Physics 317

Homework #1 – ***Due in class, Wed Sep 14***

1. Suppose *f, g, h* are functions of *x*. $f = f(x)$, $g = g(x)$, *and* $h = h(x)$. Write down expressions for
   a. $\frac{d}{dx}(f + g + h)$
   b. $\frac{d}{dx} fg$
   c. $\frac{d}{dx} f(g + h)$

2. Suppose $v = v(P, T)$. Write down expressions for
   a. $\frac{\partial}{\partial T}\frac{1}{v}$
   b. $\frac{\partial}{\partial P}\frac{1}{v}$
   c. $\frac{\partial}{\partial T}\frac{\partial v}{\partial P}$
   d. $\frac{\partial}{\partial P}\frac{\partial v}{\partial T}$
   e. $\frac{\partial}{\partial T}\left(\frac{1}{v}\frac{\partial v}{\partial P}\right)$
   f. $\frac{\partial}{\partial P}\left(\frac{1}{v}\frac{\partial v}{\partial T}\right)$

3. (n-dimensional heat kernel) Consider the heat kernel, $h(x,t) = \frac{1}{t^{1/2}} e^{-\frac{x^2}{4t}}$ and compute the derivatives
   a. $\frac{\partial h}{\partial t}$
   b. $\frac{\partial h}{\partial x}$
   c. $\frac{\partial^2 h}{\partial x^2}$
   d. $\frac{\partial^2 h}{\partial x\, \partial t}$
   e. $\frac{\partial^2 h}{\partial t^2}$

   Hint: Be smart about simplifying each expression as you work & look for patterns.

4. Provide as much information as you can about the function, *f*, given that $\nabla f(x, y, z) = (x, y, z)$

5. Systems 1, 2, and 3 are gases with coordinates P_1, V_1, P_2, V_2, and P_3, V_3, respectively. When systems 1 and 3 are in equilibrium

$$P_1V_1 - nbP_1 - P_3V_3 = 0$$

and when systems 2 and 3 are in equilibrium,

$$P_2V_2 - P_3V_3 + \frac{ncP_3V_3}{V_2} = 0$$

where n, c, and b are constants.

a. provide three functions, one for each system, which are equal to one another in thermal equilibrium. Your answer should have one function of only P_1 & V_1, another function of only P_2 *and* V_2, and a third function of only P_3 *and* V_3.

b. What is the relation expressing thermal equilibrium between systems 1 and 2?

6. An approximate equation of state of a real gas at moderate pressure is given by $Pv = RT\left(1 + \frac{b}{v}\right)$ where $b = b(T)$. Show that

a. $\beta = \frac{1}{T}\frac{v+b+Tb'}{v+2b}$ where the thermal expansion coefficient is defined as $\beta = \frac{1}{v}\frac{\partial v}{\partial T}$

b. $\kappa = \frac{1}{P}\frac{1}{1+bRT/Pv^2}$ where isothermal compressibility is defined as $\kappa = -\frac{1}{v}\frac{\partial v}{\partial P}$