

# The development of a new method for the proportioning of high-performance concrete mixtures

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Accepted 3 September 2003

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## Abstract

This report generalizes the results of research into silica fume based high-performance concrete. The strength properties and the rheological behavior of a cement–silica fume–superplasticizer system are presented. From the test results it is suggested that an optimal superplasticizer-to-silica fume ratio (1:10) provides ultra-dense packing and high fluidity of the system.

Models of high-performance concrete are developed from the experimental data. These models provide equations for calculating W/C for the required compressive strength (up to 130 MPa) and also the volume of cement paste for the required slump (within the range of 40–200 mm). For modeling purposes, concrete slump is presented as a function of the proportion of aggregates and both the volume and fluidity of cement paste. This modeling approach suggests a new method of proportioning of high-performance concrete mixtures.

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**Keywords:** High-performance concrete; Silica fume; Superplasticizer; Compressive strength; Slump; Mathematical models; Mixture proportioning

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## 1. Introduction

Nowadays, high-performance concretes (HPCs) are extensively applied in construction projects [1–3]. This new advanced concrete has been transferred from laboratory research to practical application; and it already occupies a noticeable share of the market. Based on the latest developments in concrete technology, HPC is characterized by a superior level of properties: workability, strength and durability. These advantages provided large-scale cost savings in many construction projects.

Modern HPC technology applies the concept of densified system with ultra fine particles (DSPs), which includes the effective combination of a silica fume (SF) with a superplasticizer (SP) that helps to maintain a workable cement system at a very small W/C [3]. The superfine SF particles fill the space between cement grains and the interfacial transition zone; in addition, the application of SF results in a high rate of pozzolanic reaction.

The modeling and precise proportioning of the mixture are important for extending the application of HPC. There are various methods for proportioning conventional concrete [4,5]. Generally, these methods are based on fundamental functions: water-to-cement ratio (Abrams–Bolomey–Feret’s laws), the constancy of water demand (Lyse’s law) and theory of optimum aggregates proportioning (Fauy–Bolomey–Fuller’s laws), all of which determine mixtures with the required properties. These methods can be adjusted to yield concrete mixtures in which part of the cement is replaced by mineral additives such as fly ash or silica fume. The effect of chemical admixtures, such as plasticizers or superplasticizers, can also be incorporated into existing methods [6]. However, with the new generation of HPC the problem of designing the concrete mixture becomes more sophisticated. Although the HPC mixture proportioning and optimization has been discussed [7–13], the solution to this problem is still unresolved. The most readily available HPC mixture proportioning methods are semi-analytical: they usually provide the proportioning of aggregates and the calculation of W/C using an equation of compressive strength [6–8].

Yet, the contribution of mineral additives or chemical admixtures and design for a certain workability level

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need further examination. Therefore, a better analytical method is required to design and optimize the concrete mixtures which contain a combination of chemical and mineral admixtures. The main postulates of a method dealing with different admixtures have been described [7,9].

To elucidate the importance of the modified cement system, rheological behavior is proposed as the universal approach to included in the design of HPC. Using this, an analytical mixture proportioning method is suggested, with the aid of mathematical models based on empirical results.

## 2. Experimental program

### 2.1. Materials

Normal portland cement (NPC) and silica fume (SF) were used in the experimental program. The portland cement was characterized by the following Bogue's composition:  $C_3S$ —64%,  $C_2S$ —14%,  $C_3A$ —4%, and  $C_4AF$ —14%. The SF was composed of glassy microspherical particles with a diameter of 0.1–0.2  $\mu m$ . The physical properties and chemical analysis of the cement and silica fume are shown in Table 1.

Commercially available sulfonated naphthalene formaldehyde condensate type superplasticizer was used as modifier in the experimental program.

Crushed granite with a maximum size of 20 mm and specific gravity 2.68 was used as a coarse aggregate. Locally available natural sand with a fineness modulus

of 1.95 and a specific gravity of 2.62 was used as a fine aggregate. The grading of the aggregates is given in Table 2.

### 2.2. Test procedures

The rheological characteristics, such as viscosity and shear stress of the modified cement systems, were measured with a rotary cylinder viscometer at various deformation rates (0.3566–11.4112  $s^{-1}$ ) in the time range of 5–60 min.

The strength characteristics of the modified cement systems were determined following the standard procedure of ASTM C 109 “Standard Test Method for Compressive Strength of Hydraulic Cement Mortar” and C 348 “Standard Test Method for Flexural Strength of Hydraulic Cement Mortar” at a cement-to-sand ratio 1:1 on prisms—samples  $40 \times 40 \times 160$  mm. Steam curing at 80 °C for 8 h was used to accelerate the hardening of samples. The compressive strength of mortar at  $W/C = 0.14$  was adopted as a strength factor of the system and applied to optimize the composition.

Using the absolute volumes method a computer program determined the proportioning of the concrete mix. The components of a concrete mix were dosed by weight and mixed in a rotary mixer for 5 min. The slump of a concrete mix was the measure of its workability. The compressive strength of concretes was tested using 100 mm cube-samples after 1, 3, 7 and 28 days of hardening in a curing room at 20 °C and a relative humidity of 95%.

The experimental program included some 70 compositions of modified cement systems and mortars, as well as more than 100 HPC mixtures. The processing of the experimental results helped to create and calibrate the mathematical models.

### 2.3. Modeling approach

The rules of the polystructural theory of concrete were used to develop HPC models. According to the polystructural theory, the properties of concrete are considered as a function of properties at both a macro-level (comprised of aggregates and cement paste) and micro-level (included the modified cement paste with chemical and mineral admixtures).

The micro level embodies the strength and rheological properties of the cement–silica fume–superplasticizer system.

Regression models of the strength and rheological behavior of the system were developed from the test results. The models involved second-order polynomial equations and were generated by custom-designed computer software. For some parameters the logarithmic compression of data was applied. The second-order

Table 1  
Physical properties and chemical analysis of cement and silica fume

	Portland cement	Silica fume
<i>Physical properties</i>		
Surface area, $m^2/kg$	350	20000
Specific gravity, $kg/m^3$	3.15	2.21
Setting time, h:min		
Initial	2:10	
Final	3:40	
Compressive strength, MPa		
1 day	15.8	
3 days	34.2	
7 days	44.3	
28 days	55.2	
<i>Chemical analysis</i>		
$SiO_2$	22.1	90.2
$Al_2O_3$	4.86	1.7
$Fe_2O_3$	4.92	0.4
CaO	66.87	2.1
MgO	0.42	1.7
$Na_2O$	0.15	0.7
$K_2O$	0.15	0.7
$SO_3$	4.03	0.5
L.O.I.	1.64	2.5

Table 2  
Grading of the aggregates

Aggregate type	Passing, % at a mesh width, mm							
	0.14	0.31	0.63	1.25	2.50	5.00	10.00	20.00
Fine	3.98	35.46	77.52	91.01	96.83	100.00	100.00	100.00
Coarse	0	0	0	0	0.67	2.68	35.90	100.00

polynomial equation to calculate the required property parameter ( $f_{(n)}$ ) is

$$f_{(n)} = \sum_{i=0}^n \sum_{j=i}^n b_{ij} x_i x_j \quad (1)$$

where  $n$  is the total number of parameters,  $b_{ij}$  the coefficients of the polynomial equation,  $x_i, x_j$  the parameters of the polynomial equation with  $x_0 = 1$ .

The optimal aggregate proportioning was specified with a minimal deviation from the nominal grading curve. After verification tests, the optimal fine-to-total aggregate ratio was set at 0.35; and the actual grading curve of the aggregates mixture was close to the gap grading proposed in references [10,11]. The concrete compressive strength is presented as a function of W/C. Since NPC and SF were considered as components of the resulting blended cement or binder, the W/C ratio used throughout this report actually represents the water-to-binder ratio or W/(NPC + SF). For modeling purposes, the slump of the concrete mix was considered as a function of the volume and shear stress of cement paste and aggregate proportioning.

### 3. Test results and models

#### 3.1. Rheological behavior of the modified cement pastes

For the NPC–SF–SP system it was observed that the increase of the SP dosage, from zero to some limited dosage  $SP_{lim}$ , provides a significant reduction of shear stress. Beyond this limited dosage no further decrease in shear stress was observed. The  $SP_{lim}$  can be considered as optimal; it depends on SF content and is approximately equal to 10% of SF [9].

With an SF content of 10–20% and the optimal SP dosage there is a plateau of a minimum shear stress [9]. These compositions provide the best packing of the mixture composed of NPC and SF particles. Replacement of even a small amount of NPC by SF of 1–5% makes the system denser; further, the addition of SP increases the relative thickness of water layers between cement particles (at fixed W/C), thus reducing the viscosity. So, the best packing and minimum viscosity are reached at the optimum proportions of SF from 10% to 20%. Increasing the SF content beyond 20% causes the unpacking of the system and loss of fluidity because of a surplus of the small-sized fraction.

The comparison of the rheological behavior of NPC and the modified system at various W/C helps to evaluate the effect of SF as a microfiller [9]. The ultimate fluidity of NPC paste in the time-range of up to 1 h takes place at W/C = 0.4; whereas it occurs only at W/C = 0.3 for the modified system. Furthermore, the shear stress of the modified system at 5% of SF and W/C = 0.25 is essentially higher than the one at 15% of SF and W/C = 0.2, which results in a more fluid system at low W/C.

These observations and test results were incorporated into the rheological model (using Eq. (1)), the coefficients of which are presented in Table 3. The model covers five parameters (5-factor model): the SF content, SP dosage, W/C, the period of time after mixing (time, min) and the deformation rate ( $s, s^{-1}$ ). This model was used to optimize the compositions and also to tabulate the isorheological mixtures presented in Table 4 (calculated for a deformation rate of  $1.4264 s^{-1}$  and a time parameter of 60 min).

#### 3.2. Strength of modified mortars

The strength tests confirmed the assumption that the optimum dosage of SP correlates with the SF content.

Table 3  
The coefficients of the rheological model

Description		Polynomial equation coefficients $b_{ij}$ for $i$					
$X_j$	$j$	0	1	2	3	4	5
–	0	9230.1760	–918.0004	–1042.286	–5441.179	765.4758	280.2586
ln(10SP)	1		–7.685648	88.31657	313.2570	–16.44136	–17.67504
SF/10	2			2.585297	253.1670	–28.80224	6.277071
ln(50 W/C)	3				814.4732	–262.9913	–95.0238
ln(time)	4					14.06832	11.51063
ln(10s)	5						7.582472

Table 4  
Parameters of the isorheological systems

Composition of the modified system, %		W/C required to obtain the shear stress, Pa				
Silica fume	Superplasticizer	5	10	50	100	300
5	8	0.431	0.424	0.386	0.354	0.278
5	10	0.427	0.419	0.379	0.346	0.271
5	12	0.421	0.413	0.371	0.338	0.264
10	8	0.360	0.356	0.327	0.301	0.240
10	10	0.360	0.355	0.324	0.298	0.235
10	12	0.360	0.355	0.321	0.293	0.230
15	8	0.314	0.311	0.290	0.269	0.217
15	10	0.314	0.311	0.288	0.267	0.213
15	12	0.314	0.311	0.286	0.263	0.209
20	8	0.282	0.279	0.262	0.245	0.199
20	10	0.282	0.279	0.261	0.243	0.196
20	12	0.282	0.279	0.259	0.241	0.193

Table 5  
The coefficients of the strength model

Description		Polynomial equation coefficients $b_{ij}$ for $i$		
$X_j$	$j$	0	1	2
–	0	95.51029	3.70645	1.28067
SF	1		–0.23804	0.26747
SP	2			–0.21051

The optimum SF content in the system was found to be 15%. For optimal composition, the compressive strength was 136 MPa and the flexural strength was 18 MPa.

The coefficients of the compressive strength model (according to the Eq. (1)) based on the processing of the experimental data are given in Table 5. This 2-factor strength model uses SF content and SP dosage as parameters. The model minimizes the cost of the NPC–SF–SP binders for the specific strength levels.

It was observed that the strength of the system depends solely on W/C (Fig. 1). This finding means that the pozzolanic effect of SF is approximately equivalent to the cement strength. The microfiller effect of SF can be specified as the ability to reduce the water demand of the system (to W/C = 0.14) because of better packing.

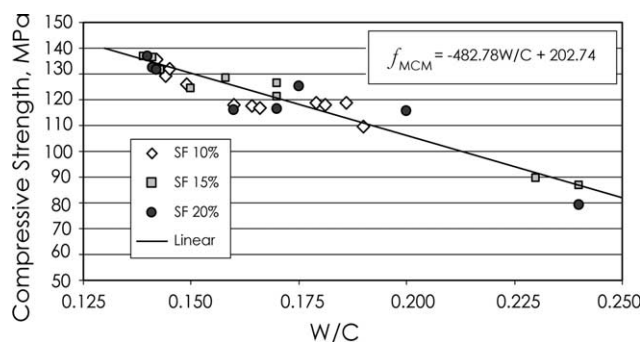


Fig. 1. Compressive strength of the modified cement mortars.

### 3.3. Properties of fresh concrete

The air content of fresh concrete mixtures was varied from 1.9% to 2.4%, with the average amount of entrained air at 2.0%. No bleeding of the HPC was observed.

It was found that for certain concrete mixtures, slump can be adjusted through a wide range (from 40 to 200 mm) only by varying the SP dosage from 4% to 18% (of SF). The constant slump value can be obtained for different SF contents (from 5% to 25%), at the optimal SP dosage and constant water content. The increase of cement content, at a constant amount of water, leads to only a slight loss of slump (20–40 mm). In this case, the increased viscosity of the system is compensated by the higher volume of cement paste. Finally, the slump of the concrete mixtures demonstrated a good correlation with the rheological parameters of the cement systems.

It was proposed that the concrete slump can be presented as a function of the proportioning of aggregates, and both the volume and rheological characteristics of the cement paste. The slump model was processed with the following parameters: fine to total aggregates ratio ( $r$ ), shear stress ( $S_s$ , Pa) and volume ( $V_{cp}$ , dm<sup>3</sup>) of the cement paste (3-factor model). In order to obtain this model the deformation rate was specified at the level of 1.4264 s<sup>–1</sup> and the time parameter was taken as 30 min. The polynomial equation coefficients of the model according to Eq. (1) are presented in Table 6.

Table 6  
The coefficients of the slump model

Description		Polynomial equation coefficients $b_{ij}$ for $i$			
$X_j$	$j$	0	1	2	3
–	0	–2246.5540	–0.463865	8.417213	47.435040
$S_s$	1		0.000013	0.000845	–0.005283
$V_{sp}$	2			–0.007401	–0.066095
$100 \times r$	3				–0.347950

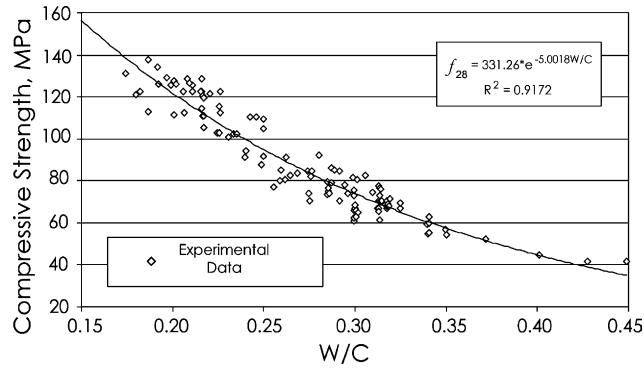


Fig. 2. Compressive strength of HPC.

### 3.4. Compressive strength of the HPC

It was found that the increase in the SP dosage from 8% to 18% (of SF) leads to a reduction of W/C from 0.31 to 0.26 (at a constant slump and cement content of 400 kg/m<sup>3</sup>), and improves the concrete 28-day compressive strength from 86 to 97 MPa.

The maximum compressive strength of 91 MPa was obtained at an SF content of 15%. The lower strength value of 90 MPa occurred at 10% and 20% of SF. HPC

strength behavior maintains its relationship with the strength of modified cement mortar.

The reduction of W/C from 0.32 to 0.19 increased the compressive strength of the concrete and resulted in super-high strength concrete (compressive strength up to 135 MPa). It was found that for a wide range of compositions the 28-day compressive strength of investigated concrete could be presented as a function of W/C (Fig. 2).

Experimental data were used for modeling HPC strength. This model has the following equation:

$$f_{28} = 331.26e^{-5.0018 W/C} \quad (2)$$

The HPC strength model can determine of W/C for the target compressive strength in the range of 50–130 MPa.

### 4. HPC mixture proportioning method

The flowchart of the proposed method for HPC mixture proportioning is given in Fig. 3. This analytical method has been used to simulate and optimize a wide range of HPC.

First, the optimal SF content and SP dosage are selected according to the strength model of modified

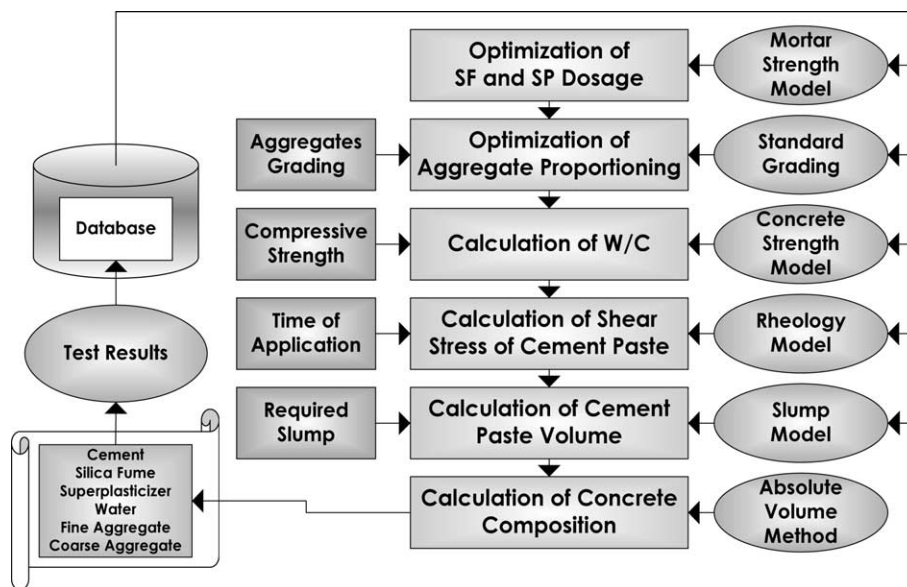


Fig. 3. Method for HPC mixture proportioning.

Table 7  
Example of HPC mixture proportioning

	SF5	SF10	SF15	SF20
<i>Mixture proportions, kg/m<sup>3</sup></i>				
Cement	426	449	468	478
Silica fume	22	50	83	120
Total cement and silica fume	448	499	550	598
Superplasticizer	2.2	5.0	8.3	12.0
Water	153	142	132	121
Coarse aggregate	1169	1155	1136	1119
Fine aggregate	630	622	612	603
<i>Mixture parameters</i>				
Silica fume, %	5	10	15	20
W/C	0.342	0.284	0.239	0.203
Shear stress, Pa	97	132	187	265
Cement paste volume, dm <sup>3</sup>	298	307	318	327
Fine to total aggregates ratio	0.35	0.35	0.35	0.35
<i>Fresh properties</i>				
Slump, mm	100	100	100	100
Fresh unit weight, kg/m <sup>3</sup>	2402	2422	2438	2453
Air content, %	2.5	2.5	2.5	2.5
<i>Mechanical properties</i>				
Compressive strength, MPa				
1 day	16.8	24.1	34.4	45.1
3 days	28.6	42.2	63.0	84.9
7 days	50.1	67.2	84.8	102.5
28 days	60.0	80.0	100.0	120.0

mortars: for optimal performance SF content is specified within 10–15% and SP dosage is set to 10% of SF. Second, the aggregates are optimized to fit a specific grading curve. Then, W/C is selected using the model of HPC strength using Eq. (2).

The contribution of different admixtures to the rheological behavior of the system can be measured or calculated using the rheological model (Tables 3 and 4). This specifies the value of shear stress of the selected composition at the required W/C. The necessary concrete slump can be obtained by solving the slump model that gives the volume of cement paste. Specifying all these parameters determines the remaining components of HPC mixture that are determined using the method of absolute volumes. Examples of the HPC mixture proportioning are presented in Table 7.

## 5. Conclusions

1. Models of the rheological behavior of modified cement systems enable the specification of the SP dosage or W/C for the required shear stress. The strength models can be used to optimize the SP dosage and SF content for the required compressive strength.
2. The concrete slump model is a function of the shear stress and volume of the cement paste at a certain proportioning of the aggregates. The proposed HPC

strength model can be given as an exponential equation involving W/C. The HPC strength model calculates the W/C for the target compressive strength of HPC within the range of 50–130 MPa.

3. The proposed method of HPC mixture proportioning can be used to design and optimize a wide-range of concrete mixtures.

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