P.I. Envelopes of Classical Simple Lie Superalgebras

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Abstract

Let $\mathfrak{g}$ be a classical simple Lie superalgebra. We describe the prime ideals P in the enveloping algebra $U(\mathfrak{g})$ such that $U(\mathfrak{g})/P$ satisfies a polynomial identity. If the factor algebra $U(\mathfrak{g})/P$ is not artinian, then it is an order in a matrix algebra over $K(z)$.

Throughout this paper we work over an algebraically closed field K of characteristic zero. All unadorned tensor products are taken over K. Let $\mathfrak{g}$ be a finite dimensional classical simple Lie superalgebra over K. A factor algebra of the enveloping algebra $U(\mathfrak{g})$ satisfying a polynomial identity is called a P.I. *envelope* of $\mathfrak{g}$. Our aim is to describe all prime P.I. envelopes of $\mathfrak{g}$. If $\mathfrak{g}$ has a nonartinian prime P.I. envelope it is not hard to show that the center of $\mathfrak{g}_0$ must be nonzero (Lemma 1.3). Thus by the classification theorem in [K1], $\mathfrak{g} = s\ell(m,n)$ with $m > n \geq 1$ or $\mathfrak{g} = osp(2,2n)$.

It was shown by Bahturin and Montgomery that when $\mathfrak{g} = s\ell(m,n)$ with $m > n > 1, \mathfrak{g}$ has a nonartinian $P.I.$ envelope. In fact the proof of [BM, Theorem 4.2] shows that this is true also when $\mathfrak{g} = osp(2,2n)$ although these algebras are omitted from the statement of [BM, Theorems 1.5 and 4.2]. If $\mathfrak{g} = s\ell(m,n)$ with $m > n > 1$ or $\mathfrak{g} = osp(2,2n)$, then $\mathfrak{g}_0 = [\mathfrak{g}_0, \mathfrak{g}_0] \oplus Kz$ where z is central in $\mathfrak{g}_0$. Furthermore as a $\mathfrak{g}_0$-module via the adjoint action

$\mathfrak{g}_1 = \mathfrak{g}_1^+ \oplus \mathfrak{g}_1^-$ a direct sum of two simple submodules. If $\mathfrak{p} = \mathfrak{g}_0 \oplus \mathfrak{g}_1^+$ then $U(\mathfrak{g}_0)$ is a homomorphic image of $U(\mathfrak{p})$ and thus any $U(\mathfrak{g}_0)$-module can be regarded as a $U(\mathfrak{p})$-module. Choose a Cartan subalgebra $\mathfrak{h}$ and a system of simple roots for $[\mathfrak{g}_0, \mathfrak{g}_0]$. Let P^+ denote the corresponding set of dominant integral weights. If $\lambda \in P^+$ let L_λ be the finite dimensional simple $[\mathfrak{g}_0, \mathfrak{g}_0]$-module with highest weight λ and set

$$V(\lambda) = U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} (L_\lambda \otimes K[z]).$$

Our main result is as follows

Main Theorem. *Set $P_\lambda = ann_{U(\mathfrak{g})} V(\lambda)$, $n = \dim_K L_\lambda$ and $N = n2^{\dim \mathfrak{g}_1^-}$. Then P_λ is a prime ideal of $U(\mathfrak{g})$ such that $U(\mathfrak{g})/P_\lambda$ is a subring of the matrix algebra $M_N(K[z])$ with Goldie quotient ring $M_N(K(z))$. In particular $U(\mathfrak{g})/P_\lambda$ is a prime P.I. algebra.*

Conversely, if P is a prime ideal in $U(\mathfrak{g})$ such that $U(\mathfrak{g})/P$ is nonartinian and satisfies a polynomial identity then $P = P_\lambda$ for a unique $\lambda \in P^+$.

If $\mathcal{C} = K[z] \backslash \{0\}$ then $\mathcal{C}$ is an Ore set of regular elements in $U(\mathfrak{g})$. A key step in the proof that P_λ is prime is to show that the localized module $V(\lambda)_{\mathcal{C}}$ is in a natural way a $U(\mathfrak{g}) - F$ bimodule where $F = K(z)$, and then a simple Kac module over $U(\mathfrak{g}) \otimes F$. For the converse we use some results of E.S. Letzter concerning prime ideals in finite extensions of Noetherian rings.

1.1 Let R and S be prime Noetherian rings. An $R - S$ bimodule M is a *bond* from R to S if M is finitely generated and torsionfree both as a left R-module and as a right S-module.

Lemma. *Suppose M is a bond from R to S. Then*

(a) R is artinian if and only if S is artinian.

(b) R is a P.I. ring if and only if S is a P.I. ring.

Proof (a) This is [J, Theorem 5.2.9].
(b) This is Remark (2) after [BS, Prop. 2.5]. We give some details for the convenience of the reader. Let $D = Fract\ R$ and $E = Fract\ S$. by [BS, Prop. 2.5] there exists an integer t such that D embeds in the ring of $t \times t$ matrices $M_t(E)$ and E embeds in $M_t(D)$. Thus if S is P.I. then R embeds

in $M_t(E)$ which is a central simple algebra by Posner's Theorem [McR, Theorem 13.6.5], so R is P.I. by the Amitsur-Levitzki Theorem [McR, Corollary 13.3.5]. Similarly if R is P.I. so is S.

1.2 Until the end of section 1.4 suppose that $R \subseteq S$ is an extension of Noetherian K-algebras of finite Gel'fand-Kirillov dimension. Assume that S is finitely generated and free as a right R-module. The following definitions are due to Letzter [L2], [L3].

i) Suppose P is a prime ideal of S and set $B = Fract(S/P)$. Let V_P be the set of prime ideals of R which are right annihilators of simple $B - R$ factor bimodules of B.

ii) Suppose Q is a prime ideal of R and set $A = Fract(R/Q)$. Let W_Q be the set of prime ideals of S which are left annihilators of simple $S - A$ factor bimodules of $S \otimes_R A$.

In addition set $J_Q = \ell - ann(S/SQ)$ and

$$X_Q = \{P \in Spec\ R | P \text{ is minimal over } J_Q\}.$$

These definitions are related by the following results.

Theorem. *(a) If $Q \in SpecR$ and $P \in SpecS$ then*

$$Q \in V_P \text{ if and only if } P \in W_Q$$

Furthermore if this condition holds there is a bond from S/P to R/Q.

(b) $W_Q \subseteq X_Q$.

Proof. (a) follows from [L2, Lemma 3.2] and [L1, Lemma 1.1], while (b) follows from the proof of [L2, Proposition 4.2].

1.3 Lemma. *Let $\mathfrak{g}$ be a classical simple Lie superalgebra such that there is a prime ideal P in $U(\mathfrak{g})$ with $U(\mathfrak{g})/P$ a nonartinian P.I. algebra. Then the center of $\mathfrak{g}_0$ is nonzero.*

Proof. We apply the results in the two previous subsections $R = U(\mathfrak{g}_0)$ and $S = U(\mathfrak{g})$. Choose $Q \in V_P$. Then there is a bond from S/P to R/Q by Theorem 1.2. Hence by Lemma 1.1, R/Q is a nonartinian P.I. algebra. The result follows from a result of Bahturin, see [BM, page 2837].

1.4 If the equivalent conditions of Theorem 1.2(a) hold we say that P *lies directly over* Q. Recall that a module over a prime Noetherian ring is *fully faithful* if every nonzero submodule is faithful. We require another result of Letzter [L3, Lemma 2.6 (iv)].

Lemma. *Suppose that P lies directly over Q and that M is a fully faithful S/P-module. Then there exists an R-submodule N of M such that $Q = ann_R N$ and N is a fully faithful R/Q-module.*

1.5 Again suppose that $\mathfrak{g}$ is classical simple. We often write U for $U(\mathfrak{g})$. The simple artinian factor rings of U correspond to the finite dimensional simple $\mathfrak{g}$-modules and these have been classified [K1, Theorem 8].

For the remainder of this paper we assume therefore that P is a prime ideal of U, such that U/P is a non-artinian P.I. algebra. By Lemma 1.3 this means that $\mathfrak{g}_0$ has nonzero center.

As noted in the introduction we have $\mathfrak{g}_0 = [\mathfrak{g}_0, \mathfrak{g}_0] \oplus Kz$. We can choose z in such a way that $[z, x] = \pm x$ for all $x \in \mathfrak{g}_1^{\pm}$. Recall that $\mathfrak{h}$ is a Cartan subalgebra of $[\mathfrak{g}_0, \mathfrak{g}_0]$. Set $\mathfrak{h}' = \mathfrak{h} \oplus Kz$, so that $\mathfrak{h}'$ is a Cartan subalgebra of $\mathfrak{g}_0$ and $\mathfrak{g}$. Fix a non-degenerate invariant bilinear form $(\ ,\ )$ on $(\mathfrak{h}')^*$. For $\alpha \in (\mathfrak{h}')^*$, we write $\mathfrak{g}^\alpha$ for the corresponding root space. There is a unique $h_\alpha \in \mathfrak{h}'$ such that $(\mu, \alpha) = \mu(h_\alpha)$ for all $\mu \in (\mathfrak{h}')^*$. Let Δ_1^+ be the set of roots of $\mathfrak{g}_1^+$. For $\alpha \in \Delta_1^+$, choose $e_{\pm\alpha}$ such that $\mathfrak{g}_1^{\pm\alpha} = Ke_{\pm\alpha}$ and $h_\alpha = [e_\alpha, e_{-\alpha}]$.

1.6 We construct a functor T between categories of left modules:

$$T : U(\mathfrak{g}_0)\text{-mod} \longrightarrow U(\mathfrak{g})\text{-mod}.$$

First set $\mathfrak{p} = \mathfrak{g}_0 \oplus \mathfrak{g}_1^+$ and $J = U(\mathfrak{p})\mathfrak{g}_1^+$. Then J is a nilpotent ideal of $U(\mathfrak{p})$ with $U(\mathfrak{p})/J \cong U(\mathfrak{g}_0)$. Thus we can regard $U(\mathfrak{g}_0)$-mod as a subcategory of $U(\mathfrak{p})$-mod and define $T(\_) = U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} (\_)$.

We use the functor T to construct some examples of P.I. envelopes of $\mathfrak{g}$. If M is any $K[z]$-module and $\lambda \in P^+$ we regard $L(\lambda, M) = L_\lambda \otimes M$ as a $U(\mathfrak{g}_0)$-module by allowing $[\mathfrak{g}_0, \mathfrak{g}_0]$ (resp. Kz) to act on the first (resp. second) factor of the tensor product. For $a \in K$, let $\mathcal{O}_a = K[z]/(z - a)$ and set

$$L(\lambda) = L(\lambda, K[z]),\ L(\lambda, a) = L(\lambda, \mathcal{O}_a),$$
$$V(\lambda) = T(L(\lambda)),\ V(\lambda, a) = T(L(\lambda, a)).$$

The natural map $K[z] \longrightarrow \mathcal{O}_a$ induces an epimorphism of $U(\mathfrak{g})$-modules

$$V(\lambda) \longrightarrow V(\lambda, a).$$

The pair $\lambda' = (\lambda, a)$ can be viewed as the element of $(\mathfrak{h}')^*$ with $\lambda'|_{\mathfrak{h}} = \lambda$ and $\lambda(z) = a$. The module $V(\lambda, a)$ is called the Kac module with highest weight λ'. By [K2, Proposition 2.9] $V(\lambda, a)$ is a simple $U(\mathfrak{g})$-module if and only if $\lambda' = (\lambda, a)$ satisfies $(\lambda' + \rho, \alpha) \neq 0$ for all odd positive roots α. Here $\rho = \rho_0 - \rho_1$, where ρ_0 (resp. ρ_1) is the half-sum of the positive even (resp. odd) roots. Since $V(\lambda)$ maps onto any Kac module of the form $V(\lambda, a)$ we call $V(\lambda)$ the *universal Kac module with highest weight* $\lambda \in \mathfrak{h}^*$.

1.7 The enveloping algebra U has a $\mathbb{Z}$-grading, $U = \oplus_{n \in \mathbf{Z}} U(n)$ extending the $\mathbb{Z}$-grading on $\mathfrak{g}$ given by $\deg \mathfrak{g}_0 = 0,\ \deg \mathfrak{g}_1^{\pm} = \pm 1$. Henceforth the adjective "graded" refers to this grading. We use this grading to construct a useful localization of U.

Suppose that $M = \oplus M(n)$ is a $\mathbb{Z}$-graded U-module which is torsionfree as a $K[z]$-module. We can make M into a $U - K[z]$-bimodule via the rule

$$mf(z) = f(z - n)m \tag{1}$$

for $m \in M(n), f(z) \in K[z]$. Let $F = K(z)$ and give $M^F = M \otimes_{K[z]} F$ the right F-module obtained by localization. In particular U^F becomes a $U - F$-bimodule in this way and we can extend the algebra structure on U to U^F by

$$(u \otimes f_1(z))(v \otimes f_2(z)) = uv \otimes f_1(z + n) f_2(z)$$

for $u \in U,\ v \in U(n)$ and $f_1, f_2 \in F$. It is now easy to verify the following.

Lemma. *The multiplicative set $\mathcal{C} = K[z] \backslash \{0\}$ is Ore in U and $U_{\mathcal{C}} \cong U^F$ with the above algebra structure. If M is a graded left U-module which is torsionfree as a $K[z]$-module, then $M_{\mathcal{C}} \cong M \otimes_{K[z]} F$ as a $U - F$ bimodule via the map*

$$f(z)^{-1} m \longrightarrow m \otimes f(z + n)^{-1}$$

for $m \in M(n), f(z) \in K[z]$. In addition if N is any graded U-submodule of $M_{\mathcal{C}}$, then N is a $U_{\mathcal{C}}$-submodule if any only if it is a $U - F$ sub-bimodule of $M_{\mathcal{C}}$.

1.8 If V is a vector space over K we write V_F for $V \otimes F$. If A is a K-algebra and M is a left A-module, then A_F is an F-algebra and M_F a A_F-module by extension of scalars.

We apply these remarks to the universal Kac module $V(\lambda) = U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} L(\lambda)$. If $\Lambda = \oplus \Lambda^n$ is the exterior algebra on $\mathfrak{g}_1^-$ then as a left $U(\mathfrak{g}_0)$-module

$$V(\lambda) \cong \oplus \Lambda^n \otimes L(\lambda).$$

By definition $L(\lambda) = L_\lambda \otimes K[z]$, and so $L(\lambda)$ and $V(\lambda)$ are in an obvious way right $K[z]$-modules. Since $\deg \mathfrak{g}_1^- = -1$ the gradings on Λ and U satisfy $\Lambda^n \subseteq U(-n)$. Observe that the $K[z]$-bimodule structure on $V(\lambda)$ satisfies (1) in 1.7.

Note also that $V(\lambda)$ is torsionfree as a left (and right) $K[z]$-module. Thus $V(\lambda)_{\mathcal{C}}$ is a $U - F$ bimodule or equivalently a left U_F-module and we have

$$\begin{aligned} V(\lambda)_{\mathcal{C}} &\cong (U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} L(\lambda)) \otimes_{K[z]} F \\ &\cong U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} (L(\lambda) \otimes_{K[z]} F) \\ &\cong U(\mathfrak{g})_F \otimes_{U(\mathfrak{p})_F} (L(\lambda) \otimes_{K[z]} F). \end{aligned}$$

Next we consider the $U(\mathfrak{p})_F$-module $L(\lambda) \otimes_{K[z]} F$. This is annihilated by $\mathfrak{g}_{1F}^+$ and so is a $U(\mathfrak{g}_0)_F$-module. In fact it is the finite dimensional simple module over this algebra whose highest weight λ' is the unique F-linear map $\mathfrak{h}'_F \longrightarrow F$ such that $\lambda'|_{\mathfrak{h}} = \lambda$ and $\lambda'(z) = z$. Thus $V(\lambda)_{\mathcal{C}}$ is a Kac module over $U(\mathfrak{g}) \otimes F$. In fact we have

Proposition.

(a) The module $V(\lambda)_{\mathcal{C}}$ is a simple Kac module with highest weight λ' over the algebra $U(\mathfrak{g}) \otimes F$.

(b) The module $V(\lambda)_{\mathcal{C}}$ is a simple module over the algebra $U(\mathfrak{g})_{\mathcal{C}}$.

Proof. It remains to show simplicity in both cases.

(a) We extend $(\ , \ )$ to an F-bilinear form $(\ , \ )_F$ on $(\mathfrak{h}')^*_F$. It suffices to show that $(\lambda' + \rho, \alpha)_F \neq 0$ for all odd positive roots α. Since the highest exterior power of $\mathfrak{g}_1^+$ is trivial as a $\mathfrak{g}_0$-module we have $(\rho_1, \beta) = 0$ for all even roots β. Thus $\mathfrak{h}^\perp = K\rho_1$. If α is an odd root, it follows that $(\rho_1, \alpha) = \rho_1(h_\alpha) \neq 0$. Hence $h_\alpha \notin (K\rho_1)^\perp = \mathfrak{h}$, so $h_\alpha - bz \in \mathfrak{h}$ for some nonzero $b \in K$. Therefore

$$(\lambda' + \rho, \alpha)_F = \lambda'(h_\alpha - bz + bz)_F + (\rho, \alpha) = bz + \lambda(h_\alpha - bz) + (\rho, \alpha),$$

which is a linear polynomial in z.
(b) Fix an order on Δ_1^+ and for $I \subseteq \Delta_1^+$ set

$$e_I = \Pi_{\alpha \in I} e_{-\alpha}$$

where the product is taken with respect to this order. Let N be a nonzero $U(\mathfrak{g})_{\mathcal{C}}$-submodule of $V(\lambda)_{\mathcal{C}}$ and suppose

$$n = \sum_I e_I n_I$$

is nonzero with

$$n_I \in L(\lambda) \otimes_{K[z]} F \quad \text{for all} \quad I.$$

Choose m minimal such that $n_I \neq 0$ for some subset I with $|I| = m$. Set $I' = \Delta_1^+ \backslash I$. Then

$$e_{I'} n = \pm e_{\Delta_1^+} n_I$$

is a nonzero homogeneous element of N, so generates a graded submodule. It follows from Lemma 1.7, that $N = V(\lambda)_{\mathcal{C}}$.

Corollary. (a) If N is any nonzero $U(\mathfrak{g})$-submodule of $V(\lambda)$ then N contains a submodule isomorphic to $V(\lambda)$.
(b) $P_\lambda = ann_{U(\mathfrak{g})} V(\lambda)$ is a prime ideal of $U(\mathfrak{g})$

Proof. (a) By the Proposition $N_{\mathcal{C}} = (V(\lambda))_{\mathcal{C}}$. Hence $(L_\lambda \otimes 1) \subseteq N_{\mathcal{C}}$ so $L_\lambda \otimes (f) \subseteq N$ for some nonzero f. The submodule of $V(\lambda)$ generated by $L_\lambda \otimes (f)$ is isomorphic to $T(L(\lambda, (f))) \cong V(\lambda)$.

(b) This follows since any nonzero submodule of $V(\lambda)$ has annihilator P_λ, by part (a).

1.9 Lemma. Identify $L(\lambda)$ with the $U(\mathfrak{p})$-submodule $1 \otimes L(\lambda)$ of $V(\lambda)$ and let $J = U(\mathfrak{p})\mathfrak{g}_1^+$. Then $ann_{V(\lambda)} J = L(\lambda)$.

Proof. We use the same notation as in the proof of Proposition 1.8(b).
For $0 \leq m \leq |\Delta_1^+|$ set

$$V(m) = \oplus_{|I|=m} e_I L(\lambda).$$

Then $V(\lambda) = \oplus_m V(m)$ and $\mathfrak{g}_1^+ V(m) \subseteq V(m-1)$. Therefore it suffices to show that if $m > 0$ then $\mathfrak{g}_1^+(\sum_{|I|=m} e_I w_I) \neq 0$ provided the $w_I \in L(\lambda)$

are not all zero. Now $L(\lambda) = L_\lambda \otimes K[z]$ has a filtration given by setting $deg(L_\lambda \otimes z^n) = n$. Choose I so that w_I has maximum degree in this filtration, choose $\alpha \in I$ and set $H = I \backslash \{\alpha\}$. Note that

$$\deg(h_\alpha w_I) = \deg(w_I) + 1.$$

Using the formula

$$[e_\alpha, ab] = [e_\alpha, a]b \ \pm \ a[e_\alpha, b]$$

for homogeneous $a, b \in U(\mathfrak{g})$ we see that

$$e_\alpha e_I w_I = \pm e_H h_\alpha w_I$$

plus a sum of terms of smaller degree. The result follows easily from this.

1.10 The next result is an easy consequence of the Artin-Wedderburn theorem.

Lemma. Let U be a K-algebra and L a finite dimensional simple U-module. Then for any field extension K' of K we have

$$End_{U \otimes_K K'}(L \otimes_K K') \cong K'.$$

1.11 Lemma. For any $\lambda \in P^+$,

$$End_{U(\mathfrak{g})}(V(\lambda)) \cong K[z].$$

Proof. By the adjoint isomorphism $f \in End_{U(\mathfrak{g})}(V(\lambda))$ is determined by

$$f_1 = f|_{L(\lambda)} \in Hom_{U(\mathfrak{p})}(L(\lambda), V(\lambda)).$$

If J is as in Lemma 1.9, then $f_1(L(\lambda)) \subseteq ann_{V(\lambda)} J = L(\lambda)$, and hence

$$f_1 \in End_{U(\mathfrak{p})}(L_\lambda \otimes K[z]) = End_{U(\mathfrak{g}_0)}(L_\lambda \otimes K[z]) \cong K[z],$$

using Lemma 1.10.

1.12 Suppose that $\mathfrak{k}$ is a semisimple Lie algebra over K and C is a commutative K-algebra. We describe the prime ideals Q of $R = U(\mathfrak{k}) \otimes C$ such that R/Q is P.I. Since C is central in R, $q = Q \cap C$ is prime in C and by replacing R by the factor ring R/Rq we can assume that $Q \cap C = 0$. There is a one-one correspondence between prime ideals Q of R such that $Q \cap C = 0$ and prime ideals of $U(\mathfrak{k}) \otimes Fract(C)$. Thus we may assume that C is a field extension of K. By the argument on page 2837 of [BM] Q is the annihilator of a finite dimensional simple module over $U(\mathfrak{k}) \otimes C$.

To apply this to our reductive algebra $\mathfrak{g}_0$ with center Kz set $\mathfrak{k} = [\mathfrak{g}_0, \mathfrak{g}_0]$, $R = U(\mathfrak{g}_0)$ and $C = K[z]$. If $q \neq 0$ then $R/Rq \cong U(\mathfrak{k})$ and R/Q is artinian. Thus if R/Q is nonartinian, then $q = 0$ and Q corresponds to the annihilator of a finite dimensional simple module over $U(\mathfrak{k}) \otimes Fract(C)$. This gives the following result.

Lemma. Suppose Q is a prime ideal of $U(\mathfrak{g}_0)$ such that $U(\mathfrak{g}_0)/Q$ is a nonartinian P.I. ring. Then for some uniquely determined $\lambda \in P^+, Q = ann_{U(\mathfrak{g}_0)} L(\lambda)$

Proof of the Main Theorem. Suppose $\lambda \in P^+$ and let $n = \dim_K L_\lambda$. By Corollary 1.8 P_λ is a prime ideal in $U(\mathfrak{g})$. Set $U_\lambda = U(\mathfrak{g})/P_\lambda$. Note that $V(\lambda)$ is a torsionfree $K[z]$-module and thus U_λ embeds in $(U_\lambda)_{\mathcal{C}}$. Since $V(\lambda)$ is a $U(\mathfrak{g}) - K[z]$-bimodule which is free of rank $N = n2^{\dim \mathfrak{g}_1^-}$ on the right U_λ embeds in $M_N(K[z])$. This embedding induces an embedding of $(U_\lambda)_{\mathcal{C}}$ into $M_N(F)$ which is surjective since $V(\lambda)_{\mathcal{C}}$ is a simple $(U_\lambda)_{\mathcal{C}}$-module of dimension N over its endomorphism ring F.

Conversely suppose P is a prime ideal of $U(\mathfrak{g})$ with $U(\mathfrak{g})/P$ a nonartinian P.I. ring. We apply the results in 1.1 and 1.2 with $R = U(\mathfrak{p})$ and $S = U(\mathfrak{g})$. If $Q \in V_P$ there is a bond from S/P to R/Q, so R/Q is a nonartinian P.I. ring, by Lemma 1.1. Lemma 1.12 implies that $Q = Q_\lambda$ for some $\lambda \in P^+$. Since $P \in X_Q$, P is minimal over $ann_S(S/SQ)$ which equals $ann_S V(\lambda) = P_\lambda$ by [BGR, Satz 10.4]. As P_λ is prime we get $P = P_\lambda$. To show that λ is uniquely determined by P it suffices to show that $V_{P_\lambda} = \{Q_\lambda\}$. However if $Q' \in V_{P_\lambda}$ then by Lemma 1.4 $Q' = ann_R N$ for some R-submodule N of $V(\lambda)$ which is fully faithful as an R/Q'-submodule. Since $J = U(\mathfrak{p})\mathfrak{g}_1^+$ is nilpotent $J \subseteq Q'$ so using Lemma 1.9 $N \subseteq ann_{V(\lambda)} J = L(\lambda)$. However every nonzero submodule of $L(\lambda)$ has annihilator Q_λ so $Q' = Q_\lambda$ as desired.

References

[BM] Y. Bahturin and S. Montgomery, PI-envelopes of Lie superalgebras, Proc. Amer. Math. Soc., 127 (1999), 2829-2839.

[BGR] W. Borho, P. Gabriel and R. Rentschler, Primideale in Einhüllenden auflösbarer Lie-Algebren, Lecture notes in Mathematics, 357, Springer-Verlag, Berlin, 1973.

[BS] K.A. Brown and S.P. Smith, Bimodules over a solvable algebraic Lie algebra, Quart. J. Math. 142 (1985), 129-139.

[J] A.V. Jategaonkar, Localization in Noetherian Rings, LMS Lecture Notes 98, Cambridge University Press, Cambridge 1986.

[K1] V.G. Kac, Lie Superalgebras, Adv. in Math., 26 (1977), 8-96.

[K2] V.G. Kac, Representations of classical Lie superalgebras, pages 597-626. Differential Geometric Methods in Mathematical Physics II, ed. K. Bleuler et al. Lecture notes in Mathematics, 676, Springer-Verlag, Berlin, 1978.

[L1] E.S. Letzter, Primitive ideals in finite extensions of Noetherian rings, J. London Math. Soc. 39 (1989), 427-435.

[L2] E.S. Letzter, Finite correspondence of spectra in Noetherian ring extensions, Proc. Amer. Math. Soc. 116 (1992), 645-652.

[L3] E.S. Letzter, A bijection of spectra for classical Lie superalgebras of Type I, J. London Math. Soc. 53 (1996), 39-49.

[McR] J.C. McConnell and J.C. Robson, Noncommutative Noetherian rings, Wiley-Interscience, Chichester 1987.