Physics 782

Homework #2 – ***Due in class, Thurs Feb 25. If you do not have access to MATLAB let me know ASAP.*** *When asked to validate your "theory" numerically show me your images and your well-commented code, which should be short.*

1. Compute Fourier coefficients for the indicator function on the rotated box, bounded by
   $y = ¼ - x$
   $y = -¼ + x$
   $y = ¼ + x$
   $y = -¼ - x$
   Evaluate $f\hat{}_{m,n}$ numerically on a 512 x 512 lattice and take the 2D IFFT to verify your result for f(x,y). What is the max(Re(f)) in your reconstruction? Where is the maximum achieved? What is going on?

2. Reconstruct subsampled 512x512 k-space data from problem #1 as follows:
   a. Compare images created by taking all 512 lines of k-space, to those created by taking every other, every 3[rd], and every 4[th] line of k-space. For example, take every other line using the command `fhatsub = fhat(1:2:512,:)`
   b. Compare images created using only the central 4,8,16, & 32 lines of k-space.

   Label the x- and y- axes of your images to give the correct spatial scaling!!!

3. Compute Fourier coefficients of the indicator of the disc of radius R<1/7, first rotated by angle θ and then shifted by $(\Delta x, \Delta y)=(1/6,1/4)$. Evaluate your coefficients numerically & take the 2D IFFT to verify your result.

4. Suppose your MRI scanner should sample data, $f\hat{}(k_m,k_n)$ at points $(k_m,k_n) = \Delta_k(m,n)$ for m,n=-N/2, -N/2+1, … N/2-1. Due to eddy currents, it actually samples at points $(m\Delta_k+\varepsilon_1, n\Delta_k+\varepsilon_2)$. How can you recover the desired function $f(x_1,x_2)$? What shortcut could you take knowing *a priori* that f represents density of hydrogen nucleii?

5. Suppose in problem 4 $(\varepsilon_1, \varepsilon_2) = m(1/3,1/2)$ for and $f(x_1,x_2) = \chi(x_1)\,\chi(x_2)$ where $\chi(x)=1$ for $|x|<1/4$ and $\chi(x)=0$ for all other x. Show the naive reconstructions obtained when sampling shifted data for m=0,1,2 and using N=512. What happens to the magnitude and phase images?

6. The file PropellerToy.mat contains 4-blade Propeller data in the variables fhat & fhatNoiseFree. Blades sample k-space on equispaced Cartesian lattices; blades are rotated by $\pi/4$ relative to each other. No motion correction is required. Reconstruct the data using the easiest method you can find.

   HINTS:
   a) MATLAB's "griddata" routine can do this
   b) Reconstruct fhatNoiseFree first, so you can debug your code.

   Brownie Points: Play around with different griddata options: nearest, linear, and cubic