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Strategic Incentives in Teams: Implications of Returns to Scale

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This article demonstrates the critical relationship between the characteristics of the production function and the strategic incentives in a team. Equilibrium effort increases in team size when substitutability is low relative to returns to scale. Effort levels are actually strategic complements when returns to scale exceed the substitutability of members' effort. Moreover, even with equal shares the well-known $\frac{1}{n}$ problem is determined by returns to scale and becomes worse as returns increase. While a target scheme can support the optimal output level as an equilibrium, it does not completely deter free riding. A team member will accommodate shirking by increasing their own effort within a remarkably large "accommodation zone" where the additional effort cost is less than the bonus. This accommodation of shirking by others exists for different returns to scale and even for very low levels of substitutability.

JEL Classification: D2, L23, D62

1. Introduction

Alchian and Demsetz (1972) define team production as situations where the marginal product of an individual's effort depends on the effort profile of others. When output is equally shared and effort cost is entirely privately borne, Holmström (1982) and Kandel and Lazear (1992) emphasize that team production is subject to the "$\frac{1}{n}$" problem. This means the team optimum is n times the Nash equilibrium level since maximizing individual payoff does not internalize the positive externalities generated by effort. Empirical studies have detailed the moral hazard problem inherent in team production in a wide variety of areas, such as medical practices (Newhouse 1973; Gaynor and Gertler 1995), airlines (Knez and Simester 2001), garment production (Hamilton, Nickerson, and Owan 2003), and professional baseball (Gould and Winter 2009). Conventional wisdom is that the $\frac{1}{n}$ problem is a result of equal output shares; however, this article shows that even with equal shares that the $\frac{1}{n}$ problem is determined by returns to scale.

Economies of scale have been recognized as an important factor in team production for some time (Newhouse 1973; Farrell and Scotchmer 1988), but the implications for strategic incentives have not been thoroughly explained. This article utilizes the most general constant elasticity of substitution (CES) production function to show that the $\frac{1}{n}$ problem does not just depend on equal shares of output or substitutability, but rather returns to scale. Increasing

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returns makes the $\frac{1}{n}$ problem much worse than $\frac{1}{n}$. This is the first article to consider the entire range of substitutability, from perfect complements to perfect substitutes, simultaneously with increasing, decreasing, or constant returns to scale. Reaction function slopes show that effort levels are strategic complements when scale dominates substitutability, and strategic substitutes otherwise.

Two recent articles provide relevant departure points for this work. Ray, Baland, and Dagnelie (2007) is the first article to allow the entire range of CES substitutability in team production, but assumes decreasing returns to scale. Their focus is on the relationship between the distribution of output shares and the resulting social surplus (that is, output minus aggregate effort cost). They find that equal shares are optimal when the elasticity of substitution is less than two, but that inequality may be Pareto improving at higher elasticities. Adams (2006) assumes constant returns CES production and equal shares, but considers only the range of substitutability where the elasticity is greater than one (i.e., between Cobb-Douglas and perfect substitutes). Adams (2006) finds that equilibrium effort levels increase in team size when the elasticity of substitution is less than two, but effort is decreasing at higher elasticities.

This article considers varying degrees of both effort substitutability and returns to scale and derives a reaction function slope. Several important results emerge. First, equilibrium effort is increasing in team size when substitutability is low relative to returns to scale (Proposition 1). Proposition 2 shows that effort levels are strategic complements for most commonly assumed production functions, even if the elasticity of substitution is very high. Specifically, effort levels are always strategic complements with increasing returns. Furthermore, only with decreasing returns and an elasticity of substitution greater than one (Cobb-Douglas) can effort levels be strategic substitutes. Gould and Winter (2009), in a framework similar to Holmström and Milgrom (1991), Kremer (1993), and Bose, Pal, and Sappington (2010), obtain strategic complements if the production function is supermodular in probability of success, and efforts are sequentially chosen. By contrast, in this article production is a deterministic function of effort levels that are simultaneously chosen. Proposition 3 shows that even with equal shares the $\frac{1}{n}$ problem is determined by returns to scale and does not depend on substitutability. Nash equilibrium effort is normalized by the team optimum to show that the "effort ratio" is $\frac{1}{n}$ only for constant returns. In fact, the effort ratio is strictly declining in returns to scale, and Proposition 4 shows that the same result holds for output.

These strategic incentives provide the foundation for implementing a target (or forcing) contract from Holmström (1982) where a bonus is paid only if the optimal output level is attained. Target schemes have been criticized (Legros and Matthews 1993; Battaglini 2006; Ray et al. 2007) as essentially requiring the destruction of output if the optimum is not obtained. This threat is not credible since it is vulnerable to renegotiation by the team members, unless there is a third party in the form of a principal. For this reason, much of this literature assumes a balanced budget constraint (all output is allocated to the team members) and focused on other methods to achieve optimality. In addition to unequal shares, some of these approaches are augmenting the objective function with peer pressure or guilt (Kandel and Lazear 1992), adding incentive contracts (Akerlof 1982; Knez and Simester 2001; Adams 2006; Heywood and Jirjahn 2009; Mas and Moretti 2009), or punishments in a repeated game (Che and Yoo 2001; Knez and Simester 2001).

Rather than these approaches, this article assumes that a principal implements a target scheme and some interesting results emerge. With team production the standard linear incentive contract (Holmström and Milgrom 1987) breaks down since only output is observable and effort cost is privately borne. Reducing effort implies a full reduction in effort cost for each team member, but only a fraction of the reduction in benefit; thus a linear contract does not discourage shirking. Not surprisingly, under the target scheme the optimal output level is obtained as an equilibrium since the reduction in benefit is much larger without the bonus for achieving the target. However, the target scheme does not necessarily deter free riding since the optimal output level is supported by a range of equilibria where one team member accommodates shirking (suboptimal effort) by another team member. Accommodation occurs up to the point where the additional effort cost is exactly equal to the bonus. The range of accommodation is normalized by the optimal effort level to define a zone of accommodation as a percentage. This approaches a high of 24% when effort levels are perfect substitutes and there are increasing returns to scale, but the range is remarkably robust for all cases other than perfect complements. Even for an elasticity of substitution as low as 0.1 the accommodation zone is 16% or higher for increasing, decreasing, or constant returns. Given the standard assumption of convex effort cost, team payoff is monotonically decreasing in effort differences. Thus, free riding and suboptimal payoffs can routinely occur in teams that achieve the optimal output level under a target scheme. The target scheme creates a strategic substitute incentive, even in regions where effort levels are strategic complements in the absence of the target scheme.

Three special cases of perfect complements, perfect substitutes, and Cobb-Douglas are presented in section 5. It is well known that the team optimum is a Nash equilibrium for perfect complements (Legros and Matthews 1993; Hvide 2001). In this case the range of accommodation is zero; however, this is not harmful since the target scheme is not needed to obtain the optimum. Simple examples show that both Cobb-Douglas and perfect substitutes have accommodation zones of 20% or more for increasing, decreasing, or constant returns.

The remainder of the article is organized as follows. Section 2 presents the model of team production, derives the Nash equilibrium, and shows how equilibrium effort responds to changes in team size. Section 3 determines the strategic behavior in teams and then relates the $\frac{1}{n}$ problem to returns to scale. Section 4 presents the target scheme and provides some numerical examples. Section 5 investigates the three special cases of perfect substitutes, perfect complements, and Cobb-Douglas. Section 6 concludes and discusses directions for future research.

2. The Model

Team output is determined by the CES production function

$$Q = \left(\sum_i^n x_i^\rho\right)^{\frac{\varepsilon}{\rho}}, \tag{1}$$

where x_i is the (non-negative) effort level chosen by player i. The degree of substitutability depends on the parameter $\rho \leq 1$, $\rho \neq 0$, where the elasticity of substitution is $\frac{1}{1-\rho}$. The special cases of Cobb-Douglas $\rho = 0$ (elasticity of substitution of one), perfect complements $\rho = -\infty$ (zero elasticity of substitution), and perfect substitutes $\rho = 1$ are presented in section 5. The production function is homogenous of degree ε; thus ε relative to one determines returns to scale.

Effort cost is convex and homogenous of degree two, as is commonly assumed in the literature (see Prendergast 1999 for a survey):

$$C(x_i) = \frac{kx_i^2}{2} \ \forall i = 1, \ldots, n. \tag{2}$$

The restriction $\varepsilon \in (0, 2)$ ensures equilibrium effort is a continuous function of the parameters and that an interior equilibrium is obtained for all relevant n and ρ. This amounts to the reasonable assumption that the degree of homogeneity of the production function is strictly less than that of the effort cost function, and therefore team payoff is a concave function. Each team member receives an equal share of output and chooses effort to maximize individual payoff:

$$\max_{x_i} \ \pi_i = \frac{1}{n}\left[x_i^\rho + \sum\nolimits_{j \neq i} x_j^\rho\right]^{\frac{\varepsilon}{\rho}} - \frac{kx_i^2}{2}. \tag{3}$$

This generates the first order condition

$$\left[x_i^\rho + \sum\nolimits_{j \neq i} x_j^\rho\right]^{\frac{\varepsilon-\rho}{\rho}} x_i^{\rho-2} = \frac{nk}{\varepsilon}. \tag{4}$$

The first-order conditions for the other $n - 1$ players imply $x_i = x_j = x$ for all $i, j = [1, \ldots, n]$; thus 4 reduces to

$$[nx^\rho]^{\frac{\varepsilon-\rho}{\rho}} x^{\rho-2} = \frac{nk}{\varepsilon}. \tag{5}$$

The Nash equilibrium effort level is

$$x^* = \left(\frac{\varepsilon n^{\frac{\varepsilon-2\rho}{\rho}}}{k}\right)^{\frac{1}{2-\varepsilon}}. \tag{6}$$

Nash equilibrium output is[1]

$$Q^* = \left(\frac{\varepsilon n^{\frac{2(1-\rho)}{\rho}}}{k}\right)^{\frac{\varepsilon}{2-\varepsilon}}. \tag{7}$$

Equilibrium payoff for each team member is

$$\pi_i^* = \left(\frac{\varepsilon}{k}\right)^{\frac{\varepsilon}{2-\varepsilon}} n^{\frac{2(\varepsilon-2\rho)}{\rho(2-\varepsilon)}}\left[n - \frac{\varepsilon}{2}\right]. \tag{8}$$

The interaction between substitutability and returns to scale determines the influence of team size. The free-rider problem would seem to imply that effort should decrease in team size. However, this is not always the case.

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[1] With constant returns to scale ($\varepsilon = 1$) Adams's (2006) results are obtained: $x^* = \frac{n^{\frac{1-2\rho}{\rho}}}{k}$ and $Q^* = \frac{n^{\frac{2-2\rho}{\rho}}}{k}$. The comparison with Ray et al. (2007) is less straightforward since they assume linear effort cost.

Proposition 1. As the team grows larger, equilibrium effort

(i) increases if $\rho \in \left(0, \frac{\varepsilon}{2}\right)$
(ii) decreases if $\rho < 0$ or $\rho > \frac{\varepsilon}{2}$.

Proof. The derivative of equilibrium effort with respect to team size is

$$\frac{\partial x^*}{\partial n} = \frac{x^*(\varepsilon - 2\rho)}{n\rho(2-\varepsilon)}.$$

The derivative is positive if $\varepsilon > 2\rho$ and $\rho > 0$. The derivative is negative for all $\rho < 0$, and if $\varepsilon < 2\rho$, which requires $\rho > 0$. *QED.*

Effort is decreasing in team size whenever the elasticity of substitution is inelastic ($\rho < 0$), regardless of returns to scale. In this case there is little substitutability, so effort falls in response to a smaller share of output as a result of the increase in team size. However, effort is also falling in team size when the elasticity of substitution is elastic ($\rho > 0$) but returns to scale are sufficiently small $\left(\rho > \frac{\varepsilon}{2}\right)$. In this case it is the relationship between the elasticity of substitution and returns to scale that matters. The incentive to reduce effort is greater when substitutability is high since the other team member can more easily accommodate this reduction and mitigate the reduction in output. However, the returns to scale effect works in the opposite direction. The marginal product of effort is increasing in ε; thus there is a smaller incentive to reduce effort in response to a smaller share of output when ε is large. If substitutability is sufficiently large relative to returns to scale $\left(\rho > \frac{\varepsilon}{2}\right)$, then output is falling in team size as own benefit from effort is not sufficient to dominate the substitutability incentive.

The surprising result is that effort can actually be increasing in team size, despite the fact that each team member receives a smaller share of output while still bearing the full effort cost. Effort is increasing in team size when the elasticity of substitution is greater than one, but not large enough to dominate returns to scale ($\varepsilon > 2\rho$ and $\rho > 0$). In this case the own benefit from effort is large enough to dominate both the smaller share of output and the substitution incentive. When the elasticity of substitution is greater than one, effort can be increasing in team size for any type of returns to scale.

This generalizes Adams (2006) who assumes constant returns ($\varepsilon = 1$), and restricts attention to positive ρ to show that $\rho = \frac{1}{2}$ defines the critical region where equilibrium effort increases in team size when the elasticity of substitution is greater than two. However, it is not an elasticity of two that is important. Rather, the scale effect must be large enough to dominate substitutability. Effort is always decreasing in team size when the elasticity of substitution is less than one, or if substitutability dominates the scale effect ($\varepsilon < 2\rho$). The next section presents the strategic implications of these results.

3. Strategic Incentives

First, a reaction function slope is obtained using the general CES production function. For teams larger than two, it is assumed that each member has a symmetric belief regarding all others' effort. Thus, a member imagines that all others vary effort, but do so by the same amount $x_j = x_k \forall_j, k \neq i$. This belief is obtained from the symmetric first-order conditions of the identical team members. With two-member teams a player considers only the effort of the other player. However, with a team of n players it is the effort profile of the other team members that determines the best response, not just the aggregate effort level of others.[2] While it is not possible to obtain an explicit solution for the reaction function, the sign of the slope can be determined. Define the first-order condition 4 as an implicit function $F(x_1, x_2, \ldots, x_n) = \frac{kn}{\varepsilon}$. Using the implicit function theorem, the best-response slope is

$$\frac{dx_i}{dx_j} = -\frac{\partial F/\partial x_j}{\partial F/\partial x_i}. \tag{9}$$

With symmetric conjectures the slope of player i's best response function is

$$\frac{dx_i}{dx_j} = (n-1)\left(\frac{x_i}{x_j}\right)^{1-\rho}\left[\frac{(\varepsilon-\rho)}{2-\varepsilon+(n-1)(2-\rho)\left(\frac{x_j}{x_i}\right)^{\rho}}\right]. \tag{10}$$

The strategic behavior can be characterized as follows.

Proposition 2. Effort levels in a team with symmetric conjectures are

(i) strategic substitutes if $\varepsilon < \rho$
(ii) strategic complements if $\varepsilon > \rho$.

Proof. Define the denominator as $\Delta \equiv 2-\varepsilon+(n-1)(2-\rho)\left(\frac{x_j}{x_i}\right)^{\rho}$. The parameter upper bounds are $\varepsilon < 2$ and $\rho = 1$; thus $\Delta > 0$. Therefore, the sign of $\frac{dx_i}{dx_j}$ is the sign of $\varepsilon - \rho$. *QED.*

With increasing returns effort levels are always strategic complements, even for the case of perfect substitutes ($\rho = 1$). Effort levels are strategic complements when returns to scale are sufficiently large such that the marginal product of effort dominates the substitutability incentive. Most commonly assumed production functions are strategic complements, that is, when scale dominates substitutability ($\varepsilon > \rho$). It is possible for effort levels to be strategic substitutes only if there are both decreasing returns to scale and an elasticity of substitution greater than one. In this case the substitutability effect dominates the returns to scale effect. The first two propositions generate interesting possibilities. When $\rho < \varepsilon < 2\rho$ equilibrium effort is falling in team size even though effort levels are strategic complements. However, if effort levels are strategic substitutes ($\varepsilon < \rho$), then effort must decrease in team size ($\varepsilon < 2\rho$).

Next, a comparison of the Nash equilibrium and the team optimum illustrates the $\frac{1}{n}$ problem. Each team member receives a $\frac{1}{n}$ share of output; thus in the Nash equilibrium team members do not consider the benefit that others receive from their effort. However, the team optimum internalizes the positive externality from effort and maximizes team payoff

$$\Pi = Q - \sum_{i=1}^{n} \frac{kx_i^2}{2}. \tag{11}$$

[2] This is a more complicated problem than an n-firm homogenous good oligopoly model or a pure public goods model. In these models players choose a best response to an aggregate quantity from all other players, and the distribution is irrelevant. With team production, the best response changes in the effort profile; thus the distribution matters, other than the perfect substitutes case ($\rho = 1$).

The optimal effort level is

$$x^o = \left(\frac{\varepsilon n^{\frac{\varepsilon-\rho}{\rho}}}{k}\right)^{\frac{1}{2-\varepsilon}}. \tag{12}$$

The "effort ratio" is defined as the Nash equilibrium normalized by the team optimum, $\frac{x^*}{x^o}$. The effort ratio equals

$$\frac{x^*}{x^o} = n^{\frac{-1}{2-\varepsilon}} \in (0,1)\ \forall \varepsilon \in (0,2),\ n \geq 2. \tag{13}$$

Even with equal output shares the effort ratio is independent of substitutability and depends only on returns to scale and team size. The limit of the effort ratio as ε approaches 2 from below is 0, and the limit as ε approaches 0 from above is $n^{\frac{-1}{2}}$. This leads to the following result.

Proposition 3. The ratio of Nash to optimal effort is decreasing in returns to scale.

Proof. Define the ratio of Nash to optimal effort as n^γ where $\gamma \equiv \frac{-1}{2-\varepsilon}$. The derivative with respect to ε is $\frac{\partial \gamma}{\partial \varepsilon} = \frac{-1}{(2-\varepsilon)^2} < 0$. *QED.*

If there are constant returns to scale (ε = 1), then the effort ratio is $\frac{1}{n}$ and the team optimum is exactly n times the Nash equilibrium. However, with increasing returns to scale the effort ratio is less than $\frac{1}{n}$, even though each team member receives a $\frac{1}{n}$ share of output. For example, if $\varepsilon = \frac{3}{2}$ then $\frac{x^*}{x^o} = \frac{1}{n^2}$. Increasing returns makes the $\frac{1}{n}$ problem worse than $\frac{1}{n}$. With decreasing returns to scale the effort ratio is greater than $\frac{1}{n}$. The intuition for this result is clear. At the Nash equilibrium individuals are internalizing all of the cost, but only $\frac{1}{n}$ of the output. Effort generates a positive externality that accrues to the other $n-1$ team members. This externality is increasing in ε and the team optimum internalizes all of the externalities, while the Nash equilibrium does not. Put differently, the effort ratio compares two locations on a convex effort cost function that become increasingly farther apart as returns increase.

Output at the team optimum is

$$Q^o = \left(\frac{\varepsilon n^{\frac{2-\rho}{\rho}}}{k}\right)^{\frac{\varepsilon}{2-\varepsilon}}. \tag{14}$$

Similarly, the output ratio is

$$\frac{Q^*}{Q^o} = n^{\frac{-\varepsilon}{2-\varepsilon}} \in (0,1) \forall \in (0,2),\ n \geq 2. \tag{15}$$

The limit of the output ratio as ε approaches 2 from below is 0, and the limit as ε approaches 0 from above is 1.

Proposition 4. The ratio of Nash to optimal output is decreasing in returns to scale.

Proof. Define the ratio of Nash to optimal effort as n^ϕ where $\phi \equiv \frac{-\varepsilon}{2-\varepsilon}$. The derivative with respect to ε is $\frac{\partial \phi}{\partial \varepsilon} = \frac{-2}{(2-\varepsilon)^2} < 0$. *QED.*

Table 1. $n = 2$ Nash to Optimal

	$\frac{x_i^*}{x_i^o}$	$\frac{Q^*}{Q^o}$	$\frac{\pi_i^*}{\pi_i^o}$
$\varepsilon = 0.5$	0.63	0.79	0.93
$\varepsilon = 1$	0.50	0.50	0.75
$\varepsilon = 1.5$	0.25	0.13	0.31

Again, the output ratio is exactly $\frac{1}{n}$ with constant returns and decreases in ε. Payoff for each team member at the optimum is

$$\pi_i^o = \left(\frac{\varepsilon}{k}\right)^{\frac{\varepsilon}{2-\varepsilon}} n^{\frac{2(\varepsilon-\rho)}{\rho(2-\varepsilon)}} \left[1-\frac{\varepsilon}{2}\right] > 0 \ \forall \varepsilon \in (0, 2). \tag{16}$$

Finally, the (individual and team) payoff ratio is

$$\frac{\pi_i^*}{\pi_i^o} = \frac{2n-\varepsilon}{n^{\frac{2}{2-\varepsilon}}[2-\varepsilon]} \in (0,1) \ \forall \varepsilon \in (0,2),\ n \geq 2. \tag{17}$$

As with the output ratio, the limit of the payoff ratio as ε approaches 2 from below is 0 and the limit as ε approaches 0 from above is 1. Thus, $\frac{\pi_i^*}{\pi_i^o} \in (0,1) \forall \varepsilon \in (0, 2),\ n \geq 2$. The output and payoff ratios have the same relationship with returns to scale as effort. Table 1 illustrates these ratios for a two-member team. The output ratio is more sensitive to returns than the effort ratio. Additionally, the payoff ratio rapidly approaches one as returns to scale decrease. Thus, an important conclusion is that the incentive problems in teams become much worse as returns to scale increase, even though effort levels are strategic complements. The next section details how a target scheme changes the strategic incentives.

4. Contracts in Teams

This section considers a contract that specifies a fixed wage plus a bonus. The contract is a target (or forcing) scheme from Holmström (1982) where the bonus is paid only when the optimal output level is obtained. The fixed wage is an equal share of the Nash equilibrium output. The bonus awards each team member an equal share of the difference between the optimal and Nash equilibrium output levels. The bonus generates a discrete jump in payoffs that can deter shirking. If shirking means losing the bonus, then a team member will internalize the cost from shirking. However, this section shows that not all shirking will necessarily be eliminated by a target scheme. One team member will accommodate some degree of shirking by another team member if the bonus is greater than the additional effort cost necessary to obtain the bonus.

With a target scheme there is a set of equilibrium effort profiles where the target is met. There is a zone of accommodation where one worker will exert effort beyond the optimum and tolerate suboptimal effort (shirking) by the other worker since the additional effort cost is less than the bonus. An accommodation rule equates the bonus with the additional effort cost and determines the upper and lower bounds on effort defining the equilibrium set. Team effort cost is increasing in asymmetry for a given level of output; thus this strategic shirking reduces team payoff. To facilitate the presentation, the slope of the marginal effort cost function k is normalized to one and two-member teams are considered in this section. The results that follow generalize to larger teams and different effort cost slopes.

It is well known that a target scheme can make the optimal effort choice an equilibrium. However, the following shows that the target scheme is still vulnerable to free riding. Under a target scheme the bonus is paid only when the output target Q^o is obtained. If both team members choose the optimal effort level 12, then the optimal result is obtained. However, the optimal effort level x^o is just one element from the set of Nash equilibria that generate Q^o, and the bonus is achieved. With team production, a peer will have an incentive to compensate for another's lack of effort when the bonus is sufficiently large. If the target is met, then the entire value of output is distributed among the two team members; thus the bonus b is half the difference in output between the Nash and optimal levels, $b(\varepsilon,\rho)=\frac{1}{2}[Q^o-Q^*]$. Proposition 4 implies the bonus is strictly positive since $\frac{Q^*}{Q^o}\in(0,1)\forall\varepsilon\in(0,2)$, $n\geq 2$. The contract under the target scheme is

$$w_i=\left\{\begin{array}{l}0 \text{ if } Q<Q^* \\ \dfrac{Q^*}{2} \text{ if } Q\in[Q^*,Q^o) \\ \dfrac{Q^o}{2} \text{ if } Q\geq Q^o\end{array}\right\}. \tag{18}$$

Using equations 14 and 7 the bonus simplifies to

$$b(\varepsilon,\rho)=\varepsilon^{\frac{\varepsilon}{2-\varepsilon}}2^{\frac{2(\varepsilon-\rho)-\varepsilon\rho}{\rho(2-\varepsilon)}}\left[2^{\frac{\varepsilon}{2-\varepsilon}}-1\right]. \tag{19}$$

The bonus creates an additional strategic incentive. In the absence of contract, Proposition 2 states that effort levels are strategic complements when $\varepsilon>\rho$. However, with a bonus for achieving the productivity target Q^o, effort levels may become strategic substitutes even if $\varepsilon>\rho$. A team member has a strategic incentive to exert higher than optimal effort, accommodating the shirking by the other team member.

Define the upper bound on effort as $\bar{x}$ and the minimum effort that will be tolerated in equilibrium as $\underline{x}$ where $\underline{x}<x^o<\bar{x}$.[3] This generates an accommodation rule and the range of effort levels where the bonus is obtained. The accommodation rule is obtained by inverting the production function and solving for the effort level that will result in the target Q^o, as a function of the other team member's effort. From the production function 1 and the team optimum 14, setting $Q=Q^o$ results in

$$\left[x_1^\rho+x_2^\rho\right]^{\frac{\varepsilon}{\rho}}=\varepsilon^{\frac{\varepsilon}{2-\varepsilon}}2^{\frac{\varepsilon(2-\rho)}{\rho(2-\varepsilon)}}. \tag{20}$$

Equation 20 can be inverted to determine the effort level for team member i as a function of the other team member j's effort that is required to obtain the optimal output level. This result is the accommodation rule 21 consistent with obtaining optimal output:

$$x_i=\left[\varepsilon^{\frac{\rho}{2-\varepsilon}}2^{\frac{2-\rho}{2-\varepsilon}}-x_j^\rho\right]^{\frac{1}{\rho}}. \tag{21}$$

[3] The one exception is the case of perfect complements, in section 5, where they are equal ($\underline{x}=x^o=\bar{x}$).

Table 2. Target Scheme Contracts for $n = 2$

	Effort	Output	Payoff	Bonus	Range	Accommodation %
$\rho = 0.5$, elasticity of substitution is 2						
	(x^*, x^o)	(Q^*, Q^o)	(π^*, π^o)	b	$(\underline{x}, \bar{x})$	$\frac{\bar{x}-\underline{x}}{x^o}$
$\varepsilon = 0.5$	(0.40, 0.63)	(1.26, 1.60)	(0.55, 0.60)	0.16	(0.57, 0.70)	21%
$\varepsilon = 1$	(1.00, 2.00)	(4.00, 8.00)	(1.50, 2.00)	2.00	(1.78, 2.24)	23%
$\varepsilon = 1.5$	(9.00, 36.0)	(216, 1728)	(67.5, 216)	756	(32.3, 39.9)	21%
$\rho = -2$, elasticity of substitution is 0.33						
	(x_i^*, x_i^o)	(Q^*, Q^o)	(π^*, π^o)	b	$(\underline{x}, \bar{x})$	$\frac{\bar{x}-\underline{x}}{x^o}$
$\varepsilon = 0.5$	(0.22, 0.35)	(0.40, 0.50)	(0.17, 0.19)	0.05	(0.33, 0.39)	19%
$\varepsilon = 1$	(0.18, 0.35)	(0.13, 0.25)	(0.05, 0.06)	0.06	(0.32, 0.40)	21%
$\varepsilon = 1.5$	(0.05, 0.20)	(0.007, 0.05)	(0.002, 0.007)	0.02	(0.18, 0.22)	19%
$\rho = -9$, elasticity of substitution is 0.1						
	(x_i^*, x_i^o)	(Q^*, Q^o)	(π^*, π^o)	b	$(\underline{x}, \bar{x})$	$\frac{\bar{x}-\underline{x}}{x^o}$
$\varepsilon = 0.5$	(0.24, 0.39)	(0.48, 0.60)	(0.21, 0.22)	0.06	(0.37, 0.43)	16%
$\varepsilon = 1$	(0.23, 0.46)	(0.21, 0.43)	(0.08, 0.11)	0.11	(0.44, 0.52)	17%
$\varepsilon = 1.5$	(0.11, 0.45)	(0.03, 0.27)	(0.01, 0.03)	0.12	(0.42, 0.50)	16%

Clearly the optimal effort x^o is obtained if team member j chooses x^o. However, if player j shirks and chooses $x_j < x^o$, then player i can accommodate this by choosing effort according 21, where $x_i > x^o$. Team member i will find it a best response to accommodate shirking by j when the bonus exceeds the additional effort cost from shirking. Exerting effort beyond the Nash equilibrium x^* is optimal when the additional effort cost, from 2, does not exceed the bonus $b(\varepsilon, \rho)$. However, this does not mean that both team members necessarily choose the optimum, as Proposition 5 makes clear.

Proposition 5. A target scheme does not fully eliminate the incentive to shirk.

Proof. Shirking is defined as any effort level below the optimum $x_j < x^o$. Suppose one team member chooses $x_j < x^o$. The other team member can choose to accommodate, i.e., $x_i > x^o$, according to (21) such that Q^o is realized and the bonus is obtained. Alternatively, if team member i does not accommodate the Nash equilibrium payoff is obtained. Effort cost is $\frac{x_i^2}{2}$; thus marginal effort cost is x. The additional effort cost necessary to achieve the bonus is strictly less than the bonus $\int_{x^*}^{x^o} x\,dx < b(\varepsilon,\rho)$; thus there is a higher payoff for a (limited) degree of accommodation than for not accommodating. The additional effort cost is $\int_{x^*}^{x^o} x\,dx = \frac{1}{2}\left[(x^o)^2 - (x^*)^2\right]$. The bonus is $b(\varepsilon,\rho) = \frac{1}{2}[Q^o - Q^*]$, and therefore the payoff difference between the Nash equilibrium and the optimum is $\pi_i^o - \pi_i^* = b(\varepsilon,\rho) - \int_{x^*}^{x^o} x\,dx > 0$ by Equation 17. *QED.*

Consider an example from Table 2 where $\varepsilon = 1$ and $\rho = \frac{1}{2}$. This results in effort and output levels $x^* = 1$, $x^o = 2$, $Q^* = 4$, and $Q^o = 8$. At the Nash equilibrium the additional effort cost for each team member is $\int_{x^*}^{x^o} x\,dx = \frac{3}{2}$ and the bonus for achieving the optimal output target is $\frac{1}{2}[Q^o - Q^*] = 2$. There is still a surplus of $\frac{1}{2}$ that generates the incentive to shirk. At the optimum a team member has an incentive to accommodate some shirking until the additional effort cost (beyond the optimal level) exceeds the bonus. Next, we turn to the upper bound on the level of accommodation and the maximum level of shirking that will be tolerated.

Integrating the marginal effort cost from the Nash equilibrium identifies the upper bound on effort $\bar{x}$ as the solution to

$$\int_{x^*}^{\bar{x}} x\,dx = b(\varepsilon, \rho). \tag{22}$$

The upper bound $\bar{x}$ is

$$\bar{x} = \left[(x^*)^2 + 2b\right]^{\frac{1}{2}} \tag{23}$$

and the lower bound $\underline{x}$ is obtained via the accommodation rule in 21. Effort cost is convex; thus team payoff is decreasing in effort differences for a given level of output. At $\bar{x}$, the maximum level of accommodation, payoff is equal to the Nash level π^* in 8. The payoff at the minimum effort level that is tolerated, $\underline{x}$, exceeds the optimal level π^o in 16. Shirking still pays for the individual, at the expense of the team, even though the target scheme obtains the optimal output level.

The range of accommodation, $\bar{x} - \underline{x}$, varies in both returns to scale and substitutability. For this reason, the range is normalized by the optimal effort level x^o to define an accommodation zone as a percentage: $\frac{\bar{x}-\underline{x}}{x^o}$. Table 2 illustrates how the bonus and the degree of accommodation respond to substitutability and returns to scale. In Table 2 effort levels are strategic complements, other than the $\rho = \varepsilon = 0.5$ example where the reaction functions are orthogonal. The next section illustrates the special cases, including a strategic substitutes example. The table shows that the zone of accommodation is highest when effort levels are highly substitutable. With increasing returns ($\varepsilon = 1.5$) and an elasticity of substitution of 2, the bonus is very large, but the zone of accommodation is smaller than with constant returns ($\varepsilon = 1.0$). The reason is the convex effort cost. With increasing returns optimal effort is 36, while Nash effort is only 9. Thus moving even further up the convex effort cost function is very costly and beyond the maximum level of accommodation 39.9 payoff is higher at the Nash equilibrium.

By contrast, with decreasing returns the additional output from the optimal effort is small; thus the bonus is small. However, the additional effort cost is also small since the difference between Nash and optimal effort is small; therefore a high degree of accommodation (21%) can be supported as a Nash equilibrium under the target scheme. This pattern holds even as the elasticity of substitution decreases to 0.33 and 0.1. It is surprising that such a high degree of accommodation (16–17%) can be supported, even as effort levels approach perfect complements. The next section shows that the degree of accommodation is zero only when the elasticity of substitution is zero (perfect complements), but Table 2 shows that even a very small degree of substitutability generates a robust degree of accommodation.

Figure 1 plots the accommodation percentage $\frac{\bar{x}-\underline{x}}{x^o}$ for $\varepsilon \in (0, 2)$ and $\rho \in (-4, 0), (0, 1)$. The accommodation percentage is highest with slightly increasing returns and high elasticity of substitution.

5. Three Special Cases

The previous sections have considered the strategic incentives for a general CES production function. There are three special cases of CES that are commonly considered. These

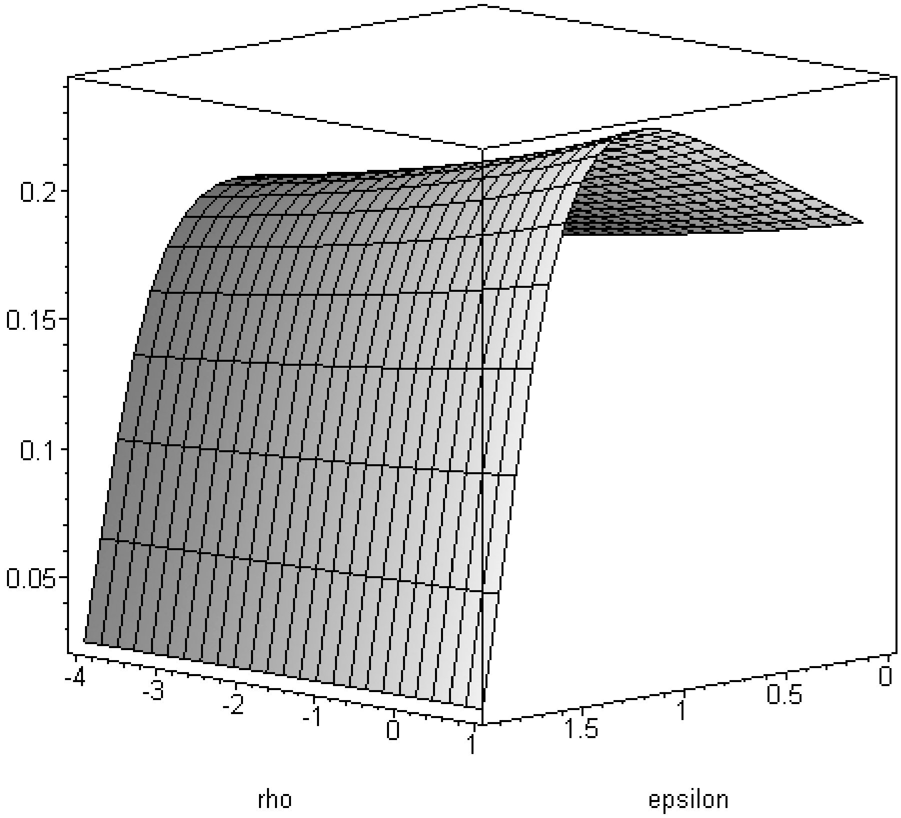

Figure 1. Accommodation Percentage

are perfect complements (limit as $\rho \to -\infty$), perfect substitutes ($\rho = 1$), and Cobb-Douglas (limit as $\rho \to 0$). These special cases are briefly presented for comparison with the results from the previous sections. The results differ only for perfect complements where no degree of accommodation exists since the full cost of shirking is internalized.

In the limit as $\rho \to 0$ the CES production function becomes Cobb-Douglas: $Q = \Pi_{i=1}^{n}\left[x_i^{\frac{\varepsilon}{n}}\right]$. The Nash equilibrium effort level is

$$x^* = \left[\frac{\varepsilon}{kn^2}\right]^{\frac{1}{2-\varepsilon}}. \tag{24}$$

Output at the Nash equilibrium is $Q^* = (x^*)^{\varepsilon}$, or

$$Q^* = \left[\frac{\varepsilon}{kn^2}\right]^{\frac{\varepsilon}{2-\varepsilon}}. \tag{25}$$

The team optimum maximizes output minus the sum of effort cost $Q - \frac{k}{2}\sum_{i=1}^{n} x_i^2$. The optimal effort for Cobb-Douglas production is

$$x^o = \left[\frac{\varepsilon}{kn}\right]^{\frac{1}{2-\varepsilon}} \tag{26}$$

and optimal output is

$$Q^o = \left[\frac{\varepsilon}{kn}\right]^{\frac{\varepsilon}{2-\varepsilon}}. \tag{27}$$

Note that while both output and effort differ from the general case in sections 2 and 3, the effort and output ratios are the same as Equations 13 and 15.

Perfect complements is obtained in the limit as $\rho \to -\infty$. The production becomes $Q = [\min\{x_i\}]^{\varepsilon}$, referred to as "O-ring" production in Kremer (1993). The best response for each player is the smallest effort level among the remaining team members. The reaction functions are rays from the origin with slope 1; thus any $x_i = x_j \forall i, j = [1, \ldots, n]$ is a Nash equilibrium. However, there is a unique effort level x^* from this set which payoff dominates all other Nash equilibria:

$$x^* = \left[\frac{\varepsilon}{kn}\right]^{\frac{1}{2-\varepsilon}}. \tag{28}$$

The team optimum is the solution to maximizing: $[\min\{x_i\}]^{\varepsilon} - \frac{k}{2}\sum_{i=1}^{n} x_i^2$. Obviously, effort levels are equated at the optimum so $x_i = x \; \forall i = [1, \ldots, n]$, and this problem is equivalent to maximizing $x^{\varepsilon} - \frac{knx^2}{2}$. The first-order condition is

$$\varepsilon x^{\varepsilon - 1} - knx = 0. \tag{29}$$

Clearly the optimum x^o is identical to the payoff dominant Nash equilibrium $x^* = x^o = \left[\frac{\varepsilon}{kn}\right]^{\frac{1}{2-\varepsilon}}$. With perfect complements no target scheme is needed. As is well known in the literature, all team members can coordinate on the highest payoff Nash equilibrium, which is the team optimum (Legros and Matthews 1993; Hvide 2001; Ray et al. 2007). Indeed, no shirking is tolerated in equilibrium since output is determined by the minimum effort level. Only with perfect complements is the cost of shirking fully internalized, and thus there is no incentive to shirk. This result holds regardless of team size and for any degree of returns to scale.

When team members are perfect substitutes, $\rho = 1$, the production function becomes $Q = \left[\sum_{i=1}^{n} x_i\right]^{\varepsilon}$, and all the results from the previous sections are valid. Table 3 provides some examples for the three special cases. Again, for clarity, the marginal effort cost slope k is normalized to one, and two-member teams are considered, but again the results easily generalize to larger teams and different slopes. As Figure 1 illustrates, the accommodation percentage is greatest with perfect substitutes and slightly increasing returns.

6. Conclusion

This article has shown the importance of returns to scale in team production. A reaction function slope is obtained from a general CES production function that allows for the entire range of substitutability and any degree of returns to scale. Equilibrium effort is increasing in team size, and effort levels are strategic complements when returns to scale dominate substitutability. Effort levels are always strategic complements when there are increasing returns to scale. The $\frac{1}{n}$ problem inherent in team production is not just a result of equal output shares, but rather is also determined by returns to scale. Increasing returns makes the $\frac{1}{n}$ problem worse than $\frac{1}{n}$ since the optimum internalizes the positive externality generated by effort, and the Nash equilibrium does not.

Table 3. Target Scheme Contracts for $n = 2$ Special Cases

	Effort	Output	Payoff	Bonus	Range	Accommodation %
$\rho = 1$, perfect substitutes						
	(x^*, x^o)	(Q^*, Q^o)	(π^*, π^o)	b	$(\underline{x}, \bar{x})$	$\frac{\bar{x}-\underline{x}}{x^o}$
$\varepsilon = 0.5$	(0.31, 0.50)	(0.79, 1.00)	(0.35, 0.38)	0.10	(0.45, 0.55)	21%
$\varepsilon = 1$	(0.50, 1.00)	(1.00, 2.00)	(0.38, 0.50)	0.50	(0.88, 1.12)	24%
$\varepsilon = 1.5$	(1.13, 4.50)	(3.38, 27.0)	(1.05, 3.38)	11.8	(4.01, 4.99)	22%
$\rho = 0$, Cobb-Douglas						
	(x_i^*, x_i^o)	(Q^*, Q^o)	(π^*, π^o)	b	$(\underline{x}, \bar{x})$	$\frac{\bar{x}-\underline{x}}{x^o}$
$\varepsilon = 0.5$	(0.25, 0.40)	(0.50, 0.63)	(0.22, 0.24)	0.07	(0.36, 0.44)	20%
$\varepsilon = 1$	(0.25, 0.50)	(0.25, 0.50)	(0.09, 0.13)	0.13	(0.45, 0.56)	22%
$\varepsilon = 1.5$	(0.14, 0.56)	(0.05, 0.42)	(0.02, 0.05)	0.18	(0.51, 0.62)	21%
$\rho = -\infty$, perfect complements						
	$x_i^* = x_i^o$	$(Q^* = Q^o)$	$(\pi^* = \pi^o)$	b	$(\underline{x} = \bar{x} = x^o)$	$\frac{\bar{x}-\underline{x}}{x^o}$
$\varepsilon = 0.5$	0.40	0.63	0.24	0	0.40	0
$\varepsilon = 1$	0.50	0.50	0.13	0	0.50	0
$\varepsilon = 1.5$	0.56	0.42	0.05	0	0.56	0

The optimum output level can obtained by a target scheme that pays a bonus when the target is met. However, the target scheme does not eliminate the incentive to shirk. One team member has an incentive to accommodate shirking by another when the bonus exceeds the additional effort cost. Thus, there exists a range of Nash equilibrium effort levels where the output target is met. The accommodation zone approaches 24% of the optimal effort level with very high elasticity of substitution and slightly increasing returns. Surprisingly, a robust range is still obtained even when the elasticity of substitution is very small and for all types of returns to scale.

While this article has examined the production function in detail, future research could extend these results by examining the role of asymmetric shares of output. Differences in productivity across team members would also justify asymmetric distributions of team output. In addition, it may be possible to exploit the strategic complementarity through sequential effort choices. With strategic complements, a leader that chooses high effort will induce followers to do the same, benefiting the entire team. Finally, adding a stochastic component to output may show that the strategic incentives change with the realization of the productivity shock.

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