Enveloping Algebras of
Lie Superalgebras: A Survey

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ABSTRACT. We survey some recent results on prime and primitive ideals in enveloping algebras of Lie superalgebras

We denote the enveloping algebra of a Lie superalgebra g by $U(g)$ (see below for definitions). We give a survey of some recent developments, mainly concerning prime (or primitive) ideals in $U(g)$ and the structure of the corresponding factor rings. We work over an algebraically closed field F of characteristic zero throughout and all Lie superalgebras will be finite dimensional over F.

There are two main reasons why these developments are of interest. Firstly much is known about prime ideals in enveloping algebras of Lie algebras (see for example [**BGR**] for the solvable case, and [**Ja**] for the semisimple case), and it is natural to attempt to generalize these results to Lie superalgebras. Secondly, it follows from the PBW Theorem [**Sch, p. 26**], that $S = U(g)$ is a finitely generated free $R = U(g_0)$-module, and there is currently much interest in the study of ring extensions $R \subseteq S$, with R Noetherian and S a finitely generated left and right R-module. Much of the groundwork for the relationship between prime ideals of R and S in this situation has been set by Letzter [**L1-L3**], building on earlier work of Warfield, [**W**].

1. Definitions and General Results

A Lie superalgebra is a $\mathbb{Z}_2$-graded algebra $g = g_0 \oplus g_1$ with a bilinear product $[,] : g \times g \to g$ satisfying

$$[g_\alpha, g_\beta] \subseteq g_{\alpha+\beta} \quad \text{for } \alpha, \ \beta \in \mathbb{Z}_2 \qquad (\mathbb{Z}_2\text{–grading})$$

$$[a, b] = -(-1)^{\alpha\beta}[b, a] \qquad (\text{graded skew-symmetry})$$

$$(-1)^{\gamma\alpha}[a, [b, c]] + (-1)^{\alpha\beta}[b, [c, a]]$$
$$+ (-1)^{\beta\gamma}[c, [a, b]] = 0 \qquad (\text{graded Jacobi identity})$$

for $a \in g_\alpha$, $b \in g_\beta$, $c \in g_\gamma$.

The enveloping algebra of a Lie superalgebra $g = g_0 \oplus g_1$ is defined in a manner analogous to the Lie algebra case. Let $T(g)$ be the tensor algebra on the vector space g and J the two-sided ideal of $T(g)$ generated by the elements

$$a \otimes b - (-1)^{\alpha\beta} b \otimes a - [a, b]$$

with $a \in g_\alpha$, $b \in g_\beta$.

The enveloping algebra of g is defined as $U(g) = T(g)/J$. As in the Lie algebra case the enveloping algebra is a very useful tool for the study of Lie superalgebras and their representations. We refer to [**Sch**] for more details and background.

We let σ be the automorphism of the Lie superalgebra g defined by $\sigma(a_0 + a_1) = a_0 - a_1$ for $a_i \in g_i$. Then σ extends to an automorphism of $U(g)$. An ideal I of $U(g)$ is graded if $\sigma(I) = I$. A graded ideal P is graded prime if $IJ \subset P$ for graded ideals I, J implies $I \subseteq P$ or $J \subseteq P$. We denote by Spec $U(g)$, Gr Spec $U(g)$, Prim $U(g)$, Gr Prim $U(g)$ the spaces of prime, graded prime, primitive and graded primitive ideals of $U(g)$ respectively.

There are a few general results relating these spaces of ideals. A Noetherian ring R is a Jacobson ring if every prime ideal is an intersection of primitive ideals. By [**CS, Theorem 1**] or [**L1, Lemma 2.5**], this property passes to finite extensions. Since $U(g_0)$ is a Jacobson ring [**Dix, Proposition 3.1.15**], it follows that $U(g)$ is also Jacobson.

Concerning the relationship between prime and graded prime (or primitive) ideals we have

$$\text{Gr Spec } U(g) = \{P \cap \sigma(P) | P \in \text{Spec } U(g)\}$$

and

$$\text{Gr Prim } U(g) = \{P \cap \sigma(P) | P \in \text{Prim } U(g)\}.$$

This follows from [**CM, Theorem 6.3**] and [**L1, Theorem 3.1**].

If $R \subseteq S$ is a finite extension of Noetherian rings it is an interesting problem to study the relationship between prime (and primitive) ideals of R and S. When S is commutative we have the classical Krull relations of lying over, going up, etc. In the noncommutative setting we say that lying over (LO) holds if for any prime ideal Q of R there is a prime P of S such that Q is minimal over $P \cap R$. There is an example where LO does not hold, [**HO**]. However, if S is finitely generated and free as either a left or right R-module, and S is an F-algebra of finite GK-dimension, (for example when $R = U(g_0) \subseteq S = U(g)$), Lenagan [**L1, Theorem 2.1**] has shown that LO holds.

When $R \subseteq S$ is a finite normalizing extension results on primitive ideals can be obtained from the facts that $S \otimes_R V$ has finite length for any simple R-module V and that any simple S-module has finite length as an R-module, [**P, Section 2**]. An example of Stafford [**S, Corollary 4.3**] shows that both these properties fail for finite extensions of Noetherian rings in general. Another example of Stafford can easily be turned into a Lie superalgebra example as follows.

If $g_0 = s\ell(2) \times s\ell(2)$, then by [**S, Theorem 4.1**] there exist simple left $R = U(g_0)$-modules V and E with E finite dimensional such that the R-module

$E \otimes V$ has infinite length. Let $g = g_0 \oplus g_1$, with $g_1 = E$ and $[g_1, g_1] = 0$ and $[x, e] = x.e$ for $x \in g_0$, $e \in E$. Let $\wedge = \bigoplus \wedge^i$ be the exterior algebra on E with its natural gradation. Then $U(g) \otimes_R V = \bigoplus (\wedge^i \otimes V)$ as g_0-modules. Furthermore $g_1(\wedge^i \otimes V) \subseteq \wedge^{i+1} \otimes V$, so if we set $W_j = \bigoplus_{i \geq j} \wedge^i \otimes V$, then $U(g) \otimes_R V = W_0 \supseteq W_1 \supseteq W_2 \supseteq \ldots$ is a descending chain of $U(g)$-modules such that g_1 acts trivially on each quotient. Furthermore W_1/W_2 is none other than $E \otimes V$ made into a $U(g)$-module by allowing g_1 to act trivially. Hence $U(g) \otimes_R V$ has infinite length.

In spite of these negative results, it is possible for certain classes of modules to be well behaved with respect to induction and restriction. This is the case for example for the categories $\mathcal{O}$ and $\tilde{\mathcal{O}}$ of $U(g_0)$ and $U(g)$-modules, when g is classical simple, see [M1].

Let $R \subseteq S$ be a finite extension of Noetherian F-algebras of finite GK-dimension over F. Letzter has found a description of the prime and primitive ideals of S in terms of those of R. We describe this result here under the additional assumption that S is a finite free left R-module.

For $Q \in \mathrm{Spec}(R)$, let $J_Q = \mathrm{ann}_S(S/QS)$ and let X_Q be the set of prime ideals of S minimal over J_Q.

THEOREM [**L3, Proposition 4.2**].
  (1) If $P \in \mathrm{Spec}(S)$ and $Q \in \mathrm{ass}_R(S/P)$, then $P \in X_Q$.
  (2) $\mathrm{Spec}(S) = \bigcup_{Q \in \mathrm{Spec}(R)}(X_Q)$.
  (3) $\mathrm{Prim}(S) = \bigcup_{Q \in \mathrm{Prim}(R)}(X_Q)$.

Finally, Letzter has obtained the following characterization of primitive ideals in $U(g)$.

THEOREM [**L1, Theorem 2.6**]. *If P is a prime ideal of $U(g)$ the following are equivalent:*

  (i) *P is right primitive*
  (ii) *P is left primitive*
  (iii) *P is rational*
  (iv) *P is locally closed in* Spec *$U(g)$.*

Here P is rational if the center of the Goldie quotient ring of $U(g)/P$ equals F, and P is locally closed in Spec $U(g)$ if the intersection of all prime ideals property containing P is different from P.

There is a similar characterization of graded primitive ideals [**L1, Theorem 3.2**].

2. The Nilpotent and Solvable Cases

In 1963 Dixmier proved that if g is a nilpotent Lie algebra then any primitive factor ring of $U(g)$ is isomorphic to a Weyl algebra. The Lie superalgebra version of this is the following, [**BM**].

THEOREM. *Let g be a nilpotent Lie superalgebra. If P is a primitive ideal of $U(g)$, then $U(g)/P \cong M_s(A_n)$ a matrix ring over a Weyl algebra. If instead P is*

graded primitive then either $U(g)/P \cong M_s(A_n)$ *or* $U(g)/P \cong M_s(A_n) \times M_s(A_n)$. *Here* $s = 2^m$ *and* $m, n \geq 0$.

If F is not algebraically closed, there is a version of this theorem where the matrix algebras are replaced by Clifford algebras.

Now if g is a solvable Lie algebra, McConnell has given a precise structure theorem for the prime factors of $U(g)$ (after some localisation) see [**McR, Chapter 14**]. We have been unable to extend this to the Lie superalgebra case, although numerous examples suggest that the following is true.

CONJECTURE (Bell-Musson). Let g be a completely solvable Lie superalgebra, and R a graded prime factor ring of $U(g)$. Then for some even eigenvector $e \in R$, we have $R_e \cong A(V, \delta, \Gamma) \otimes_F C(q, K)$. Here $A(V, \delta, \Gamma)$ is one of the algebras described by McConnell and $C(q, K)$ is the Clifford algebra of some nonsingular quadratic form defined over an extension field K of F. We remark that even in the algebraically closed case, it is impossible to avoid the use of Clifford algebras here as the following example shows.

EXAMPLE (Bell-Musson). Let $g = g_0 \oplus g_1$ where $g_0 = Fa$, $g_1 = Fx \oplus Fy$ with a central, $[x, y] = 0$ and $[x, x] = [y, y] = 2a$. Note that g is nilpotent of class two. Then $U(g)$ is a domain with center $F[a]$. If we invert the nonzero elements in $F[a]$, we obtain $U(g) \otimes F(a) = \left(\frac{a, a}{F(a)}\right)$ a quaternion division algebra over $F(a)$, see [**Lam, Theorem III, 2.7**].

We outline a technique that can be used to study enveloping algebras of solvable Lie superalgebras. If g is solvable there is a series

$$0 = g_0 < g_1 < \ldots < g_n = g$$

of graded subalgebras of g with dim $g_i = i$ such that g_i is an ideal in g_{i+1}. In this case we can study $U(g)$ by studying the sequence of ring extensions

$$F = U(g_0) \subset U(g_1) \subset \ldots \subset U(g_n) = U(g).$$

Now if g_{i+1}/g_i is even we can choose an even element $x \in g_{i+1}$ such that $g_{i+1} = g_i \oplus Fx$. Then we have $U(g_{i+1}) \cong U(g_i)[t; \delta]$, an Ore extension where $\delta = adx$ is the derivation of $U(g_i)$ defined by $\delta(a) = [a, x]$ and $at = ta + \delta(a)$ for $a \in U(g_i)$. If instead g_{i+1}/g_i is odd, choose an odd element $x \in g_{i+1}$ such that $g_{i+1} = g_i \oplus Fx$. We then have $U(g_{i+1}) \cong U(g_i)[t; \sigma, \delta]/(t^2 - h)$ with $h = \frac{1}{2}[x, x]$. Here $\delta = adx$ is a σ-derivation (i.e. $\delta(ab) = a\delta(b) + \delta(a)\sigma(b)$ for $a, b \in U(g_i)$) and we have $at = t\sigma(a) + \delta(a)$ in the Ore extension $U(g_i)[t; \sigma, \delta]$.

For g a solvable Lie superalgebra, E. Letzter [**L5**] has constructed a bijective map from Spec $U(g_0)$ to Gr Spec $U(g)$. In the case g is nilpotent this map is in fact a homeomorphism. Since $U(g_0)$ is known to be catenary in this case (any two chains of prime ideals between P and Q have the same length), it follows that $U(g)$ is catenary. Recently T.H. Lenagan [**Len**] has shown that $U(g)$ is catenary for any solvable Lie superalgebra g.

3. The Classical Simple Case

Simple Lie superalgebras were classified by V. Kac in his fundamental paper [K1], see also [Sch]. We say that g is classical simple if g_0 is reductive. The classical simple Lie superalgebras are further subdivided into those which admit an invariant bilinear form, the basic Lie superalgebras, and those that do not. The latter consists of two infinite families, denoted $P(n)$, $Q(n)$ by Kac. Classical simple Lie superalgebras behave in many ways like semisimple Lie algebras. As an example, let $g = s\ell(m,n)$ be the Lie superalgebra of block matrices of the form $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ where A is $m \times m$, D is $n \times n$, $\mathrm{Trace}(A) = \mathrm{Trace}(D)$ and B, C are the appropriate size. Let

$$g_0 = \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix}, \quad g_1 = \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix},$$

and make g into a Lie superalgebra by defining

$$[X, Y] = XY - (-1)^{\xi\eta}YX$$

for $X \in g_\xi$, $Y \in g_\eta$. The Lie superalgebra $g = s\ell(m,n)$ is classical simple if $m, n > 0$ and $m \neq n$.

Now in [M1] we have obtained a classification of the graded primitive ideals in $U(g)$ for g classical simple. We describe this result here for the case $g = s\ell(m,n)$. Let h be the set of diagonal matrices in g and n^+ (resp. n^-) the set of strictly upper (resp. strictly lower) triangular matrices in g. For $\lambda \in h^*$, let Fv_λ be the one dimensional h-module with $hv_\lambda = \lambda(h)v_\lambda$ for $h \in h$, and make Fv_λ into a module over $b = h \oplus n^+$ by setting $n^+v_\lambda = 0$. Then we form the Verma module

$$\tilde{M}(\lambda) = U(g) \otimes_{U(b)} Fv_\lambda.$$

It is easy to see that $\tilde{M}(\lambda)$ has a unique maximal submodule. The corresponding simple factor module is denoted $\tilde{L}(\lambda)$. We show in [M1] that every primitive ideal of $U(g)$ has the form ann $\tilde{L}(\lambda)$ for some $\lambda \in h^*$.

This is the analogue of a theorem of Duflo [D], [Ja, Corollar 7.4] for semisimple Lie algebras. Let $b_0 = g_0 \cap b$ and regard Fv_λ as a b_0-module by restriction. Then $M(\lambda) = U(g_0) \otimes_{U(b_0)} Fv_\lambda$ has a unique simple factor module $L(\lambda)$ and every primitive ideal of $U(g_0)$ has the form $I(\lambda) = $ ann $L(\lambda)$ for some $\lambda \in h^*$.

We remark that when $g = \mathrm{osp}(1,2)$, Pinczon has obtained a classification of primitive ideals in $U(g)$ by using the decomposition of the adjoint representation of g on $U(g)$, [Pi].

An interesting problem is to determine the relationship of primitive ideals to the center $Z(g)$ of $U(g)$. The situation for semisimple Lie algebras is well understood (see section 3 of Borho's survey [Bor]). Part of the difficulty in the Lie superalgebra case is that $Z(g)$ may not be Noetherian. In [K2], Kac constructs a Harish Chandra map $\psi : Z(g) \to S(h)^W$ for basic classical simple g. Here W is the Weyl group of the Lie algebra g_0. For α a root of g we set

$$P_\alpha = \{\lambda \in h^* \mid (\lambda + \rho, \alpha) = 0\}.$$

Here $\rho = \rho_0 - \rho_1$, where ρ_0 (resp. ρ_1) is half the sum of the positive even (resp. odd) roots, and $(\ ,\ )$ is a non degenerate symmetric W-invariant form on h^*. We say that λ is *regular* if $\lambda \notin P_\alpha$ for all roots α and λ is *typical* if $\lambda \notin P_\alpha$ for all isotropic roots α.

Now let $S(h)_0$ be the algebra

$$S(h)_0 = \{f \in S(h) \mid \text{if } \lambda \in P_\alpha \text{ for some isotropic root } \alpha,$$
$$\text{then } f(\lambda) = f(\lambda + t\alpha) \text{ for all } t \in \mathbb{C}\}.$$

Then ψ is injective and the image of ψ is $S(h)_0^W$ [**K4, Theorem 3**].

For example, if $g = s\ell(2,1)$ then $h = \text{span}\{h, z\}$ with $h = \text{diag}(1, -1, 0)$, $z = \text{diag}(1, 1, 2)$. There are two positive isotropic roots β, γ and the ideals of functions vanishing on P_β, P_γ are generated by $(z-h)$ and $(z+h+2)$ respectively. It follows that $S(h)_0 = \mathbb{C} + x\ \mathbb{C}[h, z]$ where $x = (z - h)(z + h + 2)$. The Weyl group $W = \langle \sigma \rangle \cong C_2$ acts on $S(h)$ by $\sigma(h) = -h - 2$, $\sigma(z) = z$. Hence $Z(g) \cong S(h)_0^W = \mathbb{C} + x\ \mathbb{C}[x, z]$ is not Noetherian.

For any $\lambda \in h_0^*$ and $c \in Z(g)$, c acts as a scalar $\chi^\lambda(c)$ on $\tilde{M}(\lambda)$ and we obtain the central character $\chi^\lambda : Z(g) \to \mathbb{C}$.

Now suppose that g belongs to one of the series $s\ell$, osp or Q. If λ is typical Penkov [**Pe, Théorème 3**] shows that $U(g)/\text{Ker}\chi^\lambda U(g)$ may be identified with the ring of global sections of a sheaf of twisted superdifferential operators on a superflag manifold (see [**Ma**] for geometric background). If λ is regular he obtains the stronger result that $U(g)/\text{Ker}\,\chi^\lambda U(g)$ is Morita equivalent to a primitive factor ring of the enveloping algebra of g_0.

This leads us to conjecture that if g is classical simple and $\lambda \in h_0^*$ is typical then ann $\tilde{M}(\lambda)$ is primitive. Furthermore, if λ is regular we conjecture that ann $\tilde{M}(\lambda) = \text{Ker}\ \chi^\lambda U(g)$.

It is worth remarking that when $g = \text{osp}(1,2)$ and λ is typical but not regular then ann $\tilde{M}(\lambda) \neq \text{Ker}\ \chi^\lambda U(g)$. This follows from [**Pi, Section 7**], and can be explained as follows. For any λ, $\tilde{M}(\lambda) = M_0 \oplus M_1$ where M_0, M_1 are Verma modules for $g_0 = s\ell(2)$. The Casimir elements Q and C of $U(g_0)$, $U(g)$ are related by the equation $(16C + 1) = (8Q - (8C - 1))^2$, [**Pi, Proposition 1.2**]. When λ is not regular, C acts on $\tilde{M}(\lambda)$ by the scalar $-\frac{1}{16}$, and Q acts on M_0, M_1 by the same scalar $-\frac{3}{16}$. The image of $Q + \frac{3}{16}$ in $U(g)/\text{Ker}\ \chi^\lambda U(g)$ is nonzero and generates the ideal ann $\tilde{M}(\lambda)/\text{Ker}\ \chi^\lambda U(g)$.

Turning to the structure of the primitive factor rings of $U(g)$ for g classical simple, very little is known in this case. Any such ring A has a Goldie quotient ring which takes the form $M_n(D)$ for some positive integer n known as the Goldie rank and some division ring D. As a first step we can ask for information about n and D.

When g is a semisimple Lie algebra, D turns out to be the quotient division ring of a Weyl algebra in many cases [**Ja, Kapitel 15**]. In [**M2**] we verify that this is also the case for the Lie superalgebra $s\ell(2,1)$.

One of the deepest results in the theory of semisimple Lie algebras asserts that the Goldie rank of $U(g)/I(\lambda)$ for $\lambda \in h^*$ is a polynomial in λ, [**Ja, Satz 12.6**]. If $L(\lambda)$ is finite dimensional the Goldie rank of $U(g)/I(\lambda)$ is equal to the

dimension of $L(\lambda)$ and the result cited above reduces to the well known Weyl dimension formula.

Now Kac, [**K2**] has obtained an analogue of Weyl's character formula for finite dimensional $\tilde{L}(\lambda)$ when λ is typical. Much effort has been made to obtain a corresponding result when λ is atypical, see [**HJKM1**] and [**HJKM2**] and the references given therein. However as far as we know there are still no analogues of Weyl's character formula or dimension formula for λ atypical, so this represents a major obstacle to extending the results on Goldie rank.

4. When is $U(g)$ prime?

For some questions about enveloping algebras of Lie superalgebras the corresponding question about Lie algebras is trivial or easy. It is well known for example that the enveloping algebra of a Lie algebra is a domain. In [**AL**] it is shown that for a Lie superalgebra g, $U(g)$ is a domain if and only if g contains no odd element x with $[x, x] = 0$. One direction here is easy: if $[x, x] = 0$ in g, then $x^2 = 0$ in $U(g)$. The problem of determining when $U(g)$ is prime is unsolved, see also [**Behr**], but we mention here one result of interest, [**Bell**].

THEOREM. *If g is classical simple and $g \neq P(n)$, then $U(g)$ is a prime ring.*

5. Dimensions

Without a doubt the most useful dimension for the study of enveloping algebras is Gel'fand Kirillov dimension, see [**KL**] for background. It is also the easiest to calculate. We have GK dim $U(g) = GK$ dim $U(g_0) = \dim g_0$ for any Lie superalgebra. However other dimensions are of interest, as test problems to see how well we understand the rings or modules involved and in order to see how these dimensions behave under finite ring extensions.

With regard to Krull dimension it would be interesting to know whether K dim$(R) = K$ dim(S) when $R \subseteq S$ is a finite extension of Noetherian rings. For non Noetherian rings the equality does not hold since there is an example of Bergman [**Berg**] where S is a division ring and R is not. Note also that the corresponding equality for (one sided) modules does not hold either, since there is an example of Stafford [**S**] where S has a simple module which has infinite length as an R-module.

Finally, it is shown in [**KKS**] that the injective and finitistic global dimensions of $U(g)$ are equal to $\dim g_0$, for any finite dimensional Lie superalgebra g. The global dimension of $U(g)$ is discussed in [**AL**], [**Behr**] and [**Bø**].

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