

Important constants:

$h = 6.6260755 \cdot 10^{-34} \text{ J s}^{-1}$	Planck's constant
$\hbar = h/(2\pi)$	
$c = 2.99792458 \cdot 10^8 \text{ m s}^{-1}$	speed of light in vacuum
$k = 1.380658 \cdot 10^{-23} \text{ J K}^{-1}$	Boltzmann constant
$N_A = 6.022 \cdot 10^{23} \text{ mol}^{-1}$	Avogadro constant
$\gamma_H = 2.67522 \cdot 10^8 \text{ rad T}^{-1} \text{ s}^{-1}$	proton gyromagnetic ratio
$\gamma_C = 6.7283 \cdot 10^7 \text{ rad T}^{-1} \text{ s}^{-1}$	carbon gyromagnetic ratio

$\omega = 2\pi \nu$ Relation between angular frequency and frequency

Important definitions and relations:

<u>Symbol</u>	<u>description</u>	<u>Relationship to experiment</u>
B_0	main magnetic field (static along z axis)	Field of superconducting magnet
B_1	rotating RF field, in rotating frame along x (or y)	p1 (power level of pulse)
τ_p	length of RF pulse	p1, p2, ...
ν_0, ω_0	precession frequency of spins in laboratory frame	signal that induces voltage in receiver coil
ω_{RF}	frequency of the RF field B_1 . Normally $\omega \approx \omega_0$	sfo1, set with o1p
ω_1, ν_1	rotation rate of spins during pulse about rf field B_1	determines length of pulse
Ω	free precession in rotating frame: $\Omega = \omega_0 - \omega$	Observed NMR signal after mixing with reference frequency

ω_0 depends on main field (precession in laboratory frame)

$$\vec{\omega}_0 = -\gamma \vec{B}_0 \quad \text{or} \quad \nu_0 = \frac{\gamma B_0}{2\pi}$$

ω_1 depends of strength of B_1 field (rotation during pulse)

$$\vec{\omega}_1 = -\gamma \vec{B}_1 \quad \text{or} \quad \nu_1 = \frac{\gamma B_1}{2\pi}$$

flip angle depends on strength and duration of B_1 field

$$\theta = \omega_1 \tau_p = \gamma B_1 \tau_p$$

Change in magnetization **during** pulse B_1 with length $\tau_p = p_1$ along x-axis:

$$\begin{aligned} M_z &= M_0 I_z \cos(\omega_1 \tau_p) \\ M_y &= -M_0 I_y \sin(\omega_1 \tau_p) \\ M_x &= 0 \end{aligned}$$

Free precession and relaxation of a single signal **after** a 90° pulse along x-axis:

$$\begin{aligned} \vec{M}_z(t) &= M_0 \vec{I}_z [1 - e^{-\frac{t}{T_1}}] \\ \vec{M}_y(t) &= -M_0 \vec{I}_y \cos(\Omega t) e^{-\frac{t}{T_2^*}} \\ \vec{M}_x(t) &= M_0 \vec{I}_x \sin(\Omega t) e^{-\frac{t}{T_2^*}} \end{aligned}$$

Energy of spin state:

$$E_m = \gamma \hbar B_0 m_s$$

transition energy of $m_s = +\frac{1}{2}$ to $m_s = -\frac{1}{2}$

$$\Delta E = \gamma \hbar B_0 = \hbar \omega_0$$

excess magnetization for one spin in high temperature approximation

$$\Delta N = N \frac{\gamma \hbar B_0}{2kT}$$

dwell time of receiver

$$DW = 1/(2 v_{\max}) = 1/(2 \cdot SWH)$$

acquisition time

$$AQ = TD \cdot DW = TD / (2 \cdot SWH)$$

width at half height

$$\Delta \nu_{1/2} = \frac{1}{\pi T_2^*}$$