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The merger paradox in a mixed oligopoly

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ABSTRACT

This paper examines the set of surplus maximizing mergers in a model of mixed oligopoly. The presence of a welfare maximizing public firm reduces the set of mergers for which two private firms can profitably merge. When a public firm and private firm merge, the changes in welfare and profit depend on the resulting extent of private ownership in the newly merged firm. When the government sets that share to maximize post merger welfare as assumed in the privatization literature, the merger paradox will often remain and the merger will not take place. Yet, we show there always exists scope for mergers that increase profit and increase (if not maximize) welfare. Interestingly, these mergers often include complete privatization.

1. Introduction

The rapid progress of deregulation in developed countries and of transition in developing countries has resulted in increased emphasis on economic models of mixed oligopolies.[1] These oligopolies are characterized by the presence of a public firm competing against private firms and exist in a wide variety of utility, financial, transportation and even industrial markets. The original interest of economists in studying such markets was to examine the potential for "regulation by participation". This term was meant to suggest that a public firm operating to maximize welfare might change the behavior of private firms that would otherwise exploit market power. We study two under-recognized aspects of this regulation that involve mergers. First, we explore the influence that the public firm has on the potential for merger among competing private firms. Second, we explore the consequences of a public firm that partially privatizes by selling a share of its assets to a rival private firm.

Specifically, our first research question is whether the presence of a public firm in markets with convex costs makes mergers by private firms more or less likely. This question has actual policy analogues that command attention. Thus, at the end of 2001 two private Greek airlines, Aegean and Cronus, announced their merger. The merger formed the largest, but not the only, private sector carrier in Greece. Yet, the Greek market remained mixed as the major airline Olympia retained public ownership. ECA (2004) provides a series of descriptions of European airline mergers and the antitrust stances taken by the respective governments. While important distinctions exist between airline markets within and between countries (Dadpay and Heywood, 2006), the point remains that many of the mergers involved private firms in markets with at least one partially or fully owned public carrier. In Canada, extensive provincial ownership of telephone service, financial services, utilities and energy companies, combined with recent private firm mergers in these sectors raise similar issues.

Mergers in private oligopolies have long been recognized to suffer from the merger paradox, the realization that merging rivals need to command an extremely large share of the pre-merger market for a merger to be profitable. Thus,

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[1] Indeed, a check of EconLit reveals three-dozen articles presenting mixed oligopoly models in the last decade. Among other issues, these articles have examined the result of privatization on welfare (Barcena-Ruiz and Garzon, 2005; Pal and White, 1998), the role of subsidization (Pal and White, 1998; Fjell and Heywood, 2004), issues of timing (Pal, 1998; Armel, 2004) and issues of strategic location (Matsushima and Matsumura, 2003; Li, 2006) but only one paper examines mergers. That paper (Barcena-Ruiz and Garzon, 2003) undertakes a limited examination as described later in this introduction.

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Salant et al. (1983) show that no two-firm merger will be profitable for private Cournot–Nash competitors as long as there exists at least one excluded rival. This emerges because the reduction in quantity by the merged firm designed to exploit market power causes the excluded rivals to respond by increasing their output, thereby reducing any price increase. Perry and Porter (1985) provide a well-known resolution showing that any two-firm merger can be profitable given a sufficiently steep marginal cost curve.[2] Indeed, Heywood and McGinty (2007) provide a generalization by fully characterizing the required slope of the marginal cost curve for mergers of any given share of the pre-merger market.

The role of convexity (upward sloping marginal cost curves) in resolving the merger paradox is of interest for mixed oligopoly models as they also typically assume convex (typically quadratic) production costs. Without convexity, the problem of regulation by participation reduces to a truism as the public firm facing the same linear or declining costs as private firms simply produces the competitive quantity (DeFraja and Delbono, 1990). Unexplored is whether the presence of the public firm makes merger between private firms more or less likely.

Our second research question explores merger between a private firm and the public firm. Recognizing that the public firm will presumably merge only when welfare increases and the private firm will merge only when profit increases, we examine the extent to which such mergers are themselves subject to the merger paradox. Again, such mergers also have important policy analogues. In airlines, the partially publicly owned SAS purchased its private rival Braathens. In China, the government has actively sought mergers between its state owned enterprises and private firms (China Daily, 2003). The European automobile industry also provides examples. Volkswagen's acquisition of the publicly owned Spanish firm SEAT and Renault's acquisition of Dacia represent combinations of public and private firms. To date the only examination of such mergers in a mixed market is that by Barcena-Ruiz and Garzon (2003). They study a duopoly with differentiated products and imagine a merger in which the private firm receives an exogenously given degree of ownership in the previously public firm.[3] They emphasize that merger is more likely when the two firms' goods are poor substitutes. They do not consider the more traditional market with many firms producing identical products, nor do they focus on the incentive issues associated with the merger paradox.

Importantly, theory indicates that a government should partially privatize a public firm. Specifically, if the government sells a welfare-maximizing share of the public firm, welfare with the resulting mixed ownership firm exceeds that of a fully public firm that chooses quantity to maximize welfare (Matsumura, 1998; Matsumura and Kanda, 2005). It seems sensible that potential purchasers of the privatized portion of the assets would include the private firms already competing in the market. Yet, to the best of our knowledge, no one has explored the implications of the merger paradox for such a merger.

In what follows, we isolate the equilibrium in a traditional mixed oligopoly model and then identify the new equilibrium that emerges if two private firms merge. We isolate the profit earned by merger and the marginal cost slope needed to achieve positive profit. We show that the required convexity is greater than that which emerges in an otherwise identical merger but without the presence of the public firm. The merger paradox is more restrictive. We follow this demonstration with an examination of the merger between the public and a private firm. When the private ownership share is chosen by the government to maximize post-merger welfare, welfare unambiguously increases. Yet, we show the wide variety of circumstances in which the merger paradox continues to exist as the welfare-maximizing share does not yield increased profit to private shareholders.

2. Merger between private firms

We imagine an industry of n firms as Cournot–Nash competitors in a market with a linear demand curve: $P = a - Q$, where $Q = \sum_{i=1}^{n} q_i$. One public firm produces quantity q_1 and $n-1$ private firms produce the remaining output. All firms share the same convex cost schedule: $C_i = f + (1/2)kq_i^2$ generating linear marginal cost curves with slope k. We consider a merger of two private firms resulting in $n-2$ post merger private firms. We take the original number of firms n to be exogenous which allows us to ignore the fixed cost and set $f = 0$ in the cost schedules. Indeed, as Perry and Porter (1985) make clear, adopting a positive fixed cost does not change in any way the incentives for merger because the merged firm would retain the fixed costs from each of its constituent parts.

The $n-1$ initial private firms each maximize their individual profits, $\pi_i = Pq_i - C_i$, while the public firm maximizes social welfare, the sum of consumer surplus and the profits of all n firms (including that of the public firm), $W = \frac{1}{2}Q^2 + \sum_{i=1}^{n} \pi_i$. The resulting equilibrium mimics those in the literature:

$$\begin{aligned} q_1 &= \frac{a(1+k)}{k^2+kn+k+1} \\ q_i &= \frac{ak}{k^2+kn+k+1} \quad \forall i = 2 \text{ to } n. \\ P &= \frac{ak(1+k)}{k^2+kn+k+1}. \end{aligned} \tag{1}$$

[2] Other attempts to resolve the paradox include examining issues of leadership (Daughety, 1990; Huck et al., 2001), differentiated products (Reitzes and Levy, 1995; Rothschild et al., 2000) and merged firms that sequence output decisions across plants (Huck et al., 2004; Creane and Davidson, 2004).

[3] Such circumstances can be distinguished from the selling off of government assets to a rival (such as the controversial sale of Canada's de Havilland to Boeing in 1984) but where the public retains no share of ownership in the merged firm.

The public firm produces more than any private firm as it values the consumer surplus gained from increased output. As outputs from the public and private firms are strategic substitutes, the private firms decrease their output as the public firm increases its output. The final profit for a representative private firm is:

$$\pi_i = \frac{a^2k^2(2+k)}{2(k^2+kn+k+1)^2} \quad \forall i = 2 \text{ to } n. \tag{2}$$

The total pre-merger profit for two private firms will simply be $2\pi_i$.

The post-merger equilibrium follows from similar underlying conditions. The $n-3$ private firms and the public firm are excluded from the merger and retain cost functions $C_i = (1/2)kq_i^2$. The merged firm retains two plants each with that same cost function generating the resulting composite cost function: $C_{n-1} = (1/4)kq_{n-1}^2$. This function reflects the underlying advantage of being able to direct output across two plants (Perry and Porter, 1985). Note, however, that if the output of the merged firm remained identical to that of its constituent pre-merger firms, $q_{n-1} = 2q_i$, the total cost to produce that output would be unchanged. The merger by itself does not immediately provide cost savings.

The point of the merger remains to reduce output in order to exploit market power. The equilibrium resulting from $n-3$ excluded private firms and one public firm with C_i and one merged private firm with C_{n-1} can be derived:

$$\begin{aligned} q_1^M &= \frac{a(1+k)(2+k)}{\Omega} \\ q_i^M &= \frac{ak(2+k)}{\Omega} \quad \forall i = 2 \text{ to } n-2 \\ q_{n-1}^M &= \frac{a2k(1+k)}{\Omega} \\ P^M &= \frac{ak(2+3k+k^2)}{\Omega} \end{aligned} \tag{3}$$

where $\Omega = k^3 + k^2(n+3) + 2kn + k + 2 > 0$.

The profit of merged firm can be identified as:

$$\pi_{n-1}^M = \frac{a^2k^2 2(4+k)(1+k)^2}{2\Omega^2}. \tag{4}$$

We now characterize the change in the equilibrium caused by the merger in the following proposition.

Proposition 2.1. *The output of the merged firm is smaller than that of its pre-merger constituent parts but the outputs of both the public firm and the private excluded firms increase. The total output decreases and price increases.*

Proof. All of these differences can be signed unambiguously by subtracting the magnitudes in (1) with those in (3). As an illustration, $P^M - P = \frac{ak^2 2(1+k)}{(1+k+kn+k^2)\Omega} > 0$ and the other results follow analogously. □

The intuition is that the merged firm reduces output to exploit market power, but this is partially offset by the strategic responses of the other firms. Interestingly, and important for the eventual comparison of this mixed oligopoly to one without a public firm, the increase in output from the public firm exceeds that from any one private firm (a proof is in the Appendix (A1)). This happens because the reduction in output by the merged firm increases the importance of consumer surplus to the public firm causing it to further increase output. Nonetheless, like the merger among only private firms, the merger is associated with reduced total welfare.

The gain from merger for the participating firms is

$$g(n,k) = \pi_{n-1}^M - 2\pi_i. \tag{5}$$

This expression can be set equal to zero and solved for n. The resulting expression for $n^*(k)$ identifies the largest size market for which a merger of two firms earns non-negative profit:

$$n^* = \frac{k + k^2 - 4 - k^3 + 2[k^2(2+k)(1+k)^2(4+k)]^{1/2}}{k(4+3k)}. \tag{6}$$

For a given k, markets with n^* or fewer pre-merger firms will support a profitable two firm merger. Two firms will not be able to profitably merge in markets with more than n^* firms.

The fundamental issue is to examine the difference in the environment for profitable mergers created by the presence of a public firm. The steps to derive n^* can be exactly reproduced retaining a model of n firms of which two merge but in which all of the $n-2$ excluded firms are private profit maximizing firms. This generates a $n_{pri}^*(k)$ and has been done in Heywood and McGinty (2007) as part of their generalization of the Perry and Porter results. We reproduce the resulting critical market size:

$$n_{pri}^* = \frac{8 + 2k - 2k^2 + 2[88k + 84k^2 + 32 + 32k^3 + 4k^2]^{1/2}}{2(3k+4)}. \tag{7}$$

Comparing (7) and (6) allows the following proposition:

Proposition 2.2. *For any slope of the marginal cost curve, k, the set of markets in which a two firm merger will be profitable is smaller with the presence of a public firm.*

Proof. If $k > 0$, $n^*_{pri} > n^*$.

While an illustration of the relevant magnitudes is available from the authors, the basic intuition flows from recognizing the public firm makes a larger quantity response to the reduction in quantity sought by the merging firms. The emphasis on consumer surplus means that for a given reduction in output by the merged firm, the market with a public firm will result in a smaller total reduction in market quantity. As a consequence, there will be a smaller increase in price making it more difficult for the merged firm to gain profit relative to its two constituent pre-merger firms.[4]

This logic fully carries over to the case of mergers involving more than two firms. Indeed, it can be shown that increasing the number of firms merging, m, increases the number of firms in the pre-merger market, n, for which the merger is profitable but the scope for merger remains smaller with the presence of a public firm.[5] Thus, the presence of the public firm serves to reduce the likelihood of merger and so reduce the chance that welfare decreasing combinations occur. □

3. Merger with the public firm: The case of partial privatization

We now consider the consequences of a merger between the public firm and one of the $n - 1$ private firms. This consideration requires identifying both the necessary conditions for the two potential participants to merge and the objective function of the newly merged firm. The former seems straightforward in that the private firm owners will agree to merge only if their profit increases from doing so and the public firm will agree to merge only if welfare increases as a result.

The objective function of the merged firm should reflect the two types of owners, one public and one private and the weights given to these two objectives presumably reflect the respective ownership shares. Thus, if half the assets are publicly owned and half are privately owned, the objective function becomes the average of firm profit and social welfare. This has been the convention followed in examining a mixed ownership firm (Barcena-Ruiz and Garzon, 2003). We note that Matsumura (1998) emphasizes that mixed ownership firms are common in all major countries other than the United States and that Barcena-Ruiz and Garzon (2003) stress that mixed ownership is, indeed, the likely result of merger between a public and private firm.[6] In this section we follow the common practice in the literature on partial privatization, and assume the government offers for sale the welfare maximizing ownership share of the public firm (Matsumura, 1998, Matsumura and Kanda, 2005, Jiang, 2006). As mentioned in the introduction, this share is known to increase welfare relative to a fully public firm that chooses quantity to maximize welfare.

We model a two-stage game solved by backward induction in which the ownership structure is chosen first with the realization that in the second stage the firms play a quantity game. The solution to the post-merger second stage has $n - 2$ private firms competing with a mixed-ownership firm with the following objective function:

$$E_1 = (1 - \alpha)W + \alpha\pi_1 \tag{8}$$

where α is share of private ownership that emerges from the first stage. Again, the merged firm enjoys the composite cost function.

The resulting post-merger values now become:

$$\begin{aligned} q_1^M &= \frac{2a(1+k)}{\Phi} \\ q_i^M &= \frac{a(2\alpha + k)}{\Phi} \quad \forall i = 2 \text{ to } n-1 \\ P^M &= \frac{a(2\alpha + k + 2\alpha k + k^2)}{\Phi} \end{aligned} \tag{9}$$

where $\Phi = 2 + 2n\alpha + nk - 2\alpha + k + 2\alpha k + k^2$.

This represents a generalized equilibrium in which the critical variables depend on α, the private ownership share.

Proposition 3.1. *The output of the mixed ownership firm decreases (increases) relative to its pre-merger constituent firms as* $\alpha \gtreqless k/2(1 + 2k)$.

Proof. From (9) and (1) $h = q_1^M - q_1 - q_i$. This expression is in the Appendix (A2) and is set equal to zero to yield $\alpha = k/2(1 + 2k)$. Signs in each region are checked. Moreover, as the Appendix shows, $\partial h/\partial\alpha < 0$ for all n and k. □

[4] Another way to illustrate the point is to simulate the critical slope the marginal cost curve needs to generate positive profit. Thus, imagining $n = 3$, the needed convexity to earn profit from merger in the private market is $k > 1.60$ (Heywood and McGinty, 2007) but that in the mixed oligopoly is $k > 2.67$.

[5] A demonstration of this point and simulations for mergers of many firms are available from the authors in an earlier version of this paper.

[6] Indeed, Doganis (2001) identifies 44 airlines worldwide that have mixed public and private ownership.

This result does not imply that the magnitude of the change in output is independent of n. It only says that there exists a critical ownership share that determines the *sign* of the change in output and is independent of n. As an illustration, if $k = 1$, output increases only whenever $\alpha < 1/6$. Once in this region, the value of n helps determine the size of that increase. Small degrees of private ownership encourage the merged firm to expand output. It does so while equalizing the outputs, and consequently the marginal costs, between its two plants. Eliminating this cost asymmetry increases both consumer surplus and welfare.

Note the critical condition in Proposition 3.1 also identifies the direction of change in output for a representative excluded firm. When the merged firm increases output, the private firms decrease output and when the merged firm decreases output, the private firms increase output. Thus, for large values of α output of the merged firm shrinks relative to its pre-merger constituent firms and total market output shrinks.[7] Finally, price increases as total market output shrinks and decreases as it grows.

With the first stage equilibrium identified in (9) we now return those values to the expression for post-merger social welfare, the sum of consumer surplus and all profits. The expression for post merger welfare is rather long and is given in the Appendix (A3). From that expression is subtracted the pre-merger welfare level associated with (1). The resulting welfare change associated with merger follows from (A3) and depends upon α. Continuing our illustration, when $k = 1$ and $n = 6$, welfare increases for levels of α below .741 but decreases for higher levels. This and related simulations are available from the authors but the point is merely that welfare can increase or decrease depending upon the ownership structure.

As α does not influence the pre-merger welfare, but does influence the post-merger welfare, we maximize the post merger welfare with respect to α in order to maximize the welfare gain. This can be seen as a government imposed condition on the merger or as part of surplus maximizing negotiation. We identify the value of α that maximizes welfare and note that for the merger to go forward the private firm must experience a non-negative change in profit.[8] Again, the expression for post-merger social welfare is given in the Appendix (A3) and can be maximized with respect to α yielding the resulting critical value:

$$\alpha^* = \frac{k(n-2)}{2(k^2+nk+1)}. \tag{10}$$

In our model with $n > 2$ and $k > 0$, this critical value is always positive implying the resulting firm will have some degree of privatization. Moreover, this critical value remains below $1/2$ but approaches $1/2$ as n goes to infinity. For any given n, it reaches a maximum when $k = 1$. Thus, if $n = 6$ and $k = 1$, the α that maximizes post-merger welfare is $1/4$, well short of $1/2$. We now return the optimal α^* to the changes in the two important variables, profit and welfare, to examine the consequences of merger. The resulting welfare gain after substituting in α^* is:

$$\Delta W^* = \frac{a^2k(k^3+3k^2+(5+n^2-3n)k+1)}{2(k^3+2nk^2+(1+n^2)k+2)(1+k^2+k+nk)^2} > 0. \tag{11}$$

For our model of $n > 2$, the gain in welfare is unambiguously positive. While merger is a new setting and the resulting composite cost function is unique to our consideration, this result accords with both the literature on mixed ownership (Matsumura, 1998) and that on incentive contracts (Fershtman and Judd, 1987; Barros, 1995). The intuition in our case is two fold, first partial privatization reduces the inefficiency associated with the public firm having a higher marginal cost (because it produces more than its rivals) and second, the merged firm can now allocate its output between its two plans (the pre-merger firms) and so lower production cost for any level of output. Importantly, these two influences outweigh the inefficiency associated with the loss of a competitor. Yet, if the merger increases welfare, it remains an issue whether or not it is profitable for the private firm to purchase the welfare-maximizing share of ownership.

The profit that the mixed ownership firm earns given α^* is:

$$\pi_1^M = \frac{a^2k(k^2+nk+2n-3)(k^2+nk+1)}{(k^3+2kn^2+n^2k+k+2)^2}. \tag{12}$$

Thus, the change in profit to the shareholders of the merging private firm is:

$$\Delta\pi^* = \alpha^*\pi_1^M - \pi_i. \tag{13}$$

In other words, the privatized share of the mixed ownership firm must generate enough profit to at least equal that of the pre-merger private firm.

Substituting (2), (10) and (12) into (13) yields a higher order polynomial as shown in Appendix (A4). While it defies a closed form interpretation, it can be easily characterized. First, if $n = 3$, the profit difference is unambiguously negative.

[7] This can be shown either by directly comparing total output or realizing that the aggregate reaction function of the excluded firms has a slope less than one in absolute value. The authors can provide a formal proof but it follows directly because the outputs of the merged and excluded firms are strategic substitutes.

[8] All that is required from a social point of view is that the resulting increase in welfare from the merger be large enough to compensate the private firm for any profit loss from the merger. Yet, the government may not be able to easily translate welfare gains into profit for the merging private firm and thus we require the more stringent standard that private profit be non-decreasing.

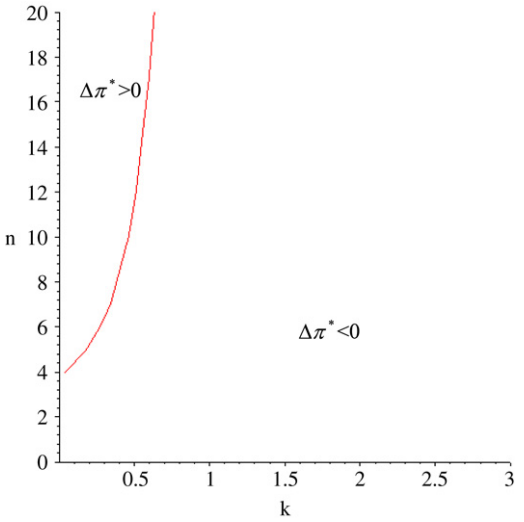

Fig. 1. The profit change for private shareholders at the welfare maximizing share.

The resulting share of the mixed firm is too small for the private owners to increase their profit. Second, by examining the factored numerator in (A4), it is clear that when $k > 0.8136$, every term in k is positive regardless of the value of n. Thus, this stands as a second sufficient condition for the profit difference to be negative. Finally, a relationship emerges between n and k when $k < 0.8136$. Fig. 1 shows that for a given k the profit differences are negative for small values of n and positive for large values of n.

We draw some of these demonstrations together in a proposition:

Proposition 3.2. *For any merger of the public and a private firm in a mixed oligopoly with n firms:* (i) *There exists a share of private ownership strictly less than* $1/2$ *that maximizes post merger welfare;* (ii) *The equilibrium quantities that emerge from that maximizing share unambiguously increase welfare;*

Proof. (i) From (10) $\alpha^* < 1/2$ for all $n > 2$ and $k > 0$. (ii) From (11) $\Delta W^* > 0$ for all $n > 2$ and $k > 0$. □

As mentioned, there is not an easy closed form solution for examining the change in profit for the private shareholders. The profit of private shareholders declines whenever n is 3 or $k > 0.8136$ as from (13) $\Delta\pi^*(n = 3) < 0$ and $\Delta\pi^*(k > 0.8136) < 0$ as shown in (A4). The relationship shown in Fig. 1 follows from solving $\Delta\pi^* = 0$ for k after substituting successive integer values of n from 3–20 into (A4) and checking the sign on either side of the solution. Thus, the merger of the public and a private firm that results in the welfare maximizing ownership shares will only occasionally increase the profit of private shareholders. Moreover, it tends to do so when n is large, the very circumstances in which the benefit of the original public firm is minimal (DeFraja and Delbono, 1990).

It is worth emphasizing the sharp difference between the merger with the public firm and the merger of two private firms. The two private firms begin with the same quantity before merger. If after merger they produce their same total pre-merger quantity, their total production cost would be unchanged. Because the public and private firms have very different pre-merger quantities, the merger allows them to reduce costs even if they do not change their combined quantities. They can equalize their production between their two post-merger plants and thereby reduce costs. Thus, the merger of the public and private firm brings about the kind of synergies that Farrell and Shapiro (1990) indicate are necessary for a merger to increase social welfare. Despite the increase in welfare, the merger will frequently not happen as the profit of private shareholders often declines at the welfare maximizing degree of privatization. Profit is likely to rise in those circumstances when k is smaller as this implies greater asymmetry in output and that the efficiency gain from combining two plants is larger.

4. Generalizing the merger

The results of the previous section do not imply a limited scope for mutually beneficial mergers, only that there is a limited scope under the traditional assumptions that governments should sell a welfare-maximizing share of the public firm, and that the profit earned by private shareholders reflects their ownership share. In this section, we make clear two important points. First, if the link between the private share of ownership and the private share of profit is broken, there always exists a welfare-maximizing merger that is profitable to private shareholders. In this critical aspect the merger between a public and a private firm differs from that of two private firms in the classic merger paradox. In the latter there is no way to divide the surplus of a two firm merger to make both participants better off. In the case of the mixed oligopoly there always exists a mechanism to make both participants better off regardless of the slope of the marginal cost curve. Second, even if the link between the share of profit and the share of ownership remains, there still exists a share that will increase surplus

for each participant even if does not maximize welfare. Importantly, this latter point leads to the realization that complete privatization through merger, selling the entire public firm to a private rival, can increase both welfare and private profit. While we will make this circumstance clear, it is important because the literature on partial privatization rarely calls for full privatization.

If we imagine that the share of profit earned by the private shareholders need not reflect their share of ownership, we can simply compare the profit of the merged mixed ownership firm to that of the pre-merger private participant.

Proposition 4.1. *There exists a share of profit in the merged firm (that firm created by selling the welfare maximizing ownership share) such that the resulting profit for private shareholders exceeds that earned without merger.*

Proof. The sum of profit earned by the mixed ownership firm is greater than that earned by the pre-merger private firm: $\Delta\pi_P^* = \pi_1^M - \pi_i > 0$ as shown in (A5). □

Thus, we know that the welfare maximizing ownership share increases welfare (Proposition 3.2) and the current proposition shows that the profit earned by the merged firm at that share of ownership is sufficient to compensate the private firm. Put somewhat differently, this proposition demonstrates that at the welfare maximizing ownership share there always exists a sufficient side payment to cause a private firm to merge. Again, that is not true in the canonical model of the merger paradox. In the mixed oligopoly, it follows from the assumption of convexity and the fact that the public and private firms initially produce different quantities. Thus, producing the same quantity as before the merger, quantities can be equalized between plants, costs reduced and profit increased. The welfare maximizing ownership structure implies the merged firm may produce a different quantity than its pre-merger firms but the increased profit remains.

As an alternative, assume that no side payment can be made but that the privatized share of both ownership and profit need not be that which maximizes post merger welfare. Thus, we identify the set of α that could emerge from a successful bargaining arrangement between the public and private merger partners. The general expression of the gain to the private shareholders is the difference between the value of their share of post merger profits and their pre-merger profits.

$$\Delta\pi_{PRI}(\alpha) = \alpha\pi_M^1(\alpha) - \pi_i. \tag{14}$$

Remembering that α is no longer chosen to maximize post merger welfare, the welfare gain becomes a function of α.

$$\Delta W(\alpha) = W_M(\alpha) - W \tag{15}$$

where the first term is the general post merger welfare that depends on α as given in (A3) and subtracted from it is the pre-merger welfare.

The values of α that make both (14) and (15) positive are a function of marginal cost slope, k, and the number of pre-merger firms in the market, n. They can be illustrated for any given n. After substituting in a specific integer value of n into (14) and (15), the expressions are set equal to zero and solved for α in terms of k. For $n = 3$, the set of potential mergers is isolated in the eye between the two loci in the top panel of Fig. 2. Thus, for example, if $\alpha = .4$ and $k = 1$, both welfare and profit will increase even though the welfare maximizing merger will not occur with three firms or with so large a k. Such a scope for merger exists in all integer values checked ($n = 3$–20).

The lower panel of Fig. 2 shows the loci for $n = 7$ and reflects the growing range of potential mergers as α increases. Interestingly, the upper extreme of $\alpha = 1$ is included in the potential mergers for certain values of k. Indeed, complete privatization through merger can be both profit and welfare enhancing for a wide variety of cases. To demonstrate this set α equal to one. We know from Proposition 4.1 that this will insure a profit gain in (14). We substitute $\alpha = 1$ into the welfare change in (15) as shown in (A6) set it equal to zero and solve for n in terms of k. The relevant root is shown in (A6). While it is clear from (A6) that for large values of k the higher order terms will assure that the welfare gain is negative, for many reasonable marginal cost slopes ($k < 4.83$) the relevant root cuts through the positive quadrant as shown in Fig. 3.

Thus, for a given k, the welfare gain from complete privatization will generally be positive for smaller n. Looked at differently, there exists a range of k such that complete privatization by merger will increase both profit and welfare for any integer n larger than five. In summary, there always exists scope for a merger that enhances both welfare and profit if a side-payment is allowed (Proposition 4.1). If the public and private firm must bargain to a common share of profit and ownership, there still exists a range of shares that generate mutual gains and for most market structures and marginal cost slopes less than 4.83, these shares include complete privatization by merger.

This result emerges because the gain implicit in a merger (cost reduction by reallocating production between plants) is not available in simple privatization. While choosing a degree of partial privatization (not complete) to maximize post merger welfare makes sense if the purchaser is outside the market, it need not make sense when the purchaser is inside the market. Put differently, in order to capture the cost advantage associated with merger, the government will often need to accept a greater than optimal degree of privatization.

5. Conclusions

This paper has explored the incentives for merger in a mixed oligopoly. When considering a merger between two private firms, the presence of a welfare maximizing public firm reduces the set of mergers that will be profitable. The degree of

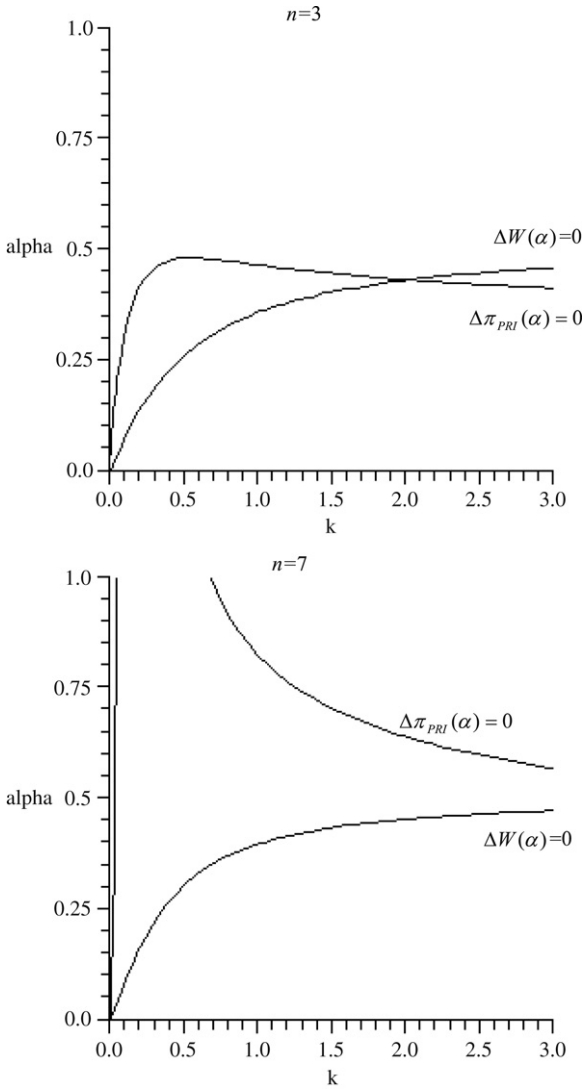

Fig. 2. The set of α that increase both private profit and welfare.

convexity required to earn profit is larger than would be the case in the absence of the public firm. Indeed, the presence of the public firm "regulates by participation".

When considering the merger of a public firm with a private firm, the critical issue becomes the resulting ownership structure that sets the objective function of the merged firm. If we follow the literature on privatization by assuming that the ownership structure maximizes post merger welfare, there will be a wide variety of circumstances in which the merger paradox holds and no private firm will want to merge. Yet, we have shown that the gain associated with coordinating production across two plants allows both profit to increase and welfare to be maximized if side-payments exist. Moreover, even when side payments are ruled out, the paradox is removed because there exist common shares of profit and ownership that increase profit and increase (if not maximize) welfare. Finally, this set of shares will often include complete privatization.

Several questions remain unanswered. First, will a second private firm be able to subsequently merge with the previously merged firm in a way that increases both profit and welfare? If so, does this process end in a single firm owning all plants but with mixed ownership? Second, interesting results may follow from the introduction of foreign firms that help generate consumer surplus but whose profit is not part of domestic social welfare (Fjell and Pal, 1996). The purpose would be to examine the scope of incentives for foreign firms to merge and for domestic and foreign firms to merge both with and without a public firm. The framework developed in this paper should allow these questions to be investigated in the future.

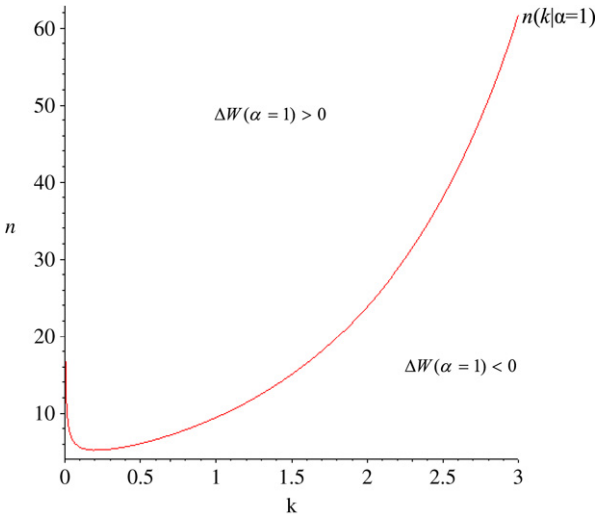

Fig. 3. Complete privatization and welfare change.

Appendix

(A1) The public firm increases output more substantially in response to a merger than do private firms. From (1) and (3): $(q_1^M - q_1) - (q_i^M - q_i) = \frac{2ak}{\Omega(k^2+kn+l+1)} > 0$.

(A2)

Proposition 3.1. *The expression for the difference in output between the post-merger mixed ownership firm and its pre-merger constituent firms:*

$$h = \frac{a(k+n-1)[k-2\alpha(2k+1)]}{(2+2n\alpha+nk-2\alpha+k+2\alpha k+k^2)(1+nk+k+k^2)}.$$

The derivative of this expression with respect to α is unambiguously negative:

$$\frac{\partial h}{\partial \alpha} = -\frac{4(n+nk+k^2-1)a}{(2+2n\alpha+nk-2\alpha+k+2\alpha k+k^2)^2} < 0.$$

(A3) Post-merger welfare:

$$W^M = \frac{a^2(4-8\alpha^2k+4n\alpha^2k+2k+4n^2k\alpha+4nk+4\alpha k^2n+2k^2n+k^3n+n^2k^2-8\alpha+4n^2\alpha^2-8n\alpha^2+8n\alpha)}{2(2+2n\alpha+nk-2\alpha+k+2\alpha k+k^2)^2}.$$

(A4) Change in profit for the private shareholders at α^*, $\Delta\pi^* = \alpha^*\pi_1^M - \pi_i$:

$$\Delta\pi^* = \frac{\begin{array}{l}-a^2k^2[(k^4+4k^3+k^2-4k)n^3+(3k^5+12k^4+11k^3+12k^2+17k-2)n^2\\ +(3k^6+16k^4+26k^3+28k^2+4k+7)n+(k^7+4k^6+6k^5+8k^4+k^3-10k^2+2)]\end{array}}{2[k^3+2k^2n+(1+n^2)k+2]^2[1+(n+1)k+k^2]^2}$$

$$\Delta\pi^*(n=3) = \frac{-a^2k^2(k^7+13k^6+69k^5+191k^4+286k^3+28k^2+57k+5)}{2[k^3+6k^2+10k+2]^2[1+4k+k^2]^2} < 0.$$

(A5) Potential profit gain for private shareholders at α^*, $\Delta\pi_P^* = \pi_1^M - \pi_i$:

$$\Delta\pi_P^* = \frac{\begin{array}{l}a^2k[k^8+(2+4n)k^7+(8n+6n^2)k^6+(8n+4n^3-12+12n^2)k^5\\ +(-25+n^4-8n+16n^2+8n^3)k^4+(2n^4-28n+8n^3-26+16n^2)k^3\\ +(-4n^2-34-4n+12n^3)k^2+(12n^2-20-8n)k-6+4n]\end{array}}{(2(2+k^3+2k^2n+(1+n^2)k)^2(1+k^2+(n+1)k)^2)} > 0.$$

Each term in parentheses is greater than zero as $n \geq 3$.

(A6) Change in welfare from merger when $\alpha = 1$

$$\Delta W(\alpha = 1) = \frac{-a^2[(-4k + k^3 - 4k^2)n^2 + (12k + 20k^2 + 13k^3 + 8k^4)n + 4 + 14k + 33k^2 41k^3 + 23k^4 + 7k^5]}{2[(2n + kn + 3k + k^2)^2(1 + nk + k + k^2)^2]}.$$

Setting $\Delta W(\alpha = 1) = 0$ and solving for n

$$n(k|\alpha = 1) = \frac{[12k + 20k^2 + 13k^3 + 8k^4 + (64k + 432k^2 + 1216k^3 + 1840k^4 + 1605k^5 + 806k^6 + 228k^7 + 36k^8)^{1/2}]}{2(+k + 4k^2 - k^3)}.$$

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