

Physics 317

Homework #5 – *Due in class, Mon Oct 17*

1. Show that thermodynamic probability can be written in a general form for all three statistics, Max Boltzmann (MB), Bose-Einstein (BE), and for Fermi-Dirac (FD).

$$\omega \propto \prod_{j=1}^n \frac{g_j(g_j-a)(g_j-2a)\dots(g_j-(N_j-1)a)}{N_j!}$$

$$\text{where } a = \begin{cases} +1 & \text{for FD} \\ 0 & \text{for MB} \\ -1 & \text{for BE} \end{cases}$$

2. A system has three energy levels, $\epsilon = 0k$, $\epsilon = 300k$, and $\epsilon = 600k$, where k has the numerical value of Boltzmann's constant, but with units of Joules. The degeneracies of the levels are 1, 3, and 5, respectively.
 - a. Calculate the partition function, the relative populations of the energy levels, and the average energy, all at a temperature of 300 Kelvin.
 - b. At what temperature is the population of the energy level at $600k$ equal to the population of the energy level at $300k$.
3. A Carnot engine operates between two heat reservoirs at temperatures of 800 K to 600 K.
 - a. If the engine receives 1200 kg calories from the hot reservoir in each cycle, how much energy does it reject to the cold reservoir?
 - b. If the engine is operated as a fridge (In reverse) and receives 1200 kg calories from the cold reservoir, how much heat does it deliver to the hot reservoir?
 - c. How much work is done by the engine in case a?
 - d. How much work is done by the refrigerator in case b?
 - e. What is the efficiency of the engine in case a?
 - f. What is the coefficient of performance of the fridge?
4. 10 kg of water at 0°C must be frozen into ice. The refrigerator is in a room with ambient temperature of 20°C . The latent heat of fusion is 3×10^5 Joules per kilogram. What is the minimum power required to freeze the water in 30 minutes?
5. Which gives the greater increase in efficiency of a Carnot engine: increasing the temperature of the hot reservoir or lowering the temperature of the cold reservoir by the same amount?
6. An ideal monatomic gas is the working substance of a Carnot engine. During isothermal expansion the volume doubles. The ratio of final volume to initial volume in adiabatic expansion is 5.7. The work output of the engine is 9×10^5 Joules per cycle. What are the temperatures of the hot and cold reservoirs?