Southern Economic Journal 2008, 74(3), 879–893

Leading and Merging: Convex Costs, Stackelberg, and the Merger Paradox

John S. Heywood* and Matthew McGinty†

This paper examines the consequences of a Stackelberg leader merging with followers when costs are convex. Such mergers are always profitable for the participants, and the followers often do better merging than remaining excluded rivals. This resolution of the merger paradox cannot be generated either by Stackelberg leadership without convex costs or by convex costs without leadership. In addition, with convex costs, a merger with the leader can actually harm excluded rivals (suggesting why they might object to the merger) and increase social welfare.

JEL Classification: L12, L13

1. Introduction

In the canonical model of the merger paradox, two firms with linear costs never have an incentive to merge as long as there remains even a single rival (Salant, Switzer, and Reynolds 1983). The reduction in quantity by the newly merged firm is outweighed by the combination of a loss of "a seat at the table" and the increase in quantity by excluded rivals such that the merger cannot be profitable. Perry and Porter (1985) show that with sufficiently convex costs, two firms can profitably merge. Yet, Heywood and McGinty (2007) emphasize that even when such a merger is profitable, the profit gained by the excluded rivals exceeds that of the merger participants. Thus, although the introduction of convex costs provides an incentive for merger, it does not eliminate the free-rider aspect of the merger paradox: namely, each firm would rather have other firms merge than do so itself.

Attempts to further resolve the paradox have moved in a wide variety of directions, but we pick up the threads of those following Daughety (1990), who examined models of Stackelberg leadership.[1] Thus, Huck, Konrad, and Muller (2001) examine firms with linear costs, showing that if a leader merges with a follower, the merged firm earns more profit than its two premerger component firms. Feltovich (2001) also produces this result and shows that such a merger results in a decrease in total welfare. These results have sufficient currency that they have found their way into current textbooks (Pepall, Richards, and Norman 2003). Yet, as is

* Department of Economics, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA; E-mail heywood@uwm.edu; corresponding author.

† Department of Economics, University of Wisconsin-Milwaukee, P.O. Box 413, Milwaukee, WI 53201, USA; E-mail mmcginty@uwm.edu.

The authors thank both reviewers and the editor for helpful suggestions that improved the paper.

Received June 2006; accepted March 2007.

[1] Other attempts to resolve the paradox include adopting Bertrand competition (Deneckere and Davidson 1985), moving the examination to spatial models (Reitzes and Levy 1995; Heywood, Monaco, and Rothschild 2001), and considering merged firms that sequence output decisions across plants (Creane and Davidson 2004; Huck, Konrad, and Muller 2004).

the case with convex costs but without leadership, the gain to remaining an excluded follower exceeds the gain from merging with the leader. Thus, the free-rider portion of the merger paradox remains stubbornly intact.

We combine for the first time the assumption of convex costs from Perry and Porter (1985) with that of Stackelberg leadership. We show that for most market structures, there is a wide range of convexity such that a merger between the leader and a follower increases profit and causes the gain to participating in the merger to exceed that of remaining an excluded follower. Thus, in comparison with the canonical model, the two firms can profitably merge, and there exists no free-rider incentive that might otherwise stop them from merging. Interestingly, there also exists a range not only in which the free-rider problem vanishes, as the profit gain from merging exceeds that from being excluded, but also a range in which the excluded firms actually suffer reduced profit because of the merger and the resulting lower price. Farrell and Shapiro (1990, p. 112) emphasize that such a result can be generated only if a merger yields cost efficiencies (synergies) and point out that such efficiencies do not exist in the case of identical firms with convex costs. Two Cournot competitors with convex costs produce equal quantities and, once merged, cannot produce their premerger output at any lower cost. Yet, a Stackelberg leader and a follower with convex costs produce very different quantities in equilibrium and, once merged, can produce their premerger quantity at lower cost by equalizing production between the two plants. Indeed, the resulting harm to the excluded firms might well result in their entreaties that antitrust officials investigate the competitive consequences of the merger. As White (1988) points out, excluded rivals are the most common source of antitrust complaints regarding mergers. In sum, the combination of convex costs and Stackelberg leadership provides a series of outcomes that help resolve important parts of the merger paradox.

Our emphasis on convex costs sets us apart from Huck, Konrad, and Muller (2004) and Creane and Davidson (2004), who examine the possibility of Stackelberg leadership among plants but within the firm. Both begin with a standard simultaneous move oligopoly but assume that after the merger, the merged firm can sequence the output decisions of its constituent parts. Although somewhat in the vein of Daughety (1990), in that the merger changes the ability to commit, these models allow a resolution of the merger paradox that retains linear costs and allows excluded rivals to be hurt. Importantly, our introduction of convex costs dramatically limits the profitability of sequencing output across plants within the firm. The differing output levels across plants that result from the internal sequencing generate a cost inefficiency with convex costs that is absent with linear costs. Thus, to focus on the importance of convex costs, we ignore the possibility of internal sequencing.

Our presentation also firmly fits within the mainstream of the merger literature by taking the original number of firms as exogenous. We exclude entry and assume that the extent of convexity is sufficient that fixed cost savings from simply closing plants do not drive merger dynamics. We recognize that the resulting model should be viewed as either short-run or as having high entry barriers. We also recognize the existence of a small literature on mergers with free entry (Cabral 2003; Spector 2003; Davidson and Mukherjee 2007).

Beyond being a theoretical exercise, the case of merger involving a leading and dominant firm commands special policy interest. Historically, the dominant market shares of the Standard Oil trust and the sugar trust were maintained through the purchase of much smaller rivals (Leeman 1956; Zerbe 1956). Later in the 1960s, the Court prohibited the takeover of even very small market share firms if the suitor was a dominant firm. Thus, Alcoa, the leading producer of aluminum conduit with nearly 30% of the market, was prohibited in 1964 from purchasing Rome Cable, which had only 1.3% of the market (Shepherd 1985, p. 231). Even today, the merger guidelines add emphasis to markets with dominant firms, as the resulting asymmetry of market shares results in a larger Herfindahl Index and so increase the chance for initial scrutiny. A recent antitrust case in the UK nicely illustrates many of the dimensions of our theoretical inquiry. The UK Competition Commission found that IMS Health Inc. was the leading provider of pharmaceutical business information services in the UK. These services enable pharmaceutical firms to monitor their competitive position, identify areas of product development, focus their marketing, and remunerate sales personnel. In 1997, IMS had a market share between 37 and 85%, depending on how narrowly the product was defined. They wished to merge with Pharmaceutical Marketing Services Inc., which had an 8% market share under the narrow definition. Excluded firm Taylor, Nelson, Sofres objected to the merger, citing that it would reduce their profits and viability. The commission found that the merger "could be expected to have adverse effects on efficiency" and "operate against the public interest" (UK Competition Commission 2006).

In the next section, we model the case of a merger between a leader and a single follower in a market in which all firms have convex costs. We isolate the profit consequences of the merger for the merger participants and for the excluded followers. We also isolate the welfare consequences. The third section generalizes the model to consider multiple followers merging with a leader. Again, the profit consequences for participants and excluded rivals are identified. We isolate the range of convexity that resolves the key components of the paradox as the merger size is varied. The welfare consequences of such multiple mergers are isolated through simulation. The final section draws conclusions, makes policy observations, and suggests future research.

2. Merger between the Leader and Single Follower

We consider a market with one leader and n followers and examine a merger between the leader and one follower. The merged firm remains the leader after merger, and all excluded followers prior to merger remain followers after merger.[2] By assuming that the roles of the firms remain what they were prior to merger, we explicitly exclude the case in which two followers merge to become a leader (Daughety 1990). As in other models of merger and Stackelberg (Huck, Konrad, and Muller 2001), we take the leadership as given. We note that a substantial literature has examined the conditions under which leadership can emerge endogenously in duopolies of otherwise similar firms (Saloner 1987; Hamilton and Slutsky 1990; Robson 1990; Pal 1996). Moreover, recent laboratory experiments have suggested that followers can emerge even among identical agents (Fonseca, Muller, and Normann 2006). Nonetheless, we recognize that leadership in our model is an assumption. We will show, however, that the profit of the leader after merging with an existing firm exceeds that earned by the leader after building a new plant.[3]

---

[2] For more details on theories of how a Stackelberg leader may emerge, see Higgins (1996).

[3] The authors thank one of the reviewers for suggesting this line of inquiry.

Following previous work, we assume a linear demand curve, $P = a - Q,$ where $Q = q_l + \sum_{i=1}^{n} q_i$ is the sum of the leader's and the n followers' output. The subscript l denotes the leader, and i denotes each identical follower. All firms have the same convex costs schedule, $C_j = (1/2)cq_j^2$, resulting in marginal cost curves that are a ray from the origin with slope c.[4] We assume that fixed costs are sufficiently small that eliminating a plant does not create an incentive for merger. In this case, nonzero fixed costs have no impact on the incentive to merge, and so we normalize them to zero.[5] Following the vast majority of work in this area, we also take n to be exogenous. The results that follow are determined by the pre- and postmerger profit comparisons of the leader, the follower included in the merger, and the excluded followers.

The premerger equilibrium price, quantities, and profits are

$$\begin{aligned} q_l &= \frac{a(1 + c)}{\Delta}, \\ q_i &= \frac{a(1 + (n + 2)c + c^2)}{(1 + n + c)\Delta} \qquad \forall i, \quad i = 1,\ldots,n, \\ P &= \frac{a\left(1 + (n + 3)(c + 1)c + c^3\right)}{(1+n+c)\Delta} \\ \pi_l &= \frac{a^2(1 + c)^2}{2(1 + n + c)\Delta}, \end{aligned} \tag{1}$$

and

$$\pi_i = \frac{a^2\left(1 + (n + 2)c + c^2\right)\left(2 + (5 + 2n)c + (n + 4)c^2 + c^3\right)}{2(1+n+c)^2\Delta^2} \qquad \forall i, \quad i = 1,\ldots,n,$$

where $\Delta = 2 + (3 + n)c + c^2$.

After a merger between the leader and a follower, there remain $n - 1$ followers. The merged leader now has two advantages relative to the remaining followers. The merged firm enjoys not only the standard Stackelberg advantage, but also the ability to allocate output across its plants. The merged firm's composite cost function becomes $C_l = (c/4)q_l^2$. The slope of the merged firm's composite marginal cost curve is cut in half because the merged firm has two different plants (Perry and Porter 1985). This function reflects the underlying advantage of being able to direct output across multiple plants. If each of the constituent plants produces its premerger output, total cost remains the same. The merger by itself does not immediately provide cost savings. Yet, the composite cost function means that the merged firm can reallocate its output to lower costs for a given level of combined output. In addition, the merged firm finds it less costly to change output than did its premerger constituent firms or than do firms excluded from the merger. Marginal costs are increasing, and so the merged firm has the advantage of spreading output changes across multiple plants, reducing costs. Note also

[4] As Perry and Porter (1985) make clear, convex costs and the resulting increasing marginal costs would follow naturally from examining a time frame in which the capital in the industry is fixed. Moreover, combining Stackelberg leadership and convex costs is a standard assumption in the long literature on mixed oligopoly in which a public firm acts as a leader but cannot produce the total market quantity because of increasing marginal costs (see DeFraja and Delbono 1990).

[5] In typical constant marginal cost models, the number of plants in the postmerger firm is irrelevant as marginal cost is invariant.

that given our assumption of sufficiently small fixed costs, merging does not close a plant, as would happen with constant marginal cost models.

We investigate the consequences of a merger between the leader and a single follower. The equilibrium is given by Equation 2, where the superscript reminds us that a single follower is merged with the leader and sets the stage for allowing merger with multiple followers.

$$\begin{aligned} q_l^1 &= \frac{2a(1+c)}{\Omega}, \\ q_i^1 &= \frac{a\left(2+(2+n)c+c^2\right)}{\Omega(n+c)}, \qquad \forall i, \quad i=1,\dots,n-1, \\ P^1 &= \frac{a\left(2+(4+n)c+(3+n)c^2+c^3\right)}{\Omega(n+c)}, \\ \pi_l^1 &= \frac{a^2(1+c)^2}{\Omega(n+c)}, \end{aligned} \tag{2}$$

and

$$\pi_i^1 = \frac{a^2\left(2+(2+n)c+c^2\right)\left(4+(6+2n)c+(4+n)c^2+c^3\right)}{2\Omega^2(n+c)^2}, \qquad \forall i, \quad i=1,\dots,n-1,$$

where $\Omega = 4 + (4 + n)c + c^2$.

Comparing Equations 1 and 2 characterizes the consequences of the merger. The additional profit generated by a merger between the leader and a single follower is $g(c,n) = \pi_l^1 - \pi_l - \pi_i$. The additional profit generated by this merger for excluded followers is $h(c,n) = \pi_i^1 - \pi_i$. These expressions are presented in Appendix 1, Equations A1 and A2, and we begin with the evaluation of $g(c,n)$.

PROPOSITION 1. It is always profitable for the leader and a single follower to merge.

PROOF. Set $g(c,n) = 0$ and solve for the critical n in terms of c. This yields four roots, but only one is real, and it is less than zero for all $c \geq 0$ and for all $n >$ critical n, it can be checked that $g(c,n) > 0$. Also, Appendix Figure 1 shows the graph $g(c,n) > 0$.[6]

Proposition 1 states that there is always a profit incentive to merge. This matches previous Stackelberg merger models assuming linear costs (Huck, Konrad, and Muller 2001) but differs from models assuming convex costs without leadership. With linear costs, the leader produces half the competitive output both pre- and postmerger. The merger increases the leader's market share and profit through a reduction in the number of followers. Nonetheless, the profit of excluded rivals rises as the increase in market concentration increases the market price. With convex costs but no leadership, merger is profitable only for sufficiently large c values (Perry and Porter 1985). Only then does the restriction in output by the merged firm bring about sufficient cost savings (through the ability to coordinate the output restriction between the two parts of the merged firm) that profit increases. The introduction of leadership makes these cost savings inherently larger. To see this, consider the absence of leadership, and imagine two firms merge and produce the same total quantity as prior to the merger. As each firm was initially producing an identical amount, there are no cost savings from the merger. If the same two firms merge and one of them is a leader, producing the same total quantity as prior to merger now

[6] The Maple 8 programs described in this and other proofs are available on request.

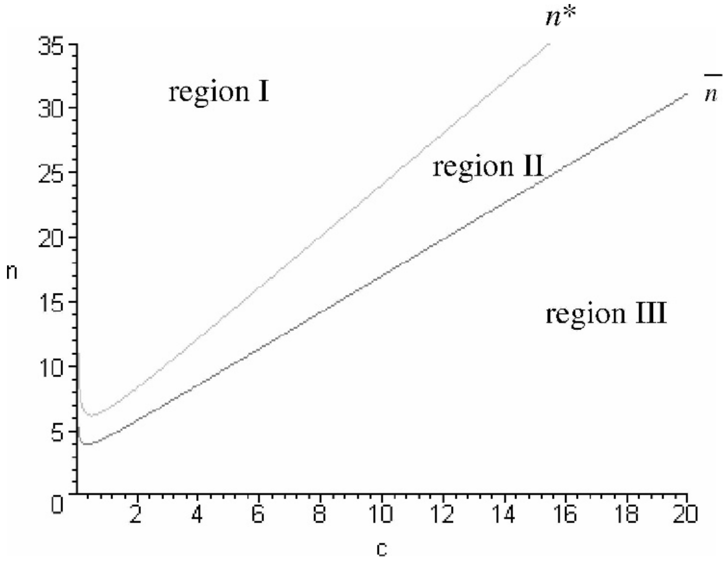

Figure 1. Critical Regions for a Merger with One Follower

allows a cost savings by reallocating that output such that the two plants of the merged firm now produce the same quantity. The asymmetry in output associated with leadership, combined with convex costs, creates a larger incentive for merger.

It might be suggested that the leader need not merge if it can simply open a new plant. Yet, opening a new plant results in a lower profit for the leader than does the merger. This can be seen by recognizing that the profit of a leader with two plants but with n followers instead of $n - 1$ (as the merged firm has) will be identical to that shown in Equation 2 after substituting $n + 1$ in for n. The value of the leader's profit after this substitution is obviously smaller.

While the profit of the merged firm always increases in our model, that of an excluded rival may either rise or fall as a result of the merger. In traditional models without merger-created efficiencies, the profit of the excluded firms increases. The merged firm attempts to increase its profit by restricting output. This restriction increases the profit of the excluded rivals. However, with convex costs, the merged Stackelberg leader may actually increase output beyond that of its premerger constituent firms. When that happens, excluded rivals see their output and profits fall. Setting the profit change of an excluded follower to zero, $h(c,n) = \pi_i^1 - \pi_i = 0$, and solving for n yields the following critical relationship:

$$n^*(c) = \frac{(2 + c)c + \sqrt{16c + 44c^2 + 36c^3 + 9c^4}}{2c}. \tag{3}$$

For values of n greater than $n^*(c)$, merger causes the profits of the excluded rivals to fall. Proposition 2 formalizes this relationship.

Proposition 2. For $n > n^*$, the profits of excluded rivals decrease as a result of the merger, and for $n < n^*$, the profit of excluded rivals increase as a result of the merger.

Proof. $\text{sgn}(n^* - n) = \text{sgn}(h)$.

Figure 1 graphs $n^*(c)$ and identifies region I as the combinations of n and c for which the profit of the excluded rivals falls as a result of the merger. Mergers in this region may well be those that fit with White's (1988) observation that excluded rivals often object to a merger and with the historical evidence from Banerjee and Eckard (1998) that excluded rivals frequently suffer financially following a merger. The function $n^*(c)$ has a minimum of 6.16 at $c = 0.498$. Thus, when there are very few firms, excluded rivals continue to benefit from the merger.

For values of n and c in region I, the merged firm increases output relative to its premerger components.[7] The introduction of convex costs gives the merged firm the advantage of spreading output changes across two plants. This advantage is greatest when the marginal cost slope is small for a given n, allowing the merged firm to increase output by a greater amount. It is also interesting to note that the total output for the market follows the output change of the leader. Thus, when the output of the merged firm increases, so does the total market output, even as that of the excluded rivals decline.

Although a reduction in the profits of the excluded rivals certainly eliminates the free-rider incentive, so would far less stringent conditions. To eliminate the free-rider problem so common in past models, the profit gain to being an excluded rival need only be smaller than the profit gain from joining the merger. To identify this condition, we define $f(c,n) = g(c,n) - h(c,n)$, the difference between the profit gain to the merging firms and that of an excluded rival. This expression is presented in Appendix 1 as Equation A3. As in the earlier propositions, we set $f(c,n)$ equal to zero and solve for n. This yields $\bar{n}(c)$, analogous to the earlier $n^*(c)$.[8] Again, for values of $n > \bar{n}(c)$, $f(c,n) > 0$, and for values of $n < \bar{n}(c)$, $f(c,n) < 0$. This allows us to formalize the conditions for the existence of the free-rider incentive.

Proposition 3. For all $n > \bar{n}$, each follower earns more as a merger participant than as an excluded rival. For all $n < \bar{n}$, each follower earns less as a merger participant than as an excluded rival.

Proof. For any given c, $\operatorname{sgn}(n - \bar{n}) = \operatorname{sgn}(f)$.

Figure 1 graphs $\bar{n}(c)$ and identifies the sum of region I and region II as the combinations of n and c for which a follower earns greater profit as a merger participant than as an excluded rival. This stands in contrast either to Stackelberg leadership without convex costs (Huck, Konrad, and Muller 2001) or to convex costs without leadership (Perry and Porter 1985; Heywood and McGinty 2007). In either of these cases, firms always prefer to remain an excluded rival, even when the merger is profitable for the participants. For $n < \bar{n}$, the merger continues to be profitable but less profitable than if remaining an excluded rival. Thus, Figure 1 identifies region III as combinations of n and c for which the standard free-rider problem remains.[9]

We confirm that the region where the output of the merged firm (and total output) increases is a subset of that for which the free-rider problem is absent.

Proposition 4. For all $c > 0$, it is the case that $n^* > \bar{n}$.

Proof. Fix $c > 0$: $n^* > \bar{n}$.

For any c, the value of n that overcomes the free-rider problem is strictly lower than that which increases the output of the merged firm.[10]

[7] The correspondence between the profit loss of the excluded firms and an increase in output for the merged firm is intuitive, but a formal proof is available on request.

[8] The actual expression for $\bar{n}(c)$ is many lines, and we have spared the reader. It is available from the authors on request.

[9] As in the original merger paradox, each firm prefers another rival to be the one that merges. This represents a type of "chicken game" that does not have an easy resolution without further refinements.

[10] Again, the expression of the difference between $n^*(c)$ and $\bar{n}(c)$ is unwieldy but available from the authors. Note that Figure 1 presents an exact graphing of both functions and makes clear the relative magnitudes.

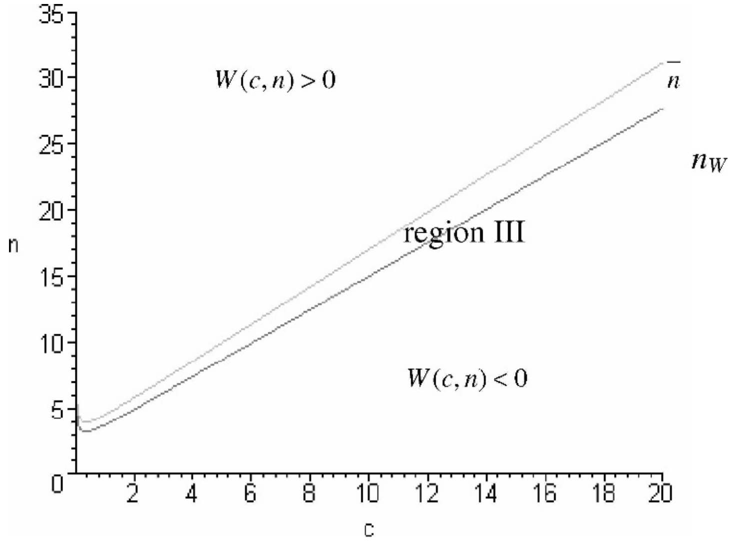

Figure 2. The Welfare Change from Merger

Finally, we examine the welfare consequences of merger. The welfare prior to merger is the sum of consumer surplus and the profits of the leader and n followers. The welfare after merger is the sum of consumer surplus and the profits of the merged leader and $n - 1$ followers. The difference between these allows an evaluation of the welfare consequences of the merger.

$$W(c,n) = \left\{ \frac{(Q^1)^2}{2} + \pi_l^1 + (n-1)\pi_i^1 \right\} - \left\{ \frac{(Q)^2}{2} + \pi_l + n\pi_i \right\}, \tag{4}$$

where $Q^1 = q_l^1 + \sum_{i=1}^{n-1} q_i^1$ from Equation 2 and $Q = q_l + \sum_{i=1}^{n} q_i$ from Equation 1.

The potential for a reduction in welfare depends on whether or not there is a restriction in output that reduces consumer surplus. If such a restriction happens, the reduction in consumer surplus must be large enough to overcome the increased profits associated with the cost-reducing properties of the merger. Substituting into Equation 4 from Equations 1 and 2 gives the welfare change as a function of only c and n. This substitution is available from the authors on request. Again, setting this expression equal to zero and solving for n yields the critical value $n_W(c)$ that allows signing the welfare change.

Proposition 5. If $n > n_W$, $W(c,n) > 0$. If $n < n_W$, $W(c,n) < 0$.

Proof. Solve $W(c,n) = 0$ for n_W and $\text{sgn}(n - n_W) = \text{sgn}(W(c,n))$. Thus, welfare may either increase or decrease depending on the slope of the marginal cost curve and on market structure. Through steps analogous to those in Proposition 4, it can be established that $n_W < \bar{n}$ for all c values. Thus, $n_W(c)$ lies entirely in region III and is plotted in Figure 2.

The welfare consequences can be summarized as follows: All mergers that overcome the free-rider problem (and so are more likely to occur) enhance welfare. Of those potential mergers that do not overcome the free-rider problem, a small share close to the critical free-rider locus $\bar{n}$ would also enhance welfare, but the remaining mergers would harm welfare. This result is unique to the

Table 1. Profit and Output Changes Resulting from Merger

Regions	g	h	f	q_g	q_h	ΔQ	W
I	+	−	+	+	−	+	+
II	+	+	+	−	+	−	+
III	+	+	−	−	+	−	±

The regions correspond to those derived in the text and identified in terms of n and c in Figure 1.
g = change in profit for the merging firms; h = change in profit for an excluded rival; $f = (g - h)$, and negative values indicate the free-rider effect; q_g = change in quantity for the merging firms; q_h = change in quantity for an excluded rival; ΔQ = change in total market quantity; W = change in welfare.

combination of leadership and convex costs and contrasts with Stackelberg with linear costs in which all mergers with followers reduce welfare (Huck, Konrad, and Muller 2001).[11]

Table 1 summarizes the main results associated with Figure 1. In region III, the merger is profitable to the participants but not as profitable as remaining an excluded follower. This result mimics that for Stackelberg leadership with constant costs. In this region, the output of the merged firm and total market output declines while that of the excluded rivals increases. In region II, the merger is profitable to the participants and is more profitable than remaining an excluded rival. The output of both the merged firm and the total market continues to decline, while that of the excluded rivals continues to increase. In region I, the merger is profitable to the participants and unprofitable to the remaining excluded rivals. The output of both the merged firm and the total market now increases, while that of the excluded rivals now decreases. Thus, not only is the free-rider problem absent but excluded rivals are actually harmed by the merger. Region I corresponds to those cases in Farrell and Shapiro (1990) in which the efficiencies (synergies) from merger are so great as to cause a reduction in price and so increase welfare. Interestingly, in this case, the excluded firms will complain of harm, but a welfare-maximizing authority should ignore their complaints, taking them as evidence of a welfare improvement.

Again, neither Stackelberg leadership without convex costs or convex costs without leadership generates the pattern of results shown in regions I and II, which stand as a reasonable resolution to the merger paradox. All mergers in regions I and II enhance welfare, while those in region III may either enhance or harm welfare. As the next section shows, with multiple merging followers, it becomes possible to simultaneously eliminate the free-rider problem and have mergers that harm welfare.

3. Merger between the Leader and Multiple Followers

Because a merger with a single follower is profitable, it would seem that a merger with multiple followers would be more profitable. The merged firm gains the cost advantage of allocating production changes across even more plants, as well as the advantage of increasing the market share of the leader. In expanding the model to allow merger with multiple followers, we note that results that could be proven analytically for a single follower must now often be demonstrated through simulation.

The resulting cost function for the merged firm created by the leader and m followers is $C_l = \{c/[2(1 + m)]\}\left(q_l^m\right)^2$. This is a straightforward generalization from the merger of two firms

[11] Of course, with linear costs, there is no possibility that merger will reallocate output in a manner that lowers total production cost.

(Heywood and McGinty 2007). Using this composite cost function, a new set of postmerger equilibrium values can be derived and are analogous to those in Equation 2.

$$
\begin{aligned}
q_l^m &= \frac{a(1+c)(1+m)}{\Psi}, \\
q_i^m &= \frac{a\left(1+m+(2+n)c+c^2\right)}{\Psi(n-m+1+c)}, \\
P^m &= \frac{a\left(1+m+(3+n+m)c+(3+n)c^2+c^3\right)}{\Psi(n-m+1+c)}, \\
\pi_l^m &= \frac{a^2(1+m)(1+c)^2}{2\Psi(n-m+1+c)},
\end{aligned} \tag{5}
$$

and

$$\pi_i^m = \frac{a^2\left(1+m+(2+n)c+c^2\right)\left(2(1+m)+(5+2n+m)c+(4+n)c^2+c^3\right)}{2\Psi^2(n-m+1+c)^2},$$

where $\Psi = 2(1+m)+(3+m+n)c+c^2$. The profit gain associated with a merger between the leader and m followers is then $g(c,m,n) = \pi_l^m - \pi_l - m\pi_i$. This is the difference between the profit of the merged firm as taken from Equation 5 and that of premerger leader and m followers as taken from Equation 1. The full expression for this profit difference is presented in Appendix 2 as Equation A4.

Proposition 6. For $n \leq 30$ and for the discrete value of c, $g(c,m,n) > 0$ for all integer values $n \geq m \geq 1$.

Proof. A grid simulation evaluated $g(c,m,n)$ for all n from 2 to 30, all m from 1 to n, and for c from 0.1 to 30 by increments of 0.1. Every value of g is strictly greater than zero.

Thus, the profit from merging is positive, and it typically grows in the number of followers that participate in the merger.[12] The gain occurs because of the ability of the merged firm to spread output among more plants and to restrict quantity. Furthermore, these simulations show (perhaps not surprisingly) that merger to monopoly ($m = n$) is always the most profitable merger.

We next examine the condition under which an excluded rival is hurt as a result of the merger. Define $h(c,m,n) = \pi_i^m - \pi_i = 0$, where π_i remains the profit of an excluded rival before a merger of the leader with m followers, and π_i^m remains the profit of an excluded rival after that merger. This difference comes directly from Equations 1 and 5 but is a large expression and so is available from the authors on request. Following previous examinations, we set it equal to zero and solve for the critical level of $n^*(c,m)$. The expression for $n^*(c,m)$ is considerably shorter and is presented in Appendix 2. It allows us to identify when excluded rivals are harmed.

Proposition 7. If $n > n^*(c,m)$, $h(c,n,m) < 0$, and if $n < n^*(c,m)$, $h(c,m,n) > 0$ and $\partial n^*(c,m)/\partial m > 0$.

Proof. $\text{sgn}(h) = \text{sgn}(n^* - n)$, where both $n^*(c,m)$ and $\partial n^*(c,m)/\partial m > 0$ are given in Appendix 2 as Equations A5 and A6.

[12] The derivate $\partial g(c,m,n)/\partial m$ is generally, but not always, positive. This ambiguity makes the simulation in Proposition 6 necessary.

Table 2. Welfare Consequences of Multiple Firm Mergers

	n		
	10	20	40
c			
0.5	+ for m = 1–5 − for m = 6–10	− for m = 1 + for m = 2–10 − for m = 11–20	− for m = 1–6 + for m = 7–21 − for m = 22–40
1	+ for m = 1–3 − for m = 4–10	− for m = 1 + for m = 2–8 − for m = 9–20	− for m = 1–6 + for m = 7–16 − for m = 17–40
3	+ for m = 1–2 − for m = 3–10	+ for m = 1–4 − for m = 5–20	− for m = 1–3 + for m = 4–9 − for m = 10–40
6	+ for m = 1 − for m = 2–10	+ for m = 1–2 − for m = 3–20	− for m = 1 + for m = 2–5 − for m = 6–40
12	− for m = 1–10	+ for m = 1 − for m = 2–20	+ for m = 1–3 − for m = 4–40

The negative sign indicates a welfare reduction from mergers in the range of m, and the positive sign indicates a welfare enhancement from mergers in the range of m.

For $n > \bar{n}(c)$, the profit from merging exceeds the profit from being an excluded rival. For $n > n^*(c)$, the profit of an excluded rival decreases as a result of merger.

For $n > \bar{n}(c)$, the profit from merging exceeds the profit from being an excluded rival. For $n > n_W(c)$, the total welfare increases as a result of merger.

Thus, there continues to exist an equivalent to region I in which the excluded rival is hurt from the merger, but this region shrinks as the size of the merger increases.

We can also identify the less stringent condition such that the profit gain from merger to the excluded follower remains less than that of a participating follower. Again, when $f(c,m,n) = g(c,m,n) - h(c,m,n) > 0$, the free-rider problem does not exist. We set $f(c,m,n) = 0$ and solve for $\bar{n}(c,m)$. While the algebra is significantly more involved, we are able to identify and plot the critical value of $\bar{n}(c,m)$ and examine this function as m increases. In general, region II increases in size as m grows, and for values of m greater than or equal to 4 (and so n greater than or equal to 5), $\bar{n}(c,m)$ never takes a positive value, implying that all mergers bring larger profits to the followers participating in the merger than those excluded from the merger.

Finally, the question remains if multiple mergers are welfare enhancing. Increasing industry concentration may offset the benefits from cost savings, thereby reducing welfare. The c,m locus for welfare-enhancing mergers is found by solving $W(c,m,n) - W(C,0,n) = 0$. The solution to this equation is a ninth order polynomial expression and intractable. However, simulations show a consistent pattern. As m increases, mergers eventually will hurt welfare. They will lower welfare for a smaller value of m as c increases. When n is very large, there may emerge another range of m in which welfare is enhanced, but this will again turn to welfare diminishing as m increases further. We have displayed a sample of the simulation results in Table 2, illustrating each of these patterns.

The important point of such simulations is to isolate the fact that it is possible to have three conditions simultaneously exist in the case of multiple mergers. First, the merging firms increase their profit. Second, the participants do not face a free-rider problem, as their profit gain exceeds that of excluded rivals. Third, the merger can hurt welfare. While there are many examples, one would be $c = 1$, $m = 4$, $n = 10$. Such cases fit commonly observed patterns that mergers are profitable, do happen, and should be objected to by welfare-maximizing antitrust authorities. Nonetheless, we recognize that simultaneous mergers involving multiple followers, four in this example, are rarely, if ever, observed.

4. Conclusion

This paper has shown that the combination of convex costs and Stackelberg leadership can largely eliminate the merger paradox. Not only do mergers between the leader and a single follower generate a profit gain but that gain often exceeds the gain earned by an excluded rival. Indeed, there exists a wide range of parameter values in which the profit of the excluded rivals actually falls. This occurs when the merging firms increase output relative to their premerger component firms. In this region, the variable cost savings generated by the merged firm's ability to allocate output across multiple locations dominates the tendency for the merged firm to restrict output. These mergers can actually increase market output. Accordingly, welfare is unambiguously improved in this region; however, firms excluded from the merger have incentive to object to the merger and complain to antitrust officials. Thus, in the single-follower mergers (far more common than multiple-follower mergers), if excluded firms complain, it can be anticipated that welfare has increased.

The impact of the Stackelberg assumption should be emphasized. In Cournot competition, Perry and Porter (1985) show that there always exists a c high enough for an initial merger to be profitable with the demand and cost specifications of our paper. With Stackelberg leadership, mergers for any c are profitable. In Cournot competition, the free-rider component of the merger paradox always exists. Each firm prefers to remain outside the merger because the additional profit of the excluded firms is greater than the additional profit generated by the merger (Heywood and McGinty 2007). With Stackelberg leadership, the free-rider component of the merger paradox often vanishes: in regions I and II for two-firm mergers and for all multiple mergers with four or more followers. Finally, some multiple-firm mergers can simultaneously be profitable to participants who overcome the free-rider issue and be harmful to social welfare. It is such mergers that might be appropriately pursued by antitrust officials.

Future studies might build on the work with multiple-firm mergers to examine the question of stability among the merging firms. This reintroduces the free-rider problem to determine when it is profitable to remain among those merging and when it is profitable for an individual firm to defect. Even more fundamentally, future studies might imagine that the leader enjoys a cost advantage. Indeed, such an advantage may be seen as the reason for leadership in the first place. In the context of our model, the cost advantage would presumably emerge as a uniformly smaller marginal cost slope for the leader (marginal cost would be a flatter ray from the origin). We anticipate two offsetting effects from introducing such an advantage. First, the cost reduction associated with combining two convex cost structures increases as the cost advantage grows (McGinty 2007). Although not originally applied to mergers, this insight certainly carries over. Second, as the cost advantage of the leader grows, the closer the premerger equilibrium resembles a monopoly with the associated greater profit. Thus, both the premerger profit and the postmerger profit can be expected to grow with the cost advantage of the leader, but it remains for future research to determine which influence dominates and whether or not the incentive to merge remains.

Appendix 1:
Critical Expressions for Merger with a Single Follower

$$g(c,n) = \pi_l^1 - \pi_l - \pi_i =$$

$$a^2\begin{pmatrix} 2c^6+(16+2n)c^5+\left(52+12n-n^2\right)c^4+\left(88+26n-4n^2\right)c^3 \\ +\left(82+24n-6n^2+n^4\right)c^2+\left(40+8n-4n^2+2n^3\right)c+8 \end{pmatrix} \Bigg/ 2\begin{pmatrix} \left(c^2+(4+n)c+4\right) \\ (n+c)\left(c^2+(3+n)c+2\right)^2(1+c+n)^2 \end{pmatrix} > 0 \tag{A1}$$

$$h(c,n) = \pi_i^1 - \pi_i =$$

$$a^2\begin{pmatrix} 4c^9+(48+14n)c^8+\left(252+140n+16n^2\right)c^7+\left(760+602n+128n^2+4n^3\right)c^6 \\ +\left(1452+1456n+423n^2+24n^3-4n^4\right)c^5+\left(1824+2172n+746n^2+48n^3-16n^4-2n^5\right)c^4 \\ +\left(1508+2056n+748n^2+32n^3-25n^4-4n^5\right)c^3+\left(792+1216n+408n^2-8n^3-18n^4\right)c^2 \\ +(240+416n+96n^2-16n^3)c+32+64n \end{pmatrix} \Bigg/ \tag{A2}$$

$$2\left(\left(c^2+(4+n)c+4\right)^2(n+c)^2\left(c^2+(3+n)c+2\right)^2(1+c+n)^2\right)$$

$$f(c,n) = g(c,n) - h(c,n) =$$

$$-a^2\begin{pmatrix} 2c^6+(16+4n)c^5+\left(54+24n+n^2\right)c^4+\left(100+52n+4n^2-2n^3\right)c^3 \\ +\left(108+48n+6n^2-4n^3-n^4\right)c^2+\left(64+16n+4n^2-4n^3\right)c+16 \end{pmatrix} \Bigg/ 2\begin{pmatrix} (1+c+n) \\ \left(2+(3+n)c+c^2\right)\left(4+(4+n)c+c^2\right)^2(n+c)^2 \end{pmatrix} \tag{A3}$$

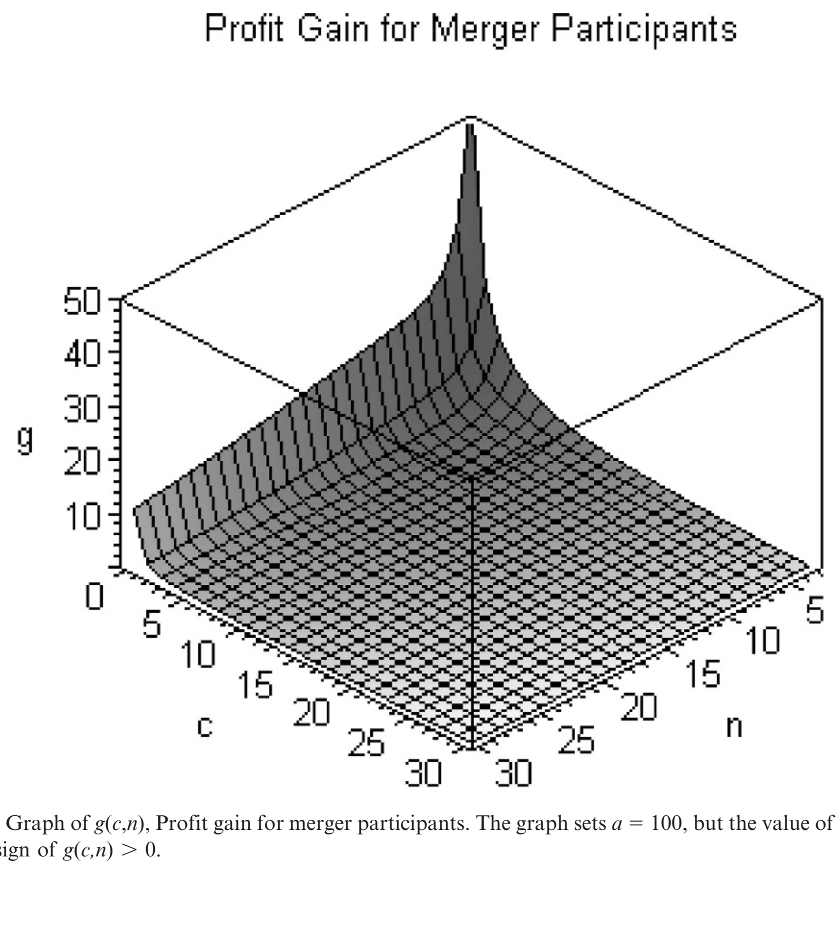

Figure 1. Graph of $g(c,n)$, Profit gain for merger participants. The graph sets $a = 100$, but the value of a does not alter the sign of $g(c,n) > 0$.

Appendix 2:
Critical Expressions for Merger with Multiple Followers

$$g(c,m,n) =$$

$$a^2 m \left(\begin{array}{l} (m+m^2)c^6 + (2m^2n+8m^2+8m)c^5 + (26m^2+12m^2n+26m-3n^2m+n^2+m^2n^2)c^4 \\ + (m^2(44+26n+4n^2)+m(44-12n^2-2n^3)+4n^2+2n^3)c^3 \\ + (m^2(41+24n+4n^2)+m(41-15n^2-4n^3)+5n^2+4n^3+n^4)c^2 \\ + (20m^2+8m^2n-6mn^2+20m+2n^3+2n^2)c+4m^2+4m \end{array} \right) \Big/ \tag{A4}$$

$$\left(2(c^2+(m+n+3)c+2+2m)(n-m+1+c)\left(c^2+(n+3)c+2\right)^2(1+c+n)^2\right) > 0$$

$$n^*(c,m) =$$

$$\left(mc^2 + 2mc + \sqrt{m^2c^4 + 4m^2c^3 + 4m^2c^2 + 20c^2 + 4c^4 + 16c^3 + 8mc + 20mc^2 + 16m}\right) \Big/ 2c \tag{A5}$$

$$\partial n*(c,m)/\partial m =$$

$$1+c/2+ \tag{A6}$$

$$\left\{(c^3 + 4c^2)(m + 2) + 2c(m + 5) + 4) \Big/ 2\sqrt{\left(c(c^3 + 4c^2)(m + 2)^2 + 4c(m^2 + 5m + 5) + 8(m + 1)\right)}\right\} > 0.$$

References

Banerjee, Ajeyo, and E. Woodward Eckard. 1998. Are mega-mergers anticompetitive? Evidence from the first great merger wave. *Rand Journal of Economics* 29:803–27.

Cabral, Luis M. 2003. Horizontal mergers with free-entry: Why cost efficiencies may be a weak defense and asset sales a poor remedy. *International Journal of Industrial Organization* 21:607–23.

Creane, Anthony, and Carl Davidson. 2004. Multidivisional firms, internal competition and the merger paradox. *Canadian Journal of Economics* 37:951–77.

Daughety, Andrew F. 1990. Beneficial concentration. *American Economic Review* 80:1231–7.

Davidson, Carl, and Arijit Mukherjee. 2007. Horizontal mergers with free entry. *International Journal of Industrial Organization* 25:157–72.

DeFraja, Giovanni, and Flavio Delbono. 1990. Game theoretic models of mixed oligopoly. *Journal of Economic Surveys* 4:1–17.

Deneckere, Raymond J., and Carl Davidson. 1985. Incentives to form coalitions with Bertrand competition. *Rand Journal of Economics* 16:473–86.

Farrell, Joseph, and Carl Shapiro. 1990. Horizontal merger: An equilibrium analysis. *American Economic Review* 80:107–26.

Feltovich, Nick. 2001. Mergers, welfare and concentration: Results from a model of Stackelberg-Cournot oligopoly. *Atlantic Economic Journal* 29:378–92.

Fonseca, Miguel A., Wieland Muller, and Hans-Theo Normann. 2006. Endogenous timing in duopoly: Experimental evidence. *International Journal of Game Theory* 34:443–56.

Hamilton, Jonathan H., and Steven M. Slutsky. 1990. Endogenous timing in duopoly: Stackelberg of Cournot equilibria. *Games and Economic Behavior* 2:29–46.

Heywood, John S., and Matthew McGinty. 2007. Convex costs and the merger paradox. *Economic Inquiry* 45:342–9.

Heywood, John S., Kristen Monaco, and R. Rothschild. 2001. Spatial price discrimination and merger: The N-firm case. *Southern Economic Journal* 67:672–84.

Higgins, Richard S. 1996. An economic theory of leader choice in Stackelberg models. *Journal of Economic Studies* 23:79–95.

Huck, Steffen, Kai A. Konrad, and Wieland Muller. 2001. Big fish eat small fish: On merger in Stackelberg markets. *Economics Letters* 73:213–7.

Huck, Steffen, Kai A. Konrad, and Wieland Muller. 2004. Profitable horizontal mergers without cost advantages: The role of internal organization, information and market structure. *Economica* 71:575–87.

Leeman, Wayne A. 1956. The limitations of local price-cutting as a barrier to entry. *Journal of Political Economy* 64:329–34.

McGinty, Matthew. 2007. International environmental agreements among asymmetric nations. *Oxford Economic Papers* 59:45–62.

Pal, Debashis. 1996. Endogenous Stackelberg equilibria with identical firms. *Games and Economic Behavior* 12:81–94.

Pepall, Lynne, Daniel J. Richards, and George Norman. 2003. *Industrial organization: Contemporary theory and practices*. Cincinnati, OH: South-Western College Publishing.

Perry, Martin K., and Robert H. Porter. 1985. Oligopoly and the incentive for horizontal merger. *American Economic Review* 75:219–27.

Reitzes, James D., and David T. Levy. 1995. Price discrimination and mergers. *Canadian Journal of Economics* 28:427–36.

Robson, Arthur J. 1990. Duopoly with endogenous strategic timing: Stackelberg regained. *International Economic Review* 31:262–74.

Salant, Stephen W., Sheldon Switzer, and Robert J. Reynolds. 1983. Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot-Nash equilibrium. *Quarterly Journal of Economics* 98:185–99.

Saloner, Garth. 1987. Cournot duopoly with two production periods. *Journal of Economic Theory* 42:183–7.

Shepherd, William G. 1985. *Public policies toward business*, 7th edition. Homewood, IL: Richard D. Irwin Publishers.

Spector, David. 2003. Horizontal mergers, entry and efficiency defenses. *International Journal of Industrial Organization* 21:1591–600.

UK Competition Commission. "IMS Health Inc. and Pharmaceutical Marketing Services Inc.: A Report on the Merger Situation." Accessed 28 November 2006. Available www.competition-commission.org.uk/rep_pub/1999/425ims.htm.

White, Lawrence. 1988. *Private antitrust litigation: New evidence, new learning*. Cambridge, MA: MIT Press.

Zerbe, Richard O. 1956. The American Sugar Refinery Company, 1887–1914: The story of monopoly. *Journal of Law and Economics* 12:339–75.