Physics 782

Homework #4 – ***Due in class, Monday March 29.***

1. Compute the PSF of FBP reconstruction with the following combinations:
   - $N_\theta = N_s$ = 2^3, 2^5, 2^7, 2^9 and θ = [0:180/ N_θ:360.999]
   - Filter kernel = Ram-Lak, Shepp-Logan, Cosine, Hamming

Plot

   a. Different filter kernels – label Fourier axis/"k-axis" clearly
   b. profiles through the center of the PSF, focusing on the "interesting" region

2. Shift the detector by -1, 0, 1,2,3 channels, keeping $N_\theta = N_s = 2^9$.
   a. What happens to the PSF?
   b. What happens to reconstructions of 2D Radon transform, $R\chi_{|(x,y)|<1/4}(\theta,s)$ of the indicator function of a disc of radius ¼?

For problems 3-5 take ideal 2D Radon transform, $R\chi_{|(x,y)|<1/4}(\theta,s)$ of the indicator function of a disc of radius ¼ with $N_\theta = N_s = 2^9$, where $\theta \in [0,2\pi)$.

3. (Fourier filtering – fine point)
   a. Reconstruct using both iradon.m and iradonSKP.m. Plot profiles through the center and point out the difference between the results.
   b. Compare Ram-Lak filter kernels, and plot them zooming in on the region where they differ most.

4. (FBP vs. Fourier/regridding) Reconstruct using FBP and also Fourier reconstruction (ala Projection Slice Theorem) using "griddata.m". Rescale the images so they have approximately the same (window,level)=(w,l), *i.e. inside the disc should be 1, outside should be 0.*
   a. Display FBP w/Ram-Lak filtering vs Fourier reconstruction with linear interpolation in k-space using MATLAB's (w,l) = (0.9, 1.1)
   b. Do Fourier reconstruction using interpolation methods: 'linear','cubic', and 'nearest'. Display images using MATLAB's (w,l) = (0.9, 1.1). Plot center profiles on the same plot for easy comparison of the effect of k-space interpolation.
   c. Do Fourier reconstruction as above but with and without
      i. Zero-padding sinogram by factor of 2 in s-direction
      ii. Upsampling gridding mesh in k-space by a factor of 2
   d. Why are these are different and what you could easily do to fix low frequency errors??

5. Add random noise with σ=0.025 to the channels closest to *s* = -0.256, -0.127, 0. Use MATLAB's randn command as follows:

```
SinoNoisy(BadChans(m),:) = SinoNoisy(BadChans(m),:)+σ*randn(1,Ntheta);
```

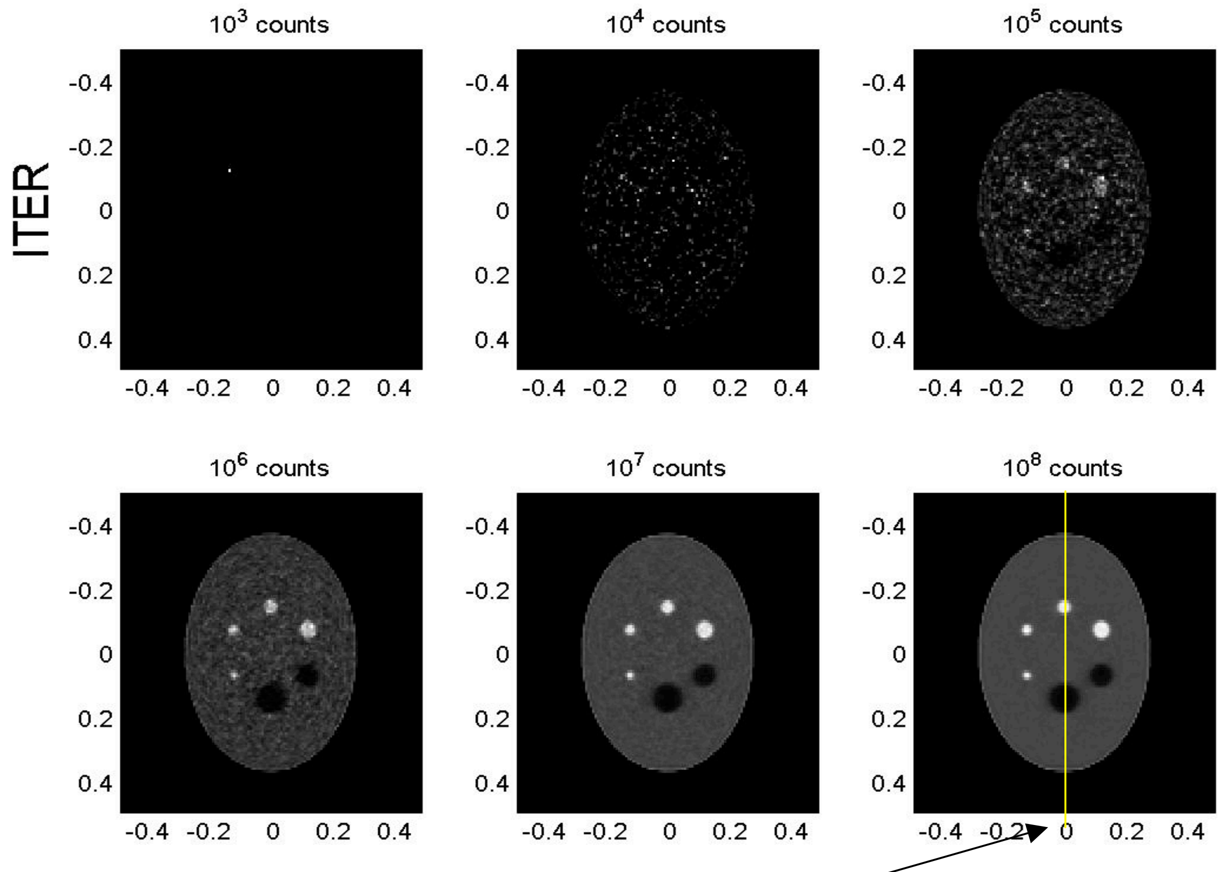
   a. Reconstruct the same sinogram using FBP with "iradon.m" and also Fourier reconstruction using "griddata.m". Rescale the images so they have approximately the same (window,level)=(w,l), *i.e. inside the disc should be 1, outside should be 0.*
   b. Display FBP w/Ram-Lak filtering vs Fourier reconstruction with linear interpolation in k-space using MATLAB's (w,l) = (0.9, 1.1) Zero-pad the sinogram and upsample in k-

space for the Fourier reconstruction.

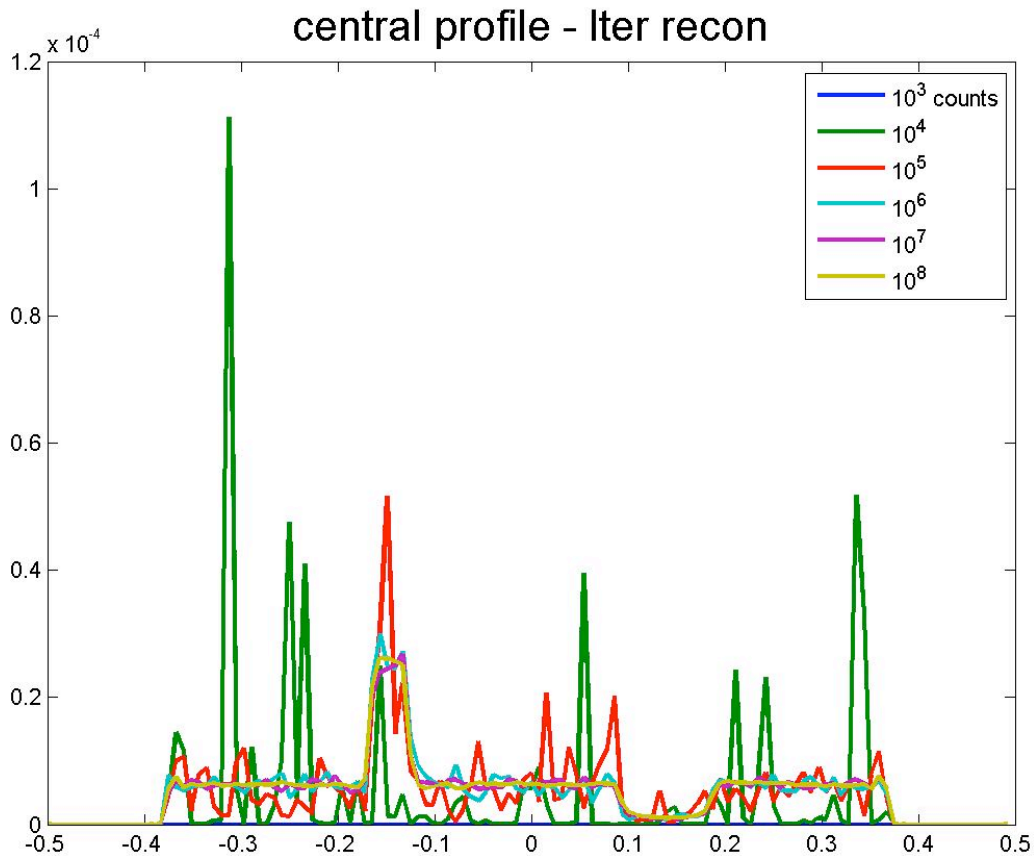
6. Reconstruct the sinogram in HWK4.mat using both FBP and Fourier reconstruction. How do the motion artifacts manifest themselves differently?

7. (PET attenuation) In PET, many events are lost due to attenuation within the patient. Reconstruct simulated PET data that accounts for attenuation and compare to the "CT" image resulting from an attenuation free sinogram. These are in HWK4PET.mat and you should reconstruct a a 128x128 image using FBP. *This data is "ideal" & corresponds to very high count rates.*

8. (Iterative vs FBP for PET) Iterative reconstruction algorithms can account for attenuation suffered in PET data and tends to be more robust to noise than FBP. Reconstruct the same sinograms using FBP and display the results similarly.



Profiles along the center vertical line in yellow are plotted below:



as of 9:46 PM on 3/13/10