Oxford Economic Papers 59 (2007), 45–62 **45**
doi:10.1093/oep/gpl028

International environmental agreements among asymmetric nations

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This paper generalizes the benchmark model of self-enforcing international environmental agreements (IEAs) by allowing for all possible coalitions of n asymmetric nations. Asymmetries introduce gains from trade in pollution permits, reducing the incentive to deviate from a properly designed agreement. Coalitions are stable when the aggregate payoff to members is greater than the sum of individual payoffs from leaving the coalition. A benefit-cost ratio rule is proposed which distributes any remaining surplus after each coalition member receives their payoff as a non-signatory. Simulations of 20 asymmetric nations illustrate that even when the gains to cooperation are large, IEAs can achieve substantial emissions reductions. For example, when the benefit-cost ratio is one, stable coalitions can result in 47% of the difference between the full and no cooperation outcomes, compared with 5% for symmetric nations. Furthermore, 72% of the global payoff difference is obtained, relative to 9% for symmetry.

JEL classifications: H41, Q20, F02.

1. Introduction

The theoretical literature on international environmental agreements (IEAs) reaches pessimistic conclusions. IEAs suffer from the free-rider problem since reduced emissions are a global public good. Furthermore, the free-rider problem becomes more severe as the potential gains to an IEA increase (Barrett, 1994). Given this theoretical foundation, the difficulties implementing the Kyoto Treaty are hardly surprising.

This paper reverses some of that pessimism. It argues that the source of the conventional wisdom is the convenient, but highly unrealistic, assumption that nations are identical. When that assumption is relaxed by letting the marginal costs and benefits of abatement vary across nations, we find that a much higher level of abatement can be sustained by an IEA. This paper is the first to provide a solution to the benchmark model of IEAs (Hoel, 1991; Carraro and Siniscalco, 1993; Barrett, 1994) for all possible coalitions of n asymmetric signatories.

Coalitions are stable when the aggregate payoff to members is greater than the sum of individual payoffs from leaving the coalition. This means that a zero-sum system of transfers exists such that no member has an incentive to leave the coalition. A benefit-cost ratio rule is proposed which distributes any remaining surplus after each coalition member receives their payoff as a non-signatory. This rule is extremely simple and directly incorporates internal stability, two improvements over cooperative game theory rules such as the Shapley value. The transfers are implemented through a system of tradable pollution permits among members similar to the Kyoto Treaty.

Simulations of 20 asymmetric nations illustrate that even when the gains to cooperation are large IEAs can achieve substantial emissions reductions. For many parameter constellations stable coalitions result in approximately ten times the abatement obtained with symmetric nations. When the high benefit share nations have flat marginal cost curves coalitions with eight members are stable, compared with the largest coalition of three symmetric nations. The conventional wisdom that self-enforcing IEAs can not achieve substantial gains when the gains to cooperation are large does not hold when nations are asymmetric.

This paper extends the results of Barrett (1994) and Barrett (1997). Barrett (1994) performs simulations on the symmetric benchmark model of IEAs. He finds that self-enforcing IEAs can not substantially improve on the non-cooperative outcome. When abatement benefits are concave there is an inverse relationship between the number of signatories and the gains to cooperation. Concave benefits result in downward sloping reaction functions. An increase in abatement by signatories is met by a reduction in non-signatory abatement. For IEAs this situation is referred to as carbon leakage, but more generally as the free-rider problem. While nations would prefer to have others provide abatement, it is rational to provide an amount greater than the non-cooperative level when others' provision is low. Alternatively, linear benefit functions result in orthogonal reaction functions so changes in abatement by signatories to an IEA do not influence non-signatory behavior (Mäler, 1987; Hoel, 1992; Barrett, 1999). However, the standard free-rider problem still emerges in which each player chooses the dominant strategy of not cooperating.

Additional research has investigated if certain types of asymmetry can improve the success of IEAs (Barrett, 1997; Botteon and Carraro, 2001). Barrett (1997) extends his 1994 paper by allowing for two types of nations and testing 20 coalitions for stability. Still, he concludes that an IEA can achieve the least when the gains to cooperation are large, supporting the earlier symmetric model. Botteon and Carraro (2001) investigate IEA stability in a model of five asymmetric players. Abatement under the IEA is determined for two burden sharing rules: Shapley value and the Nash bargaining solution. They find higher cooperation under the Shapley value rule with transfer payments in a model calibrated using parameter estimates from Musgrave (1995). Their findings call for additional research providing a general solution to the asymmetric model, while allowing for transfers among a greater number of nations.

Recently, a great deal of attention has been given to coalition formation in IEAs (Finus, 2003; Ray and Vohra, 1999; Carraro and Marchiori, 2003; Finus and

Rundshagen, 2003). IEAs typically use the concept of internal and external stability from oligopoly literature (D'Aspremont *et al.*, 1983). This notion of stability means that no non-signatory has an incentive to join an IEA and no signatory has an incentive to leave. Coalition formation in IEAs can take various forms. In open membership games (Finus and Rundshagen, 2003) any nation may join the IEA. By contrast, in exclusive membership IEAs existing members may block the accession of a new nation. Furthermore, there may be a single coalition or multiple coalitions with different IEAs. In the single coalition framework all non-signatories behave as singletons maximizing individual payoffs. This paper analyses an open membership single coalition game similar to the Kyoto Treaty.

Transfers have been shown to increase participation when nations can commit to IEAs (Hoel, 1992; Carraro and Siniscalco, 1993). However, the problem with this approach is that commitment is not a best-response and thus violates individual rationality. Even without commitment, transfers allow nations to be compensated when they have an incentive to leave the IEA. Barrett (1992) considers pollution permit trading schemes as a system of side-payments. Nations that abate an amount greater than their requirement under the agreement receive a positive transfer, while nations that purchase permits meet their requirements at a lower cost. The required abatement under the agreement is an allocation of permit rights, defining a system of transfers. Transfers allow the signatory maximization problem to be unconstrained by the payoffs members would earn outside the IEA (Hoel, 1992). Barrett (2001) highlights the incentives for high benefit nations to provide transfers to low benefit nations to induce participation, called 'cooperation for sale'. The model presented below utilizes pollution permits as a zero-sum system of transfers among signatories.[1] A simple rule is specified which divides the remaining surplus once each coalition member receives their payoff as a single defector from the IEA. This rule specifies the required abatement, and thus transfers, given the efficient level of abatement. High benefit nations will contribute more to the total cost of abatement, while high cost nations can meet their required abatement levels through permit purchases.

The remainder of the paper is organized as follows. Section 2 derives the non-cooperative and full participation outcomes, as well as global abatement for any coalition of signatories. Section 3 provides the conditions under which any given coalition of signatories is stable, including a zero-sum system of transfers implemented through tradable pollution permits. Simulations in Section 4 illustrate the main results of the paper. Section 5 concludes and suggests additional research.

2. No, full, and partial cooperation

Let $N = \{1, \ldots, n\}$ denote the set of asymmetric nations. Global benefit is assumed to be a concave function: $B(Q) = b(aQ - (Q^2/2))$ of the worldwide quantity of

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[1] The model does not consider multiple issues as in Folmer *et al.* (1993) or Cesar and de Zeeuw (1996). They show that linking cooperation across different issues can serve the same role as side-payments.

abatement $Q = \sum_{i \in N} q_i$. Greenhouse gases tend to mix uniformly in the upper atmosphere making the location of abatement irrelevant. Parameters a and b are strictly positive so the marginal benefit of the first unit of abatement is ab and the marginal benefit of the ath unit is zero. Nation $i \in N$ receives benefit share α_i, where $\alpha_i > 0 \ \forall i \in N$ and $\sum_{i \in N} \alpha_i = 1$.[2] Benefit for nation i is: $B_i(Q, \alpha_i) = b\alpha_i(aQ - (Q^2/2))$.

All nations are assumed to have convex abatement cost functions: $C_i(q_i, c_i) = (c_i(q_i)^2/2)$ generating marginal abatement cost (MAC) curves with asymmetric slopes $c_i > 0$. The atmosphere is a global public good that currently has no price to use. Since nations may freely emit effluents, MAC curves begin at the origin. However, the rates of increase vary immensely. Nations that currently use a relatively clean mix of fuels, such as nuclear or natural gas, face a much greater c_i due to fewer substitution possibilities. Nations which use a relatively high amount of coal can undertake substantial abatement at very low cost. For example, estimates by Ellerman, *et al.* (1998) suggest that the rate of marginal cost increase for Japan is more than ten times greater than the United States and more than 50 times greater than China. Differences of this magnitude suggest large gains from an IEA that efficiently allocates abatement levels. Net benefit for nation i is: $\pi_i = B_i - C_i$.

$$\pi_i(\alpha_i, c_i, q_i, Q) = b\alpha_i\left(aQ - \frac{Q^2}{2}\right) - \frac{c_i(q_i)^2}{2} \tag{1}$$

2.1 Non-cooperative outcome

In the non-cooperative outcome each nation chooses q_i to maximize π_i taking as given the sum of the others abatement, $Q_{-i} \equiv \sum_{j \neq i \in N} q_j$. The reaction function for nation i is:

$$q_i = \frac{b\alpha_i(a - Q_{-i})}{c_i + b\alpha_i} \tag{2}$$

Defining the benefit share MAC slope ratio as $\theta_i \equiv (\alpha_i/c_i)$ the slope of the reaction function is: $\partial q_i/\partial Q_{-i} = -1/(1 + 1/b\theta_i) \in (-1, 0)$. The slope of the reaction function approaches -1 as θ_i approaches ∞. Such a nation outside an agreement will reduce abatement by exactly the increase in abatement from the IEA, in other words complete carbon leakage. Carbon leakage is greatest when high benefit, low cost nations are outside an IEA. Conversely, as θ_i approaches 0 the reaction function becomes orthogonal and there is no carbon leakage.

[2] Assuming each person receives equal benefit from abatement implies α_i is global population share. GDP share is an alternative interpretation of α_i.

Abatement is a function of the coalition of signatories to an IEA. Let $K = 2^n$ be the set of all possible coalitions, and k an element of K.[3] Global abatement as a result of coalition k is $Q(k)$. The non-cooperative (or empty set) global level of abatement $Q(\emptyset) = \sum_{i\in N} q_i(\emptyset)$ is:

$$Q(\emptyset) = \frac{ab\sum_{j\in N}\theta_j}{1 + b\sum_{j\in N}\theta_j}$$
$$q_i(\emptyset) = \frac{ab\theta_i}{1 + b\sum_{j\in N}\theta_j} \tag{3}$$

The non-cooperative level of abatement is increasing in $a, b,$ and $\sum_{j\in N}\theta_j$. With identical[4] nations $\theta = 1/nc$ and $\sum_{j\in N}\theta_j = 1/c$. However, with mean-preserving asymmetry $\sum_{j\in N}\theta_j$ may be either greater or smaller than $1/c$. Typically, asymmetry increases $Q(\emptyset)$ when the high benefit share nations are also low cost. In general, $Q(\emptyset)$ is decreasing in the covariance of α and c. Negative (positive) covariance typically indicates a non-cooperative level of abatement that is greater (lower) than symmetry, however counter-examples exist.

Equation (3) also implies any two nations abate in proportion to their $\theta' s$: $q_i/\theta_i = q_j/\theta_j$. An efficient allocation of abatement levels occurs when the MAC on the last units of abatement in each nation is equalized. Symmetry imposes allocative efficiency: $c_i q_i = c_j q_j$ in the non-cooperative outcome. Any benefit share asymmetry leads to an inefficient allocation of abatement at the non-cooperative level.

2.2 Full cooperation outcome

Full cooperation is obtained when all nations are signatories to an IEA. Abatement by one nation results in a positive externality that accrues to all other $n - 1$ nations. The grand coalition level of abatement internalizes this externality. Full cooperation maximizes the global net benefit function: $B(Q) - \sum_{i\in N} C_i(q_i)$ given the grand coalition MAC curve: $Q/\sum_{i\in N}(1/c_i)$. Abatement by the grand coalition $Q(N)$ is:

$$Q(N) = \frac{ab\sum_{i\in N}\frac{1}{ci}}{1 + b\sum_{i\in N}\frac{1}{ci}}$$
$$q_i(N) = \frac{ab}{c_i\left(1 + b\sum_{j\in N}\frac{1}{c_j}\right)} \tag{4}$$

[3] The empty set and singletons are elements of K.

[4] Defining $\gamma \equiv c/b$ the symmetric $Q(\emptyset) = a/(1+\gamma)$.

$Q(N)$ is increasing in a, b and $\sum_{i\in N}(1/c_i)$. For all mean-preserving distributions $\sum_{i\in N}(1/c_i)$ is minimized for symmetric c_i (proof in the Appendix). With cost asymmetry the global MAC curve is flatter, therefore $Q(N)$ is strictly greater when nations are asymmetric. The full cooperation level of abatement for each nation is unique and independent of the benefit share distribution. The full cooperation outcome is strictly greater than the non-cooperative level since $\sum_{i\in N}(1/c_i) > \sum_{i\in N}\theta_i$ for all $0<\alpha_i<1$, $\sum_{i\in N}\alpha_i = 1$.

The simulations of the symmetric model in Barrett (1994) show that the critical parameters are b and c, while the parameter a does not affect the number of signatories to an agreement. The potential gains to an IEA is the difference between the non-cooperative and full cooperation abatement levels. This difference is:

$$Q(N) - Q(\emptyset) = \frac{ab\left[\sum_{i\in N}\frac{1}{c_i} - \sum_{i\in N}\theta_i\right]}{\left[1 + b\sum_{i\in N}\frac{1}{c_i}\right]\left[1 + b\sum_{i\in N}\theta_i\right]} \tag{5}$$

For a given distribution of MAC slopes and benefit shares, the slope of the global marginal benefit curve determines this difference. $Q(N) - Q(\emptyset)$ is strictly positive, achieves a maximum at $b = 1/\sqrt{\sum_{i\in N}(1/c_i)\sum_{i\in N}\theta_i}$, and is decreasing (increasing) in b for $b > (<) 1/\sqrt{\sum_{i\in N}(1/c_i)\sum_{i\in N}\theta_i}$. Of course, $Q(N) - Q(\emptyset)$ is strictly increasing in a.

2.3 Coalition outcomes

This sub-section determines abatement for any arbitrary partition of signatories and non-signatories to an IEA. For an arbitrary coalition, abatement by signatories is $Q_s(k) = \sum_{s\in k} q_s$, by non-signatories is $Q_t(k) = \sum_{t\notin k} q_t$, and total abatement is $Q(k) = Q_s(k) + Q_t(k)$. Coalition formation follows the Finus and Rundshagen (2003) open membership single coalition game described in the introduction. Signatories choose abatement to maximize collective net benefits and set abatement in each nation to minimize cost. Transfer payments allow the abatement choice in each signatory nation to be unconstrained by the payoff earned outside an IEA. Non-signatories behave as singletons, maximizing individual payoffs.

The reaction function for any non-signatory (2) can be decomposed into abatement by signatories $Q_s(k) = \sum_{s\in k} q_s$, and other non-signatories $Q_{-j}(k) = \sum_{t\neq j\notin k} q_t$. The aggregate reaction function of non-signatories to abatement undertaken by coalition k is:

$$Q_t = \frac{b(a - Q^s)\sum_{j\notin k}\theta_j}{1 + b\sum_{j\notin k}\theta_j} \tag{6}$$

Recognizing the reaction of non-signatories the level of abatement by signatories to an IEA is:

$$Q_s(k) = \frac{ab\sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i}{\left[1+b\sum_{j\notin k}\theta_j\right]^2 + b\sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i}$$
$$q_s(k) = \frac{ab\sum_{i\in k}\alpha_i}{c_s\left(\left[1+b\sum_{j\notin k}\theta_j\right]^2 + b\sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i\right)} \tag{7}$$

Non-signatory abatement is:

$$Q_t(k) = \frac{ab\sum_{j\notin k}\theta_j\left(1+b\sum_{j\notin k}\theta_j\right)}{\left(1+b\sum_{j\notin k}\theta_j\right)^2 + b\sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i}$$
$$q_t(k) = \frac{ab\theta_t\left(1+b\sum_{j\notin k}\theta_j\right)}{\left(1+b\sum_{j\notin k}\theta_j\right)^2 + b\sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i} \tag{8}$$

Global abatement given coalition k is $Q(k) = Q_s(k) + Q_t(k)$:

$$Q(k) = \frac{ab\left(\sum_{j\notin k}\theta_j\left[1+b\sum_{j\notin k}\theta_j\right] + \sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i\right)}{\left(1+b\sum_{j\notin k}\theta_j\right)^2 + b\sum_{i\in k}\frac{1}{c_i}\sum_{i\in k}\alpha_i} \tag{9}$$

3. Coalition stability

The efficient level of signatory abatement $q_s(k)$ may not result in a higher payoff than leaving the agreement. Transfers among signatories can overcome this problem when there is an internal surplus among signatories. First, transfers are specified for a stable coalition according to a simple rule for dividing the surplus. Then coalition stability shows when this surplus exists.

3.1 Transfers

Transfers are implemented through a system of tradable pollution permits which will equate the MAC on the last unit of abatement with the permit price $p(k)$. Under an IEA, signatory s will abate until $p(k) = c_s q_s(k)$. Transfers among signatories are distributed by abatement requirements $q_r(k)$ relative to the efficient level $q_s(k)$. Signatory s supplies $q_s(k) - q_r(k)$ permits, receiving a transfer of $\tau_s(k) = p(k)[q_s(k) - q_r(k)]$. A zero-sum system of transfers equates the sum of required abatement to the aggregate signatory level for that particular coalition, $\sum_{r\in k} q_r = Q_s(k)$.

3.2 Allocation rule and its justification

For all stable coalitions there is a non-negative surplus among signatories where q_r determines the distribution of this surplus. Previous researchers (Botteon and Carraro, 2001; Barrett, 1997) have used burden sharing rules from cooperative game theory to determine q_r. The core, Shapley value, and Nash bargaining solution allocate payoffs, but are rigid and do not directly address the stability of an IEA.

The allocation should distribute the surplus in accordance with the open-membership single coalition game. Existing coalition members may not bar accession by a potential signatory. The core is the set of imputations remaining after eliminating those blocked by all possible sub-coalitions. In an open membership single coalition game members can not block accession, and therefore the core is not an appropriate solution concept.[5] The Shapley value is the weighted average of the marginal contributions of that nation across all sub-coalitions. The Shapley value is unique and always exists, but is inflexible and does not address stability. There is no reason to expect the Shapley value, or the Nash bargaining solution, to be greater than the payoff as a single deviator for all members. Furthermore, the Shapley value is difficult to compute since the value of 2^n coalitions must be determined, as well as each nations individual payoff after leaving each coalition. Similarly, the Nash bargaining solution maximizes the product of differences from threat points, rather than focusing on stability. Botteon and Carraro (2001) illustrate this point by showing that the set of stable coalitions depends on the choice of Shapley or Nash bargaining rules.

The optimal allocation rule is such that transfers are just sufficient to quell any incentive to deviate from the agreement. The rule proposed here is remarkably simple: Distribute the surplus among signatories in proportion to their benefit-cost ratio θ. Each member receives a transfer just sufficient to make coalition

[5] Chander and Tulkens (1995) propose two characteristic functions α and γ, with associated cores. The shortcoming is that they both rely on non-credible threats. The α core assumes coalition members will choose abatement to minimize the payoff of a deviator, and the γ core assumes that coalitions break up into singletons if one member deviates, even if sub-coalitions provide a higher payoff. As stated by Finus (2003), in the γ core members behave as masochists, in the α core they behave as sadists and masochists.

membership incentive compatible and the remaining surplus is distributed by θ share. Coalitions are stable when the aggregate payoff to members is greater than the sum of individual payoffs from leaving. The theta proportion rule distributes any remaining surplus after each coalition member receives their payoff as a single deviator from the agreement. The theta proportion rule is both flexible and simple.

Surplus share for signatory s is: $x_s(k) = (\theta_s / \sum_{s\in k}\theta_s)[\sum_{s\in k}\pi_s(k) - \sum_{t\in k}\pi_t(k\backslash\{t\})]$, where $\pi_t(k\backslash\{t\})$ is the payoff signatory s would earn outside coalition k choosing $q_t(k\backslash\{t\})$ from equation (8). Using this surplus sharing rule $q_r(k)$ solves $\pi_s(k) - \pi_t(k\backslash\{t\}) + \tau_s(k) = x_s(k)$.

$$q_r(k) = q_s(k) + \frac{\pi_s(k) - \pi_t(k\backslash\{t\}) - x_s(k)}{p(k)} \tag{10}$$

3.3 Stability

Let $k\backslash\{i\}$ denote the remaining coalition when signatory i leaves the IEA, and $k\cup\{j\}$ denote the coalition when non-signatory j accedes to the IEA. The subscripts s and t denote signatories choosing abatement level $q_s(k)$ and non-signatories choosing abatement level $q_t(k)$ respectively. The following conditions define individual and coalition stability.

$$\begin{aligned} \pi_s(k) - \pi_t(k\backslash\{t\}) + \tau_s(k) &\geqslant 0, \forall s = t \in k \\ \pi_t(k) - \pi_s(k\cup\{s\}) - \tau_s(k\cup\{s\}) &> 0, \forall t = s \notin k \end{aligned} \tag{11}$$

$$\Phi = \left\{ k \in K : \sum_{s\in k}\pi_s(k) - \sum_{t\in k}\pi_t(k\backslash\{t\}) \geqslant 0 \right\} \tag{12}$$

The conditions for internal and external stability are given by eq. (11). Internal stability ensures each signatory at least as high a payoff choosing $q_s(k)$ with transfer $\tau_s(k)$ than as a single defector from the IEA choosing $q_t(k\backslash\{t\})$. External stability indicates that no non-signatory earns a higher payoff joining coalition k and choosing $q_s(k\cup\{s\})$ with transfer $\tau_s(k\cup\{s\})$. Equation (12) defines the set of all stable coalitions: $\Phi \subset K$. This set consists of members such that the sum of the signatory payoffs exceeds the sum of payoffs as a single defector from the IEA. For all elements $\phi \in \Phi$ there exists a zero-sum system of transfers that simultaneously satisfies the internal stability condition for all signatories $s \in k$, defined by abatement requirements (10).

Permits have multiple purposes. First, they allow each nation to meet their required abatement at the lowest cost. Second, they act as a transfer scheme given the abatement requirements under the IEA. Therefore, $q_r(k)$ is a required contribution to total cost, not required domestic abatement. Condition (12) defines the existence of a self-enforcing IEA with a credible system of side-payments. Non-signatories are excluded from permit trading. Furthermore, (12) shows that there is no incentive to make transfers to nations outside the coalition in exchange for an increase in abatement as in Hoel and Schneider (1997). If the coalition had an incentive to do so, they would simply bring that nation into the coalition. The remaining issues are the size and membership of coalitions, and how much of the difference between the full and no cooperation outcomes can be obtained by stable coalitions.

4. Simulations

Two different simulations show the effectiveness of stable coalitions. First, seven nation simulations on all 2^7=128 possible coalitions are presented to compare with the results of Barrett (1997). Mean-preserving asymmetry investigates the impact of allowing all nations to be asymmetric, rather than simply of two types. The largest abatement stable coalition is determined with transfer payments given by the theta proportion rule at the pollution permit price. Second, twenty nation simulations are conducted on all $2^{20} = 1{,}048{,}576$ possible coalitions for different values of b and c. The twenty nation simulations provide further insight into the role of asymmetry. They allow for differences in the gains to cooperation, and the covariance between cost and benefits, for a far greater number of coalitions.

4.1 Seven nation simulations

Barrett (1997) investigates an asymmetric IEA for two types of nations.[6] Four nations are low benefit, low cost (type 1) and the remaining three are high benefit, high cost (type 2). With side payments determined by Shapley values he finds that two out of 20 potential coalitions are stable, one with three type 1 nations and zero type 2 nations and the other with one type 1 nation and two type 2 nations.

One way to measure IEA performance is the proportion of the difference between the no and full cooperation outcomes. Barrett (1997) finds: $(Q(k) - Q(\emptyset))/(Q(N) - Q(\emptyset)) = 30\%$ and 18% for the two coalitions respectively. However, the importance of this percentage depends on how different the full and no cooperation outcomes are. An IEA that closes a large share of a very small difference may be irrelevant. The effectiveness of an IEA accounts for both the percentage gain and the magnitude of the difference between full and no cooperation: $[(Q(k) - Q(\emptyset))/Q(N) - Q(\emptyset))][Q(N)/Q(\emptyset)]$. Abatement effectiveness for the two coalitions is then: 0.52 and 0.32. Alternatively, an IEA can be judged by the proportion of the global payoff difference and payoff effectiveness obtained. The global payoff gain is: $(\Pi(k) - \Pi(\emptyset))/(\Pi(N) - \Pi(\emptyset)) = 46\%$ and 30%, and payoff effectiveness is $(\Pi(k) - \Pi(\emptyset)/\Pi(N) - \Pi(\emptyset))(\Pi(N)/\Pi(\emptyset)) = 0.56$ and 0.37,

[6] The high benefit nations are high cost in Barrett (1997). Three type 1 nations have $c_1 = 50$ and four type 2 nations have c_2=100 so $\sum_{i \in N} c_i = 500$ and $\bar{c} = 71.43$. The type 1 nations have $\alpha_1 = 9/66$ and the type 2 nations have $\alpha_2 = 10/66$, so $\sum_{\alpha \in N} = 1$ and $b = 66$. The parameter a is normalized to one.

Table 1 Symmetry

	α_i	c_i	θ_i	$q(\emptyset)$	$q(N)$	$\pi(\emptyset)$	$\pi(N)$
$i = 1, \ldots, 7$	0.1429	71.43	0.0020	0.0686	0.1237	3.273	4.083
$\sum_{i \in N}$	1	500	0.0140	0.4802	0.8661	22.91	28.58

Table 2 Symmetric coalition stability

Elements $\in k$	Q	$\pi_s(k)$	$\pi_t(k \backslash \{t\})$
1	0.4640	3.304	3.273
2	0.4945	3.279	3.181
3	0.5663	3.376	3.350
4	0.6562	3.553	3.710
5	0.7419	3.752	4.084
6	0.8124	3.933	4.359
7	0.8661	4.083	4.526

respectively, where $\Pi = \sum_{i \in N} \pi_i$. Allowing for any degree of asymmetry will increase both effectiveness measures.

The seven nation simulations that follow compare symmetry and mean preserving asymmetry of the Barrett (1997) parameters where $c_i = ic_1$ and $\alpha_i = i\alpha_1$ for $i = 1, \ldots, n$. Table 1 presents abatement and payoffs at full and no cooperation for the symmetric parameterization. The difference between c_i and c_{i+1} is $d = 15$, and the difference between α_i and α_{i+1} is $\delta = .0442$. The high benefit nations are also high cost increasing the difference between non-cooperative and grand coalition abatement. For this parameterization asymmetry reduces the non-cooperative and increases the grand coalition levels of abatement.

Including the null set there are eight coalitions of symmetric nations. Let ϕ^* denote the largest abatement stable coalition: $\phi^* \in \Phi \subset K$. Table 2 shows ϕ^* consists of three members resulting in $Q(\phi^*) - Q(\emptyset)/Q(N) - Q(\emptyset) = 22\%$ of the global abatement difference and $\Pi(\phi^*) - \Pi(\emptyset)/\Pi(N) - \Pi(\emptyset) = 36\%$ of the global payoff difference. Global payoff for the three member IEA is $\Pi(\phi^*) = 24.97$. Abatement effectiveness is 0.40 and payoff effectiveness is 0.45.

Table 3 illustrates the impact of allowing for full asymmetry. Under asymmetry the no cooperation abatement and payoffs are lower and the full cooperation levels of abatement and payoffs are greater. Furthermore, $q(N)$ is less than $q(\emptyset)$ for the two highest cost and benefit nations. In the absence of side payments the two lowest cost and benefit nations are worse off at the grand coalition outcome. Both of these results are impossible under symmetry and highlight the role of transfers.

Using eq. (12) all $2^7 = 128$ potential coalitions are tested for stability. Table 4 shows that all coalitions containing one, two and three members are stable, while no coalitions containing more than four members are stable. The largest abatement stable coalition is $\{1, 2, 3, 7\}$ with global abatement 0.6894.

Table 3 Asymmetry, $d = 15$, $\delta = 0.0442$

i	α_i	c_i	θ_i	$q(\emptyset)$	$q(N)$	$\pi(\emptyset)$	$\pi(N)$
1	0.0102	26.43	0.0004	0.0141	0.2725	0.2312	-0.6483
2	0.0544	41.43	0.0013	0.0479	0.1738	1.200	1.149
3	0.0986	56.43	0.0017	0.0638	0.1276	2.146	2.757
4	0.1429	71.43	0.0020	0.0730	0.1008	3.084	4.295
5	0.1871	86.43	0.0022	0.0790	0.0833	4.018	5.800
6	0.2313	101.4	0.0023	0.0832	0.0710	4.950	7.286
7	0.2755	116.4	0.0024	0.0863	0.0619	5.880	8.761
$\sum_{i\in N}$	1	500	0.0123	0.4472	0.8909	21.51	29.40

Table 4 Asymmetric coalition stability

Elements $\in k$	**Stable coalitions Φ**	**Largest abatement**
1	all	0.4438, {1}
2	all	0.5382, {1, 7}
3	all	0.6399, {1, 6, 7}
4	{1, 2, 3, 7}	0.6894, $\phi^* = \{1, 2, 3, 7\}$
5	none	
6	none	
7	none	

The abatement and payoff increases are: $(Q(\phi^*) - Q(\emptyset))/(Q(N) - Q(\emptyset)) = 55\%$ and $(\Pi(\phi^*) - \Pi(\emptyset))/(\Pi(N) - \Pi(\emptyset)) = 80\%$. Coincidently, abatement and payoff effectiveness are both 1.09. Asymmetric effectiveness is higher due to both a higher proportion and a greater difference. The conventional wisdom from both symmetric models and those with two types of nations is that a self-enforcing IEA can not substantially improve on the non-cooperative outcome when the gains to cooperation are large. However, introducing full asymmetry has caused the effectiveness to more than double.

ϕ^* consists of the highest cost and benefit nation (7) and the three lowest cost and benefit nations (1, 2, 3). Table 5 shows the system of side payments $\tau_s(k) = p(k)\{q_s(k) - q_r(k)\}$ consistent with the theta proportion rule from Section 3. The pollution permit price that equates the marginal cost on the last unit of abatement for each coalition member is: $p(\{1, 2, 3, 7\}) = 6.311$. In Table 5 the high benefit nation purchases permits from all three low cost nations, and all four coalition members receive a higher payoff than they would as a single deviator from the agreement.

Additional simulations (not reported) show the reducing the degree of asymmetry by setting $d = 10$ and $\delta = 2/49$ reduces both the abatement gain: $(Q(\phi^*) - Q(\emptyset))/(Q(N) - Q(\emptyset)) = 47\%$, and the global payoff gain: $(\Pi(\phi^*) - \Pi(\emptyset))/(\Pi(N) - \Pi(\emptyset)) = 72\%$, but does not change the stable coalitions in Table 4. Smaller asymmetry reduces abatement and payoff effectiveness to 0.90

Table 5 Largest abatement asymmetric coalition: {1, 2, 3, 7} and transfers

Elements $\in \phi^*$	$\pi_s(\phi^*)$	$\pi_t(\phi^*\backslash\{t\})$	$q_s(\phi^*)$	$q_r(\phi^*)$	$\tau_s(\phi^*)$	$\pi_s(\phi^*)+\tau_s(\phi^*)$
1	−0.4494	0.2785	0.2388	0.1227	0.7327	0.2833
2	1.142	1.505	0.1523	0.0923	0.3792	1.521
3	2.588	2.705	0.1118	0.0899	0.1388	2.727
7	8.043	6.763	0.0542	0.2524	−1.251	6.793
$\sum_{s\in\phi^*}$	11.32	11.25	0.5572	0.5572	0	11.32
$t\notin\phi^*$	$\pi_t(\phi^*)$		$q_t(\phi^*)$			
4	4.199		0.0410			
5	5.493		0.0444			
6	6.785		0.0468			
$\sum_{t\notin\phi^*}$	16.48		0.1321			
$\sum_{i\in N}$	27.80		0.6894			

and 0.95. In general, increasing the variance of the α and c distributions increases both ϕ^* abatement and payoff gain.

4.2 20 nation simulations

Next, twenty nation simulations for various combinations of b and c test the generality of the previous results.[7] Two types of mean-preserving asymmetry are compared with symmetry for all $2^{20} = 1,048,576$ potential coalitions. For the HiLo simulations the high benefit share nations are low cost. The HiLo simulations have: $c_i = ic_1$ and $\alpha_1/i = \alpha_i$ for $i = 1,\ldots,n$. The HiHi simulations reverse the benefit shares making the high benefit nations high cost: $c_i = ic_1$ and $\alpha_i = i\alpha_1$ for $i = 1,\ldots,n$. Furthermore, in order to mimic symmetry the mean preserving spread of benefits and costs in the HiHi simulations is such that $\theta_i = (1/nc)\ \forall i \in N$. Constant thetas increase the gains to cooperation since $Q(\emptyset)$ is the same as symmetry while $Q(N)$ is greater.[8]

Table 6 shows the largest abatement stable coalition ϕ^*. For HiLo simulations ϕ^* is typically not the stable coalition with the highest number of signatories. For example, along the main diagonal where $b/\bar{c} = 1$ many eight member coalitions are stable, but global abatement is less than the stable five member coalition. This is not the case for the HiHi ϕ^*, where the largest abatement stable coalition always contains the greatest number of signatories. In the HiLo case ϕ^* always contains the two highest benefit and lowest cost nations $(1, 2)$. Similarly, the HiHi ϕ^* always contains the highest benefit (20) and lowest cost $(1, 2, 3)$ nations.

[7] Barrett (1997) also conducts n=100 simulations without transfers for 97 type 1 nations and three type 2 nations. With full asymmetry there are $2^{100} = 1.27\times10^{30}$ possible coalitions for each of the nine combinations of b and c.

[8] In order to conserve on space, stability results for the more than 18 million coalitions tested in Table 6 are omitted. The full simulations were conducted in Maple and are available from the author by request. Furthermore, the programs may be augmented to test any value of n.

Table 6 N = 20 ϕ^* largest abatement stable coalitions

		$\bar{c}$		
		0.01	**1**	**100**
		Symmetry		
	0.01	{3 members}	{2 members}	{2 members}
	1	{17 members}	{3 members}	{2 members}
	100	{N}	{17 members}	{3 members}
		Asymmetry: HiLo		
	0.01	{1,2,3,10,20}†	{1,2,5}††	{1,2,3}††
b	1	{N}	{1,2,3,10,20}†	{1,2,5}††
	100	{N}	{N}	{1,2,3,10,20}†
		Asymmetry: HiHi		
	0.01	{1,2,3,4,20}	{1,2,3,5,20}	{1,2,3,5,20}
	1	{N}−{2,18,19}	{1,2,3,4,20}	{1,2,3,5,20}
	100	{N}	{N}−{2,18,19}	{1,2,3,4,20}

† (††) denotes coalitions of up to and including eight (five) members are stable but with $Q\{k\} < Q(\phi^*)$.

Table 6 shows that asymmetry increases the number of signatories to a self-enforcing IEA, but participation falls short of the grand coalition when the gains to cooperation are large.

The asymmetric abatement and effectiveness levels on or above the main diagonal of Table 7 are remarkable. When $b/\bar{c} = 1$ a coalition of symmetric nations results in only 5% of the difference between the full and no cooperation outcomes. When the high benefit nations are also high cost (HiHi) a stable coalition results in 47% of the difference, even though the full cooperation level of abatement is greater. The effectiveness measure multiplies the percentage difference by the ratio of full to no cooperation. The effectiveness measure for HiHi is 10 times the symmetric result, capturing the fact that the asymmetry results in a larger percentage of a larger difference. The noncooperative level is much greater when the high benefit nations are low cost (HiLo). For HiLo $b/\bar{c} = 1$ the difference falls to 34% and the effectiveness to 0.45, even though abatement is higher. Surprisingly, the more effective agreement is when the gains to cooperation are larger. The HiHi simulations have more effective agreements due to greater gains from permit trading.

When $b/\bar{c} = 0.01$ symmetry results in 1% of the difference between no and full cooperation. The difference is 11% for both types of asymmetry, and the effectiveness measures are dramatically higher since the grand coalition is nearly twice the level for symmetry. Below the main diagonal there is essentially no difference between $Q(\emptyset)$ and $Q(N)$ so asymmetry has little impact. Consequently, the effectiveness measures below the main diagonal are essentially the same as $(Q(\phi^*) - Q(\emptyset))/(Q(N) - Q(\emptyset))$, and have no importance.

Table 7 N = 20 ϕ^* global abatement for symmetry and asymmetry

		$\bar{c}$								
		0.01			**1**			**100**		
		Sym	**HiLo**	**HiHi**	**Sym**	**HiLo**	**HiHi**	**Sym**	**HiLo**	**HiHi**
		50.00	73.53	50.00	0.990	2.703	0.990	0.010	0.028	0.010
		52.23	81.69	72.06	1.086	5.451	3.805	0.011	0.062	0.040
	0.01	95.24	97.42	97.42	16.67	27.42	27.42	0.200	0.376	0.376
		(5%)	(34%)	(47%)	(1%)	(11%)	(11%)	(1%)	(10%)	(8%)
		(0.09)	(0.45)	(0.91)	(0.10)	(1.13)	(2.95)	(0.11)	(1.34)	(3.06)
		99.01	99.64	99.01	50.00	73.53	50.00	0.990	2.703	0.990
		99.06	99.97	99.43	52.23	81.69	72.06	1.086	5.451	3.805
b	1	99.95	99.97	99.97	95.24	97.42	97.42	16.67	27.42	27.42
		(5%)	(100%)	(44%)	(5%)	(34%)	(47%)	(1%)	(11%)	(11%)
		(0.05)	(1.00)	(0.44)	(0.09)	(0.45)	(0.91)	(0.10)	(1.13)	(2.95)
		99.99	100.0	99.99	99.01	99.64	99.01	50.00	73.53	50.00
		100.0	100.0	100.0	99.06	99.97	99.43	52.23	81.69	72.06
	100	100.0	100.0	100.0	99.95	99.97	99.97	95.24	97.42	97.42
		(100%)	na	(100%)	(5%)	(100%)	(44%)	(5%)	(34%)	(47%)
		(1.00)	na	(1.00)	(0.05)	(1.00)	(0.44)	(0.09)	(0.45)	(0.91)

The entries in each cell are $Q(\emptyset)$, $Q(\phi^*)$, $Q(N)$, $(Q(\phi^*) - Q(\emptyset))/(Q(N) - Q(\emptyset))$ and $((Q(\phi^*) - Q(\emptyset))/(Q(N) - Q(\emptyset))(Q(N)/Q(\emptyset)))$.

The same pattern emerges for the global payoff gains presented in Table 8. When $b/\bar{c} = 1$ HiHi asymmetry results in 72% of the payoff gain, compared to 9% with symmetry. For all cells on or above the main diagonal the asymmetric differences and effectiveness substantially improve the abatement and payoff gains attainable by an IEA. Contrary to conventional wisdom, when the gains to cooperation are largest, asymmetry results in substantially more abatement and higher payoffs. IEAs can substantially improve on the non-cooperative outcome when the gains to cooperation are large.

5. Conclusion

This paper generalizes the benchmark model of IEAs by incorporating different MAC slopes and benefit shares. Dramatically more encouraging results emerge from relaxing the convenient, but unrealistic, assumption of symmetry. Allowing for asymmetry does not overturn the fundamental result that there is a trade-off between the gains to an IEA and the number of signatories. However, symmetric models may vastly understate the degree of abatement achievable by a self-enforcing IEA regardless of the size of the gains to cooperation. In general, when the covariance between the benefit shares and the MAC slopes is negative the non-cooperative level and stable coalition abatement is greater than symmetry would indicate. The full cooperation level of abatement is unambiguously higher when nations differ.

Table 8 N = 20 ϕ^* global payoffs for symmetry and asymmetry

		$\bar{c}$								
		0.01			1			100		
		Sym.	HiLo	HiHi	Sym.	HiLo	HiHi	Sym.	HiLo	HiHi
		36.88	45.70	36.88	0.961	2.558	0.961	0.010	0.027	0.010
		37.88	47.23	45.35	1.049	4.732	3.495	0.011	0.055	0.037
	0.01	47.62	48.71	48.71	8.333	13.71	13.71	0.100	0.188	0.188
		(9%)	(51%)	(72%)	(1%)	(19%)	(20%)	(1%)	(17%)	(15%)
		(0.12)	(0.54)	(0.95)	(0.10)	(1.04)	(2.84)	(0.11)	(1.23)	(2.97)
		4997.06	4998.47	4997.06	3688	4570	3688	96.07	255.8	96.07
		4997.10	4998.68	4998.37	3788	4723	4535	104.9	473.3	350.0
b	1	4997.50	4998.68	4998.68	4762	4871	4871	833.3	1371	1371
		(10%)	100%	(81%)	(9%)	(51%)	(72%)	(1%)	(19%)	(20%)
		(0.10)	(1.00)	(0.81)	(0.12)	(0.54)	(0.95)	(0.10)	(1.04)	(2.84)
		499997	499999	499997	499706	499847	499706	368750	456982	368750
		499998	499999	499999	499710	499868	499837	378788	472269	453455
	100	499998	499999	499999	499750	499868	499868	476190	487106	487106
		(100%)	na	(100%)	(10%)	(100%)	(81%)	(9%)	(51%)	(72%)
		(1.00)	na	(1.00)	(0.10)	(1.00)	(0.81)	(0.12)	(0.54)	(0.95)

The entries in each cell are $\Pi(\emptyset)$, $\Pi(\phi^*)$, $\Pi(N)$, $(\Pi(\phi^*) - \Pi(\emptyset))/(\Pi(N) - \Pi(\emptyset))$ and $(\Pi(\phi^*) - \Pi(\emptyset))/(\Pi(N) - \Pi(\emptyset))(\Pi(N)/\Pi(\emptyset))$.

The size and scope of an agreement is increased by exploiting asymmetries, which introduce the possibility of IEAs that can substantially overcome the free-rider problem given the appropriate set of abatement requirements. Coalitions are stable when the aggregate payoff to members exceeds the sum of payoffs from individually leaving the coalition. Nations receive a net transfer sufficient to remain a signatory. The theta proportion rule is a simple method of distributing any remaining surplus, which determines required abatement for a given coalition. Transfers are zero-sum and coalition specific. No assumptions have been made regarding which stable coalitions will arise. However, external stability implies coalitions will continue to expand membership when it is profitable to do so, reducing the set of expected outcomes. Regardless, nothing ensures that the maximum abatement stable coalition will form from this subset of stable coalitions. This suggests an important role for international institutions.

Despite the encouraging results from including asymmetry, the difficulties in implementing the Kyoto Treaty remain evident. Even with asymmetry there exists the fundamental trade-off between the gains to an IEA and the number of signatories. Moreover, the abatement requirements under the Kyoto Treaty are not conducive to promoting internal stability. Under Kyoto, nations are required to reduce emissions of greenhouse gases by a minimum of 5.5% from their 1990 levels. A nation such as the United States, which accounts for roughly 1/5 of world emissions, would be responsible for 1/5 of total abatement. If the benefit shares are determined by population, it is not difficult to find parameter values where the Kyoto requirements violate incentive compatibility for the United States.

Alternative coalition formation rules is a potential topic for future research. With the single coalition open membership game (Finus and Rundshagen, 2003) there is no way to ensure that the largest abatement stable coalition ϕ^* is obtained. Other coalition formation sequences may occur that do not lead to ϕ^*. Exclusive membership rules may lead to higher abatement in this situation. It is also possible that the formation of multiple coalitions can achieve more than a single IEA. Alternatively, linking emissions reductions with other issues has been shown by Cesar and de Zeeuw (1996) and Folmer *et al.* (1993) to increase the degree of cooperation sustainable by self-enforcing agreements. Linkages in conjunction with side-payments in the form of pollution permits should prove to be more effective than either alone. Increasing the variance of the benefit shares and marginal cost slopes is another area for future research. The highest benefit nation has 20 times the share of the lowest nation in these simulations. This is a relatively small dispersion given empirical estimates (Ellerman *et al.*, 1998). Increasing variance should lead to even higher levels of abatement and payoffs attainable by a self-enforcing IEA, even if the number of signatories does not dramatically increase.

Acknowledgements

This work derives from the first chapter of my Ph.D. dissertation, completed June, 2002 at the University of California, Santa Cruz. This paper has greatly benefited from comments by Dan Friedman, John Heywood, seminar participants at the University of Minnesota's Applied Economics series, and anonymous referees.

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Appendix

The $\sum_{i\in N}(1/c_i)$ is minimized for identical c_i.

Proof Let d_i be the deviation from the mean $\bar{c} \equiv \sum_{i\in N}(c_i/n)$. A mean preserving distribution of c_i implies the constraint: $\sum_{i\in N} d_i = 0$. The Lagrangian is: $\min_{d_1,\ldots,d_n} \left\{L = \sum_{i\in N}(1/(\bar{c}+d_i)) + \lambda(0 - \sum_{i\in N} d_i)\right\}$, subject to $\sum_{i\in N} d_i = 0$. The n first order conditions are of the form: $-1/(\bar{c}+d_i)^2 = \lambda$. For all $c_i > 0$ the second order condition for a minimum is satisfied. Any two first order conditions imply $1/(\bar{c}+d_i) = 1/(\bar{c}+d_j)$, therefore the sum is minimized for identical deviations, $d_1 = d_2 =, \ldots, = d_n$. The constraint $\sum_{i\in N} d_j = 0$, implies $d_1 = d_2 =, \ldots, = d_n = 0$, and the sum $\sum_{i\in N} 1/(\bar{c}+d_i)$ is minimized for identical c_i. □