Physics 782, Spring 2010 – HWK #1

Due in class Mon Feb 15.

1. Compute Fourier coefficients for the function defined on the interval [-½, ½]

$$\chi_{|x|<1/4}(x) = \begin{cases} 1 & \text{for } |x| < 1/4 \\ 0 & \text{for } |x| \geq 1/4 \end{cases}$$

2. Plug these coefficients into the Fourier series inversion formula and evaluate $S_n \ \chi_{|x|<1/4}(x)$ over the interval [-½, ½] for *n*=8, 32, 128, 512. Plot the results using your favorite SW package, *being certain to display the important features of the partial sums.*

3. Insert zeros between every other coefficient computed for n=128 and plug into the Fourier series inversion formula for n=256. Repeat, so that 3 of 4 Fourier coefficients is zero and reconstruct for n=512. What happens to the resulting function?

4. Consider the function defined over the interval [-½, ½]

$$f_n(x) = \begin{cases} +1 & \text{for } nx \in [2m, 2m+1) \\ -1 & \text{for } nx \in [2m+1, 2m+2) \end{cases} \qquad \textit{for integers m \& n}$$

   For *n=8,*

   a. Plot discretely sampled versions of f_8, sampled at 1, 2, 3, 4, 6, 8 evenly spaced points in the interval [-½, ½]
   b. Determine the minimum number of sampling points required to capture all 8 of this function's oscillations. (Nyquist sampling)
   c. Using your favorite SW package, evaluate f_8 at 512 evenly spaced points and take the FFT to approximately recover 512 Fourier coefficients. Replace high frequency coefficients with zeros leaving only *q*=1,2,3,4,6,8,12 nonzero coefficients and take IFFT. Plot results on a single plot for comparison. Repeat for *q*=8,16, 64,128, 256 & 512 nonzero coefficients.

   For consistency please discretize the interval [-½, ½] at *p* points as follows:

   -1/2, -1/2+1/p, -1/2+2/p, ... , -1/2+(p-1)/p=1/2-1/p

   Note that for *p* even, the p/2+1 point should sample at the origin.