consummate a transaction once a trading partner is located. One reason to focus on the equilibrium in which buyers search and sellers do not is that all trade is monetary—it looks like a "cash-in-advance" economy.

REFERENCES

Aiyagari, S. Rao and Wallace, Neil. "Fiat Money in the Kiyotaki-Wright Model." *Economic Theory*, October 1992, *2*(4), pp. 447–64.

Burdett, Kenneth; Coles, Melvyn; Kiyotaki, Nobuhiro and Wright, Randall. "Buyers and Sellers." CARESS Working Paper No. 93-03, University of Pennsylvania, 1993.

Casella, Alessandra and Feinstein, Johnathon S. "Economic Exchange During Hyperinflation." *Journal of Political Economy*, February 1990, *98*(1), pp. 1–27.

Clower, Robert W. "A Reconsideration of the Microfoundations of Monetary Theory." *Western Economic Journal*, December 1967, *6*(4), pp. 1–8.

Diamond, Peter A. "Money in Search Equilibrium." *Econometrica*, January 1984, *52*(1), pp. 1–20.

Diamond, Peter A. and Yellin, Joel. "Inventories and Money Holdings in a Search Economy." *Econometrica*, July 1990, *58*(4), pp. 929–50.

Gale, Douglas M. "A Strategic Model of Trading with Money as the Medium of Exchange." CARESS Working Paper No. 86–04, University of Pennsylvania, 1986.

Kiyotaki, Nobuhiro and Wright, Randall. "A Contribution to the Pure Theory of Money." *Journal of Economic Theory*, April 1991, *53*(2), pp. 215–35.

______. "A Search-Theoretic Approach to Monetary Economics." *American Economic Review*, March 1993, *83*(1), pp. 63–77.

Li, Victor E. "Inventory Accumulation in a Search-Based Monetary Economy." *Journal of Monetary Economics*, 1995 (forthcoming).

Matsuyama, Kiminori; Kiyotaki, Nobuhiro and Matsui, Akihiko. "Towards a Theory of International Currency." *Review of Economic Studies*, April 1993, *60*(2), pp. 283–307.

Trejos, Alberto and Wright, Randall. "Search, Bargaining, Money, and Prices." *Journal of Political Economy*, February 1995, *103*(1), pp. 118–41.