Physics 782

Homework #3 – ***Due in class, Thurs March 11.***

1. Suppose your MRI scanner should sample data, $\hat{f}(k_m,k_n)$ at points $(k_m,k_n) = \Delta_k(m,n)$ for m,n=-N/2, -N/2+1, … N/2-1. Due to eddy currents, it actually samples at points $(m\Delta_k+\varepsilon_1, n\Delta_k+\varepsilon_2)$. How can you recover the desired function $f(x_1,x_2)$? What shortcut could you take knowing *a priori* that $f(x_1,x_2)$ represents density of hydrogen nucleii?

2. Suppose in problem 1 $(\varepsilon_1, \varepsilon_2) = \varepsilon\,(1/3,1/2)$ for and $f(x_1,x_2) = \chi(x_1)\,\chi(x_2)$ where $\chi(x)=1$ for $|x|<1/4$ and $\chi(x)=0$ for all other x. Show the naive reconstructions obtained when your k-space trajectories are shifted by $\varepsilon=0,1,2$. Use N=512 with the image field of view (FOV) normalized to 1. ($\Delta_k=1$ and $\Delta_x=1/512$) What happens to the magnitude and phase images for each $\varepsilon=0,1,2$?

3. (1/4 detector offset) Assume a linear xray CT detector array with individual channel width Δs and 100% fill factor.
   a) How would you use this detector to sample at the Nyquist rate?
   b) How would you use a detector with 67% fill factor?

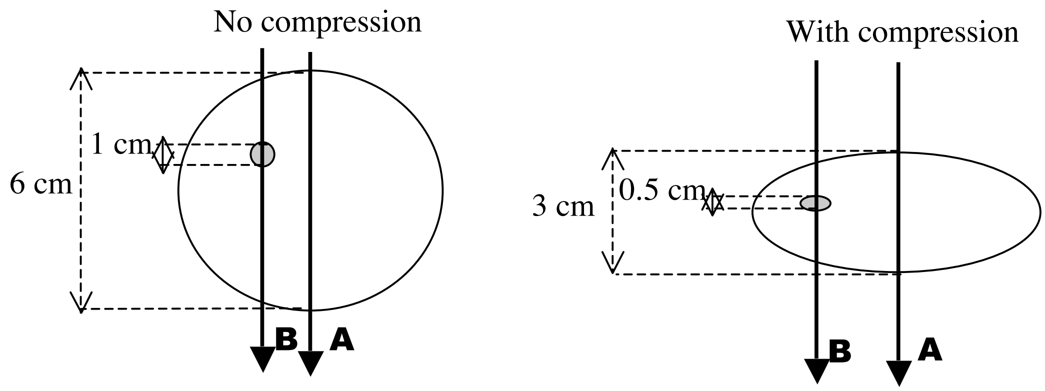
4. Copy & paste the following lines into MATLAB simply to get familiar with radon.m

```
P = phantom(128);
R = radon(P,0:179);
I1 = iradon(R,0:179);
subplot(1,2,1), imshow(P), title('Shepp-Logan Orig Image'), colorbar,
subplot(1,2,2), imshow(I1), title('Filtered backprojection'), colorbar,
```

5. (Detector averaging w/o noise.) Suppose you measure the 2D Radon transform of the indicator function on a disc of radius ¼ with an xray detector covering s∈[-1/2, ½) with Δs=1/128 and 100% fill factor.
   a. In the same graph, plot data measured by this averaging detector and an ideal detector, *being certain to display the regions of "interesting" behavior*.
   b. What is the Fourier transform of the measured and ideal projections? Plot them on the same graph to highlight differences.
   c. For a given Δs what $\Delta\theta$ would you set?
   d. Use MATLAB's Radon.m routine to numerically create a sinograms of the indicator function. Use 2D image array sizes *nxn* = 8x8, 32x32, 128x128, 512x512. Create and turn in images of both your function χ and sinogram.
   e. Evaluate the analytic/ideal sinogram numerically in MATLAB using the exact same sampling rate as created in problem d. Compare analytic vs. numerical sinograms. Why do results agree better for large *n*? (ignore scaling factors)
   f. Reconstruct sinograms from parts A & B using MATLAB's iradon.m & compare results. Show me images and profiles.

NOTE: for problems e & f be sure to sample at the same points as MATLAB.

6. Mammography is typically performed while the breast is compressed. In this question we will look at how compression improves the contrast-to-noise ratio of an infiltrating ductal carcinoma (IDC) in the breast. A schematic of the breast with and without compression is shown in the following figure. Let's assume the breast is glandular tissue with a single lump of IDC.



Contrast-to-noise ratio (CNR) is defined as:

$$CNR = \frac{N_A - N_B}{Noise}$$

which can be approximated as

$$CNR = \frac{N_A - N_B}{\sqrt{N_A}}$$

where N_A and N_B are the output number of x-ray photons for rays A and B respectively, where ray A travels only through glandular breast tissue and ray B travels through both the breast tissue and the tumor.
The linear attenuation coefficient of glandular tissue is $\mu_g = 0.8\text{cm}^{-1}$ and
the linear attenuation coefficient of IDC $\mu_{idc} = 0.9\text{cm}^{-1}$.
Using these numbers and the initial number of photons $N_o = 1000$, calculate the CNR of both the uncompressed and compressed breast. What is the percent improvement in CNR with compression?

7. Beam Hardening – Toy Problem. Assume the following xray spectra and attenuation coefficients

$$I_o(E) = \cos\left((E - E_o)\pi\delta\right)\chi_{|(E-E_o)\delta|<1/2}(E) \qquad \textit{and}$$

$$\mu(\mathbf{x}, E) = \mu_o \;\; \chi_{|\mathbf{x}|<1/4}(\mathbf{x}) \;\left(1 - \varepsilon\sin\left((E - E_o)\pi\delta\right)\chi_{|(E-E_o)\delta|<1/2}(E)\right)$$

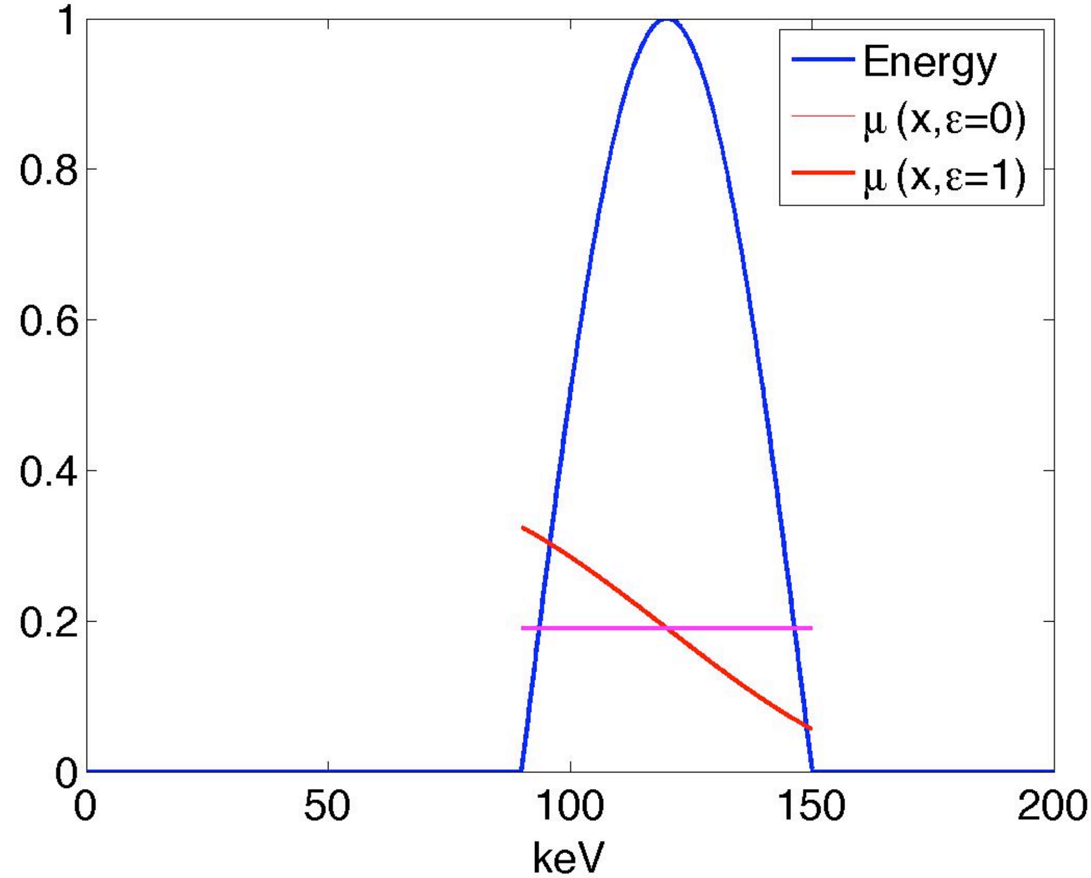
Start with E_o = 120keV, $\delta = 2/E_o$. Note that μ is independent of **x** and when ε=0, $\mu(\mathbf{x},E) = \mu_o$ (see plot below). Compute the CT projection of the disc of radius 1/4, allowing now for beam hardening according to the spectra (I_o) and LAC, $\mu(\mathbf{x},E)$, given above. You must integrate over energy as below

$$I_{meas} = \int I_o(E) \;\; e^{-\int_L \mu(x,E)\;dx} dE$$

Do the integral analytically for this toy example.
For both ε=0 and ε=1 compute the measured *projections,* which requires taking

$$-\ln\left(\frac{I_{meas}}{\int I_o(E)\;\; dE} \right)$$

Reconstruct using iradon.m and compare images with and without beam hardening.


Energy
μ (x,ε=0)
μ (x,ε=1)
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
keV