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Cross-border mergers in a mixed oligopoly

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ABSTRACT

This paper identifies the unique strategic issues of cross-border mergers in a mixed oligopoly showing that the presence of a welfare maximizing public firm increases the incentive for such mergers. The well-known merger paradox that two-firm mergers are rarely profitable is substantially relaxed in the cases of both linear and convex production costs. The ability to identify profitable two-firm mergers in this context takes on added importance as the recent cross-border merger wave often involved industries with public firms.

1. Introduction

Cross-border mergers have become increasingly common and raise unique economic issues. Over the decade of the 1990s, the number of cross-border mergers increased more than six-fold and rose to represent nearly one-third of all mergers (Kang and Johansson, 2000). Such mergers are particularly common in the increasingly integrated European market (OECD, 2003, p. 158) and are increasing rapidly in the liberalizing Asian-Pacific economies (Chen and Findlay, 2003). Indeed, in their examination of the determinants of European mergers, Zademach and Rodrigeuz-Pose (2007) found that from 1998-2003 the number of cross-border mergers involving at least one European firm exceeded the number of domestic mergers in which both firms were from the same European country.

Economic theory often views cross-border mergers as an alternative to greenfield investment for a foreign firm wishing to produce in a local market. At least two broad explanations model this choice. Drawing heavily from the management literature, one explanation stresses the heterogeneity of firms. In this view the merger exploits complementary "capabilities." Thus a foreign firm with mobile capabilities such as "intangible technological advantages" benefits from merging with a local firm with country specific capabilities such as marketing, distribution and country-specific institutional competency (Caves, 1996; Anand and Delios, 2002; Nocke and Yeaple, 2007). The second explanation draws from the industrial organization literature on endogenous mergers viewing market power as the driver of cross-border mergers (Head and Ries, 1997; Horn and Persson, 2001; Bjorvatn, 2004 and Neary, 2007). This view often stresses that increased integration of international economies gives rise to greater opportunities for profitable cross-border mergers.

While our contribution more nearly follows this second explanation, we emphasize that cross-border mergers raise the possibility of strategic interactions with public firms (firms owned by the government).[1] In particular, cross-border mergers change the "nationality" of at least one firm and in the presence of a public firm this change in ownership can make mergers between a domestic firm and foreign firm uniquely profitable. We make clear that neither a public firm in a domestic market nor the presence of foreign firms without a public firm is sufficient to generate such profitable mergers. Our demonstration of profitability is important as the canonical model by Salant, Switzer and Reynolds (1983) shows that with linear costs and demand, Cournot-Nash rivals can never participate in a profitable two firm merger if there remains even a single excluded rival (this result is called the "merger paradox"). While there have been many other attempts to modify or reverse this paradox, none have modeled the influence of a public firm in a market with cross-border mergers.[2]

We show that the acquisition of a foreign private firm by a domestic private firm generates two competing influences on the welfare objective of the public firm. First, the total number of firms

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[1] Such firms exist in both important mature economies and most transition economies (Bortolotti et al., 2004).

[2] These attempts include, among others, allowing the merger to generate a Stackelberg leader (Daughety, 1990), allowing for convex costs (Perry and Porter, 1985; Heywood and McGinty, 2007), considering managerial delegation (Ziss, 2001; Gonzalez-Maestre and López-Cuñat, 2001), examining multi-divisional firms (Creane and Davidson, 2004; Mialon, 2008), considering models of spatial price discrimination (Rothschild et al., 2000) and allowing for free-entry (Davidson and Mukherjee, 2006).

and, so total output, is reduced. This increases the price, reduces consumer surplus and encourages the public firm to increase output. Second, the acquisition of the foreign firm by a domestic firm repatriates profit that otherwise would have gone out of the country and not contributed to domestic welfare. This greater emphasis on profit encourages the public firm to increase domestic profit by reducing output. We demonstrate that that the net effect in models with both linear and convex costs is for the second influence to dominate. The public firm reduces output in response to the merger. Moreover, when the total number of firms in the market is relatively small, this reduction is sufficient to generate profit for the merger participants in contrast to the canonical merger paradox.

The economic modeling of a public firm designed to improve the functioning of a private oligopoly is nearly 50 years old (see Merrill and Schneider, 1966). Yet, considering such a mixed oligopoly in a context that includes foreign firms began with a highly cited article by Fjell and Pal (1996).[3] They show that as the number of foreign firms increase, and as the profit that is domestic falls, a domestic welfare maximizing public firm responds by concentrating more on consumer surplus and increasing its output. As Fjell and Pal recognized, this has implications for cross-border acquisition. If foreign nationals (foreign buyers not currently in the industry as opposed to a foreign firm) purchase a domestic firm, the output of the public firm will increase. If domestic nationals (as opposed to a domestic firm) purchase a foreign firm, the output of the public firm will decrease. Yet, despite many articles building on the framework of allowing foreign firms in a mixed oligopoly, none have pursued further the issue of mergers.[4] We emphasize that our examination differs from the work of Fjell and Pal as we examine a merger that combines two firms and thereby reduces the total number of firms. Fjell and Pal were concerned only with changing the nation of ownership for a fixed number of firms and so never address the merger paradox.

In what follows, we posit a public firm that chooses output to maximize domestic welfare. It has both domestic private rivals and foreign private rivals. We examine the profit incentive for a cross-border merger between private firms focusing on the strategic interaction with the public firm. The next section outlines the case of linear cost but with an inefficient public firm. The third section outlines the case of convex costs and an equally efficient public firm. In both cases, profitable two firm mergers result because of the strategic interaction with the public firm. A penultimate section considers an extension that examines the merger of two foreign firms and a final section emphasizes the policy relevance of our results and concludes.

2. The Case of Linear Costs

We imagine that there exist m domestic and n foreign private firms and one public firm competing in a single domestic market. All $T=m+n+1$ firms produce a homogenous product. Let q_0 be the output of the public firm, q_i be the output of the private firm i ($i=1,\ 2,\cdots,\ m+n$). All firms have linear cost functions but the public firm produces inefficiently: cost per unit of c for all private firms and $c+\varepsilon$ for the public firm.

$$\begin{aligned} c_i(q_i) &= cq_i \qquad \forall\, i=1 \text{ to } n+m \\ c_o(q_o) &= (c+\varepsilon)q_o \end{aligned} \tag{1}$$

As we ignore issues of entry, the number of firms is fixed without loss of generality. There exists an inverse demand curve, $P=a-Q$, yielding consumer surplus (CS) as $CS=\frac{1}{2}Q^2$ where $Q=q_o+\sum_{i=1}^{m+n} q_i$.

Thus the profit of each firm can be written as:

$$\pi_i = q_i(a-Q)-c_i(q_i), \qquad i=0,1\ldots\ m+n \tag{2}$$

The private firms each maximize their own profit while, following the work since Fjell and Pal (1996), the public firm maximizes domestic welfare (W), the sum of consumer surplus and all domestically earned profit (the profits of foreign firms are expatriated):

$$W = CS+\pi_0+\sum_{i=1}^{m}\pi_i \tag{3}$$

The firms play a simultaneous Cournot-Nash game in quantities. The resulting equilibrium is given by:

$$\begin{aligned} q_o &= (a-c-\varepsilon)-\left(\frac{m}{n+1}\right)\varepsilon \\ q_i &= \frac{\varepsilon}{n+1} \qquad \forall\, i=1,2,..m+n \\ Q &= (a-c-\varepsilon)+\left(\frac{n}{n+1}\right)\varepsilon \\ P &= (c+\varepsilon)-\left(\frac{n}{n+1}\right)\varepsilon \\ \pi_i &= \frac{\varepsilon^2}{(n+1)^2} \qquad \forall\, i=1,2,..m+n \end{aligned} \tag{4}$$

As is clear, if there are no foreign firms ($n=0$), the private firms each produce ε and the public firm reduces its output by the same ε for each of the private firms in the market. Thus, regardless of the number of private domestic firms, the total output in the market remains $Q=a-c-\varepsilon$. In such an environment, the merger of two domestic firms does not change the total output or final price meaning that the merger cannot be profitable. Yet, the behavior of the public firm changes with the presence of foreign firms as the public firm no longer reduces its output unit for unit for the output of the domestic firms. To do so would maintain a higher price and greater profits, a portion of which would now flow out of the country. As a consequence, the output of the public firm decreases less than unit for unit with an increase in the number of domestic firms but also increases with an increase in the number of foreign firms. At an extreme, if there are no domestic firms ($m=0$), the public firm produces the maximum quantity of $a-c-\varepsilon$ regardless of the number of foreign firms. In essence, it is concerned primarily with consumer surplus as virtually all profit leaves the country.[5]

We now consider the acquisition of a foreign private firm by a domestic private firm. The newly merged firm attempts to increase its profit by reducing output. As all costs are linear, the merged firm enjoys no cost advantage over its rivals. Absent the public firm, the fact that the merged firm is identical to its rivals, and the fact that the

[3] Indeed, Google Scholar identified 140 citations to Fjell and Pal (1996) as of June 2010.

[4] Nakamura and Inoue (2007) consider a merger between a public firm and private firm in strictly domestic oligopoly. Arz et al. (2009) consider both merger between two private firms in the face of a public firm and the merger of a public and private firm but again in a domestic oligopoly. Mergers with a public firm are profitable because the firms initially produce different amounts and so a merger allows two post merger plants to produce the same total quantity more cheaply given the assumption of convex costs. Indeed, Heywood and McGinty (2008) confirm a profitable merger between a Stackelberg leader and a follower in a private oligopoly for exactly the same reason. These models deviate from the traditional merger paradox in that the profitability is generated by a productivity improvement (see Farrell and Shapiro, 1990). Barcena-Ruiz and Garzon (2005) study a duopoly and imagine a merger in which the private firm receives an exogenously given degree of ownership in the previously public firm. They emphasize that merger is more likely when the two firms' goods are poor substitutes.

[5] Note that in this circumstance, the public firm will earn losses as the additional output of the foreign firms reduce the price below the cost of the public firm even as they increase consumer surplus by a greater amount.

rivals increase output in response to the merged firm's reduction will make a profit gain impossible (Salant et al., 1983). The difference here is that the acquisition reduces the number of foreign private firms causing a reduction in output by the public firm. Specifically, there remain m domestic private firms but now n-1 foreign private firms. This gives rise to the following post-merger equilibrium identified with the superscript M:

$$\begin{aligned} q_o^M &= (a-c-\varepsilon)-\left(\frac{m}{n}\right)\varepsilon \\ q_i^M &= \frac{\varepsilon}{n} \qquad \forall\, i=1,2,..m+n-1 \\ Q^M &= (a-c-\varepsilon)+\left(\frac{n1}{n}\right)\varepsilon \\ P^M &= (c+\varepsilon)-\left(\frac{n-1}{n}\right)\varepsilon \\ \pi_i^M &= \frac{\varepsilon^2}{n^2} \qquad \forall\, i=1,2,..m+n-1 \end{aligned} \tag{5}$$

The consequences of the merger can be evaluated by subtracting the values in (4) from those in (5) and gives rise to the following proposition.

Proposition 1. The acquisition of a foreign firm by a domestic firm will cause i) the output of the merged firm to fall relative to its two pre-merger constituent firms; ii) the output of the public firm to decrease; iii) the output of all excluded rivals to increase and iv) total output in the market to decrease.

Proof.
i) $q_i^M - 2q_i = -\frac{(n-1)\varepsilon}{n(n+1)} \le 0$ as $n \ge 1$
ii) $q_o^M - q_o = \frac{-m\varepsilon}{n(n+1)} < 0$ as $n > 0$
iii) $q_i^M - q_i = \frac{\varepsilon}{n(n+1)} > 0 \quad \forall i = 1,\ 2...(m+n-2)$
iv) $Q^M - Q = \frac{-\varepsilon}{n(n+1)} < 0$ as $n > 0$

Thus, in contrast to the private oligopoly in which all rivals increase output in response to the reduction in output resulting from the merger, here the public firm decreases output. The size of the decrease in output by the public firm varies inversely with the number of foreign firms but directly with the number of domestic firms. At issue is whether this decrease in output is sufficient for the merged firm to earn profit.

First, it is apparent that all excluded private rivals have an increase in profit as their quantity has increased and the price has increased following the decline in total market quantity. Second, this decrease in total quantity proves sufficient for the profit of the private firms to exceed the profit of two pre-merger firms when the number of foreign firms is sufficiently small.

Proposition 2. The acquisition of a foreign private firm by a domestic private firm causes i) the profit of excluded rivals to increase and ii) the profit of the merged firm to be larger than its pre-merger constituent firms when the number of foreign firms is either two or one.

Proof.
i) $\pi_i^M - \pi_i = \frac{\varepsilon^2(2n+1)}{n^2(1+n)^2} > 0$
ii) $\pi_i^M - 2\pi_i = \frac{\varepsilon^2(2n+1-n^2)}{n^2(1+n)^2} > 0$ iff $n = 1$ or 2

Thus, if a domestic firm purchases a foreign firm when there are two or fewer foreign firms, the restriction in output by the public firm is sufficiently large that the profit to the remaining firms exceeds that of two pre-merger firms. Importantly, this is true regardless of the number of domestic firms. When the number of foreign firms is large the decline in output by the public firm is too small to generate profit from the merger. Nonetheless, it is the combination of the welfare maximizing public firm and the presence of the foreign firms that allows this partial resolution of the merger paradox.[6]

The welfare change resulting from the merger can be divided into the domestic profit increase and the decline in consumer surplus. This change is unambiguous for any merger regardless of whether or not the merging parties see a profit increase.

Proposition 3. The acquisition of a foreign private firm by a domestic private firm increases domestic welfare.

Proof. Substitute the values of (5) into (3) and from (4) into (3) using (2) as needed. Subtracting the welfare expressions: $W^M - W = \frac{\varepsilon^2[1+2mn+2(m-1)n^2]}{n^2(1+n)^2} > 0$ as $m \ge 1$.

The critical point is that the profit increase exceeds reduction in consumer surplus. The profit increases not only because the price increases but also because of the shift in production away from the inefficient public firm and toward the efficient private domestic firms.

To help understand the intuition of these results one could imagine as an alternative the acquisition of a private domestic firm by a private foreign firm. In this case, the pre-merger equilibrium in (4) is replaced by a post-merger equilibrium in which there are m-1 domestic firms but there remain n foreign firms. The output of the public firm increases in response to the merger while the remaining private firms each continue to produce $q_i = \frac{\varepsilon}{n+1}$. Indeed, the increase in the public firm's output exactly offsets the output lost by the merger. The price doesn't change and there can be no profit from the merger. Thus, the response of the public firm to the merger is critical and encourages the acquisition by domestic firms but not by foreign firms.

3. The Case of Convex Costs

The alternative assumption of convex costs has a long history in both the literature on the merger paradox and that on mixed oligopolies. Perry and Porter (1985) show that a merger of any number of firms could be profitable with sufficient convexity. Yet, Heywood and McGinty (2007) emphasize that in many cases the needed convexity is unrealistically large. The size of the needed convexity is especially relevant in a mixed oligopoly in which the most common assumption has been that the costs of the public and private firms are identical and convex. Under this assumption, the public firm produces more than the private firms but its increasing marginal cost prohibits it from expanding to provide the entire market (DeFraja and Delbono, 1989; Fjell and Pal (1996). In this section, we adopt the assumption of identical convex costs and determine the extent to which the merger paradox can be reversed and mergers become profitable.[7]

This fundamental change yields a modification of (1):

$$C_i(q_i) = (1/2)kq_i^2 \qquad \forall\, i = 0,1...n+m \tag{6}$$

Here k is the degree of convexity as measured by the slope of the marginal cost curve. We continue to ignore issues of entry and take the

[6] As all private firms earn the same after the merger, it is clear that the increase in profit for an excluded private rival will always exceed the gain to the merger participants.

[7] Bhaumik and Dimova (2004) support the assumption of similar costs by showing that public banking firms in India responded to entry with savings that brought cost equality with private banking firms. On the other hand, Zhang et al. (2001) present evidence that Chinese state-owned enterprises are less efficienct regardless of the extent of competition. More generally, Hodge's (2000) meta-analysis of privatization shows the actual effects on cost are as likely to be negative as positive. Despite mixed evidence, the assumptions of convex and identical costs prohibit the possibility of the public firm producing all of the output and achieving first best welfare (Claude and Hindriks, 2005).

number of firms to be exogenous and, as a consequence, normalize fixed costs to zero. The demand and objective functions remain as before in (2) and (3). This results in the following pre-merger equilibrium:

$$\begin{aligned} q_o &= \frac{a(n+1+k)}{k^2+k(m+n+2)+n+1} \\ q_i &= \frac{ak}{k^2+k(m+n+2)+n+1} \qquad \forall\, i=1,2...m+n \\ Q &= \frac{a[(m+n+1)k+n+1]}{k^2+k(m+n+2)+n+1} \\ P &= \frac{ak(1+k)}{k^2+k(m+n+2)+n+1} \\ \pi_i &= \frac{a^2k^2(2+k)}{2\left[k^2+k(m+n+2)+n+1\right]^2} \qquad \forall\, i=1,2...m+n \end{aligned} \tag{7}$$

This equilibrium exactly mimics that presented by Fjell and Pal (1996). We now consider a merger of a domestic private firm with a foreign private firm.

The resulting cost function of the merged firm is $C(q_M^M)=(1/4)k(q_M^M)^2$ (Perry and Porter, 1985; Heywood and McGinty, 2007). As before, the superscript indicates post merger and the new subscript identifies the merged firm. As the merged firm consists of the two pre-merger plants, its total cost of production is identical to that of its two pre-merger constituent firms only if it produces the same output after merger. Yet, the merged firm will attempt to earn additional profit by reducing its output. The new equilibrium cannot be computed from simply reducing the number of foreign firms in (7) as was done with linear costs. Instead, the new equilibrium must recognize the new cost function of the merged firm. That equilibrium is as follows:

$$\begin{aligned} q_o^M &= \frac{a(n+k)(2+k)}{\Omega} \\ q_i^M &= \frac{ak(2+k)}{\Omega} \qquad \forall\, i=1,2...(m+n-2) \\ q_M^M &= \frac{2ak(1+k)}{\Omega} \\ Q^M &= \frac{a\left[k^2(n+m+1)+k(3n+2m)+2n\right]}{\Omega} \\ P^M &= \frac{ak\left(n+k^2+3k+2\right)}{\Omega} \\ \pi_i^M &= \frac{a^2k^2(k+2)^3}{2\Omega^2} \qquad \forall\, i=1,2...(m+n-2) \\ \pi_M^M &= \frac{a^2k^2(k+1)^2(k+4)}{\Omega^2} \end{aligned} \tag{8}$$

where $\Omega=k^3+k^2(m+n+4)+k(2m+3n+2)+2n$.

Several comparisons follow immediately:

Proposition 4. Given convex costs, the merged firm produces more and earns more profit than excluded private rivals (either foreign or domestic).

Proof. i) $q_M^M-q_i^M=\frac{ak^2}{\Omega}>0$

ii) $\pi_M^M-\pi_i^M=\frac{a^2k^3(6+6k+k^2)}{2\Omega^2}>0$

This differs from the case of linear costs because the merged firm now has a cost incentive to retain both plants resulting in an output that exceeds that from single plant firms.

Proposition 5. The acquisition of a foreign firm by a domestic firm in the case of convex costs will cause i) the output of the merged firm to fall relative to its two pre-merger constituent firms; ii) the output of the public firm to decrease; iii) the output of all excluded rivals to increase and iv) total output in the market to decrease.

Proof.

i) $q_M^M-2q_i=-\frac{2ak[k(m+n+k-1)+n-1]}{\Omega[k^2+k(m+n+2)+n+1]}<0$ as $n\geq 1$

ii) $q_o^M-q_o=-\frac{ak[k(n+m-1)+n-1]}{\Omega[k^2+k(m+n+2)+n+1]}<0$ as $n\geq 1$

iii) $q_i^M-q_i=\frac{ak(3k+2)}{\Omega[k^2+k(m+n+2)+n+1]}>0$

iv) $Q^M-Q=-\frac{ak\left(3k^2+5k+2\right]}{\Omega[k^2+k(m+n+2)+n+1]}<0$

Thus, despite convex costs that limit the ability of the public firm to replace private sector output, the basic intuition from the linear case carries over. The public firm reduces its output in response to the merger. The fact that the domestic profit increases causes the public firm to give less emphasis to consumer surplus and decrease its output. This dominates the loss of a firm from the market that would otherwise cause the public firm to increase its output. Indeed, it is easy to show that the acquisition of a domestic firm by a foreign firm would cause both of these effects to move the same direction and the public firm would, indeed, increase its output. Moreover, the fact that both the merged firm and the public firm reduce output results in the total output falling and prices increasing.

Again, the possibility of a profitable merger exists because of the reduction in quantity by the public firm. Critically, this can happen independently of the extent of convexity, k. This independence differs from private oligopolies in which only for very high levels of k can two firms profitably merge (Heywood and McGinty, 2007).

Proposition 6.A. With convex costs, the acquisition of a foreign private firm by a domestic private firm will be profitable regardless of the degree of convexity when $n=1$ and $m=1$, 2, 3, 4, or 5 and when $n=2$ and $m=1$, 2 or 3.

Proof.

$$\pi_M^M-2\pi_i=\frac{a^2k^2\cdot g(k,m,n)}{\Omega^2\left[k^2+k(m+n+2)+n+1\right]^2}$$

where

$$\begin{aligned} g(k,m,n) = {} & 3k^5+24k^4-\left(3n^2+3m^2+6nm-8n-8m-59\right)k^3 \\ & -\left(10n^2+4m^2+14nm-26n-18m-58\right)k^2 \\ & -\left(11n^2+8nm-26n-8m-25\right)k-4n^2+8n+4 \end{aligned}$$

Substitution yields unambiguous profit gains for the cases identified. For example $g(k,m=3,n=2)=3k^5+24k^4+24^3+4k^2+9k+4>0\Rightarrow \pi_M^M-2\pi_i>0\,\forall\, k>0$.

Thus, for the small numbers cases that are arguably the most important for policy, the merger is unambiguously profitable.

Also note that for large k, high degrees of convexity, the higher order terms in g will dominate and that again implies a positive gain to merging. This is merely the mixed oligopoly equivalent of the demonstration by Perry and Porter (1985) that for sufficient convexity any two firm merger will be profitable even in a private oligopoly.

At issue is whether for a given degree of convexity the mixed oligopoly with foreign firms makes merger more profitable. To make this comparison we start with the results from Heywood and McGinty (2007) who isolate the critical relationship between convexity and the total number of firms in a private market, T, such that a two firm merger is profitable. They use an identical cost function to that in (6) and derive the $T^*(k)$ locus such that the profit from merger equals zero. Thus, for n smaller than this locus or for k larger than this locus, profit from merger is positive. We reproduce this zero profit condition in the Appendix A

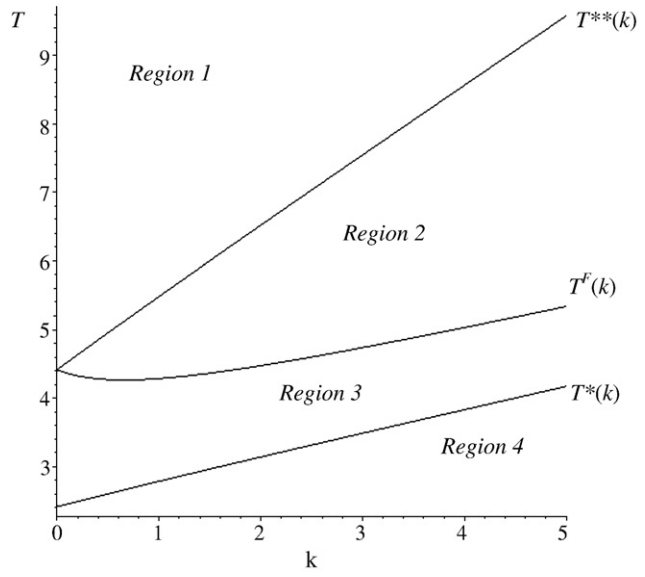

Fig. 1. $T^*(k)$, $T^{**}(k)$ and $T^F(k)$. $T^{**}(k)$ is the locus such that the profit of the least profitable merger in a mixed oligopoly is zero. $T^F(k)$ is the locus such that the profit of two foreign firms merging in a mixed oligopoly is zero. $T^*(k)$ is the locus such that the profit of a two firm merger in a private oligopoly is zero. Region 1 – no mergers are profitable. Region 2 – only the acquisition of a foreign firm by a domestic firm in a mixed oligopoly is profitable. Region 3 – both the acquisition of a foreign firm by a domestic firm and the merging of two foreign firms in a mixed oligopoly are profitable. Region 4 – acquisition of a foreign firm by a domestic firm, the merging of two foreign firms in a mixed oligopoly and the merging of two private firms in a private oligopoly are profitable.

and graph it Fig. 1. We wish to compare this locus to that which emerges from the *least profitable* potential merger in our model.

Lemma. For any given number of total firms, T, the least profitable merger in our model involves a pre-merger market of 1 public firm, 1 foreign private firm and T-2 domestic private firms.

Proof. Holding $T=m+n+1$ constant, increase the number of domestic firms by one and reduce the number of foreign firms by one: $g(k,m,n)-g(k,m-1,n+1)>0$. This implies that $\pi_M^M-2\pi_i$ for (m and n) $>\pi_M^M-2\pi_i$ for ($m-1$ and $n+1$). As $m=1$, is the smallest potential value for the pre-merger number of foreign firms, this is the least profitable merger.

This lemma allows us to set $g(k, m=1, n=T\text{-}2)=0$ and solve for a condition $T^{**}(k)$ analogous to that for the private oligopoly $T^*(k)$. Comparing the two conditions leads to the following:

Proposition 6B. The least profitable acquisition of a foreign firm by a domestic private firm in a mixed oligopoly requires less convexity to be profitable than the comparable two firm merger in a private domestic oligopoly.

Proof. $T^{**}(k)>T^*(k)$ for all k. See Appendix A for details.

Thus the range of profitable mergers is larger in a mixed oligopoly with foreign firms. For a given degree of convexity, the critical number of firms, T, for which two firm mergers are profitable is larger in the mixed oligopoly than in a private oligopoly. Equivalently, the required degree of convexity for a two firm merger to be profitable is smaller in the mixed oligopoly (a domestic firm acquiring a foreign firm) than in a private oligopoly.

Table 1 gives an idea of the magnitudes. Column one identifies the number of firms assumed to be in the oligopoly T. The second column reproduces the critical levels of convexity above which a two firm merger is profitable in a private oligopoly of these numbers of firms. The entries come simply from setting $T^*(k)$ from Heywood and McGinty (2007) and shown in (A1) equal to the number of firms in column one and solving out for k.[8] The third column presents the critical level of convexity from our mixed oligopoly with foreign firms assuming the least profitable merger (those in a market in which there is only a single foreign firm). These are taken directly from $T^{**}(k)$ in (A2). The fourth column presents the critical level of convexity from our mixed oligopoly with foreign firms assuming the most profitable merger. This results in a market of T firms in which there is only one domestic private firm, as shown by the proof of the lemma.[9] Thus, consider the market with five firms ($T=5$). The merger of two firms in a five firm private oligopoly will be profitable only when the marginal cost curve has a slope greater than 7.45 (column 2). In the cross-border mixed oligopoly, the merger of the single domestic private firm with one of the three foreign firms (the fifth firm is the public firm) is profitable whenever the marginal cost curve slope exceeds 0.54 (column 3). In the cross-border mixed oligopoly, the merger of one of three domestic private firms with the one foreign firm is always profitable (column 4). Thus, the presence the cross-border merger in the mixed oligopoly is far more likely to be profitable than the private market equivalent. More generally, and as the entries reflect, cross-border mergers in the mixed oligopoly will routinely be profitable even when the degree of convexity is too low for mergers in private oligopolies to be profitable.

Table 1
Critical Values of Convexity, k, for a Profitable Merger with a Foreign Firm (values above the critical values generate positive profit from merger).

Number of Firms	Private Oligopoly	Mixed Oligopoly	Mixed Oligopoly	Mixed Oligopoly	Mixed Oligopoly
T		$m=1$	$n=1$	$m=1$	$n=2$
		Domestic Acquires		Foreign Acquires	
3	1.60	0.00	0.00	NA	0.00
5	7.45	0.54	0.00	3.89	3.06
7	13.98	3.45	0.00	10.16	9.26
10	22.38	5.42	4.05	19.24	18.30
15	37.36	10.39	9.21	4.28	33.82

** The numbers shown are the critical values for which all larger degrees of convexity yield positive profit. Cases with one domestic firm ($m=1$) are the least profitable two-firm mergers in a market with T firms and cases with one foreign firm ($n=1$) are the most profitable two-firm mergers in a market with T firms.
NA – Not applicable as it does not allow an excluded domestic private firm and a public firm.

It is worth briefly comparing the linear cost case identified in Section 2 and the results just derived for the convex cost case. Most obviously, the profitability in the linear case simply depends on the number of foreign and domestic firms without the added parameter of the degree of convexity. Thus, for the linear case the cross-border merger is profitable regardless of the number of domestic firms as long as the number of foreign firms is either one or two (Proposition 2). This implies that at times, the merger in the linear cost case will occur even when the merger in the convex case is unlikely. Thus, as an extreme, when $T=15$ with 1 public, 1 foreign and 13 domestic private firms, the merger is profitable in the linear case but requires a very steep marginal cost curve to be profitable in the convex case (9.21 as shown in Table 1). On the other hand, when there are relatively more foreign firms, this comparison can be reversed. Consider $T=5$ with 1 public firm, 1 domestic and 3 foreign firms. Here the cross-border merger is not profitable for the linear case but requires only a modestly steep marginal cost to be profitable in the convex cost case (0.54 as shown in Table 1). The intuition for these differences is that only the convex cost case brings a cost savings to the merged firm by using two plants to produce a reduced output. The relative benefit of this cost saving is greater with a small total number of firms. Thus, small degrees of convexity can

[8] MAPLE 9 was used for generating these entries and all of those in Table 1.
[9] While a merger in such a market will always be profitable with $k=0$, at higher levels of k, the merger need not generate profit.

generate profit for the merging parties even when the number of foreign firms exceeds two and mergers are not profitable in the linear cost case.

4. The Merger of Two Foreign Firms

The gain in domestic profit generated by the merger plays a crucial role in generating the results identified in the last section. This gain causes the public firm to concentrate more on domestic profit and so reduce its own quantity. With convex costs the gain in domestic profits comes from two sources. First, there are fewer firms and the reduced total output increases prices and domestic profits. Second, the merged domestic firm is the most profitable of the remaining private firms (proposition 4). This differs from the case of linear costs in which the merged firm is merely equally profitable as all other remaining private firms. This difference has implication for the profitability of two foreign firms merging.

In the case of linear costs, the entire development of Section 2 is merely replicated. It makes no difference if the foreign firm is acquired by a domestic firm or by another foreign firm. Both acquisitions retain the same number of domestic firms and reduce the number of foreign firms by one. As the merged firm has identical profit to all other private firms, its location is irrelevant as long as the number of domestic firms is unchanged. Thus, the change in output by the public firm is also identical in these two cases as are all of the profit implications. As a consequence, when there are two foreign firms, they can profitably merge regardless of the number of private domestic firms (as seen from proposition 2ii).

With convex costs, the model must be derived again for the merger of two foreign firms. One the one hand, the reduced number of firms increases the profit of domestic firms. On the other hand, the merged firm that is most profitable is now a foreign firm. These influences work against each other in the output decision of the public firm. To see the implications modify the model so that there are m domestic firms and $n-2$ foreign firms with the original cost function from (6). Also include one foreign merged firm with the composite cost function $C(q_M^{MF})=(1/4)k(q_M^{MF})^2$ where the superscript MF indicates the equilibrium after the merger of two foreign firms. The resulting post merger equilibrium is:

$$
\begin{aligned}
q_o^{MF} &= \frac{a\left(k^2+kn+3k+2n\right)}{\Phi}\\
q_i^{MF} &= \frac{ak(2+k)}{\Phi} \qquad \forall\, i=1,2\ldots(m+n-2)\\
q_M^{MF} &= \frac{2ak(1+k)}{\Phi}\\
Q^{MF} &= \frac{a\left[k^2(n+m+1)+k(3n+2m+1)+2n\right]}{\Phi}\\
P^{MF} &= \frac{ak\left(n+k^2+3k+2\right)}{\Phi}\\
\pi_i^{MF} &= \frac{a^2k^2(k+2)^3}{2\Phi^2} \qquad \forall\, i=1,2\ldots(m+n-2)\\
\pi_M^{MF} &= \frac{a^2k^2(k+1)^2(k+4)}{\Phi^2}
\end{aligned}
\tag{9}
$$

where $\Phi=k^3+k^2(m+n+4)+k(2m+3n+3)+2n$.

Proposition 7. The merger of two private foreign firms causes the public firm to i) reduce its output but ii) to do so by a smaller amount than happens when a domestic firm acquires a foreign private firm.

Proof. i) $q_o^{MF}-q_o=-\frac{2akm}{\Phi[k^2+k(m+n+2)+n+1]}<0$

ii) $q_o^{MF}-q_o^{M}=-\frac{ak^2\left[k^2+k(n+2m+3)+2(n+m)\right]}{\Phi\Omega}<0$

Condition ii) is sufficient because the pre-merger quantity of the public firm from (7) is identical regardless of which two firms merge.

Thus, the public firm responds to a foreign firm merger with a smaller reduction in output but it remains sufficient to increase the scope of profitable mergers relative to those in a private oligopoly of the same number of firms.

Proposition 8.A. With convex costs, the merger of two foreign firms will be profitable regardless of the degree of convexity only when $n=2$ and $m=1$.

Proof.

$$\pi_M^M-2\pi_i=\frac{a^2k^2\cdot h(k,m,n)}{\Phi^2\left[k^2+k(m+n+2)+n+1\right]^2}$$

where

$$
\begin{aligned}
h(k,m,n) = {} & k^5-(2n+2m-12)k^4-\left(3n^2+3m^2+6nm+2n-38\right)k^3\\
& -\left(10n^2+4m^2+14nm-10n-10m-48\right)k^2\\
& -\left(11n^2+8nm-18n-8m-25\right)k-4n^2+8n+4
\end{aligned}
$$

Substitution yields unambiguous profit gains for the case identified: $g(k,m=3,n=2)=k^5+6k^4+7k^3+6k^2+9k+4>0\Rightarrow \pi_M^{FM}-2\pi_i^{FM}>0\,\forall\, k>0$.

Thus, for only the single case of two foreign and one domestic firm, the smallest number of total firms that allows a foreign merger and excluded domestic public and private firms, the result is unambiguous. For all other cases, the extent of convexity determines whether or not the merger is profitable. Again, for k large enough the higher order terms dominate and the difference is necessarily negative.

Comparing the required degree of convexity follows from a lemma in every way analogous to that in Section 3 that shows that the least profitable merger of two foreign firms happens when there is only one private domestic firm and that with a constant number of total firms increasing the number of domestic firms increases the profitability of two foreign firms merging. This lemma allows us to set $h(k,\ m=1,\ n=T\text{-}2)=0$ and solve for a condition $T^F(k)$ analogous to that for the private oligopoly $T^*(k)$ and the acquisition by a domestic firm $T^{**}(k)$. Comparing the conditions gives the following:

Proposition 8B. The least profitable merger of two foreign firms in a mixed oligopoly requires less convexity to be profitable than the comparable two firm merger in a private domestic oligopoly but more convexity than the least profitable acquisition of a foreign firm by a domestic firm.

Proof. $T^{**}(k)>T^F(k)>T^*(k)$ for all $k>0$. See Appendix A for details.

As before, the critical degree of convexity has been backed out for the least profitable merger and the most profitable merger for a given total number of firms in the market. These are shown in columns 4 and 5 of Table 1. They are unambiguously smaller than those shown for the private firm merger in the same size market but are substantially larger than the values associated with a domestic firm acquiring a foreign firm. The exception to the latter point is when $k=0$. As described early in this section, in that case the profitability from the two types of mergers in a mixed oligopoly are identical. Thus the locus for $T^F(k)$ and $T^{**}(k)$ in Fig. 1 converge at $k=0$.

5. Conclusion

The examination of cross-border mergers in mixed oligopolies is timely because the recent wave of cross-border mergers has often involved industries and countries with public firms. We show that cross-border mergers in a mixed oligopoly can be profitable when otherwise similar mergers will not be profitable. This occurs because

the loss of foreign firm through merger (to either a domestic firm or to another foreign firm) results in the public firm reducing its output. While this reduction in output by the government firm might have been expected given earlier work (Fjell and Pal, 1996), the showing that it sufficient to make a private merger profitable represents a unique contribution. Indeed, we stress that in a mixed oligopoly without foreign firms, the merger of two firms causes the public firm to increase its output. In this respect, the domestic mixed oligopoly mirrors the private oligopoly in which all excluded rivals increase output in response to a merger. The acquisition of the foreign firm by a domestic firm causes a larger reduction in the output of the public firm because the increase in domestic profit is greater than for the merger of two foreign firms.

The context we examine is pertinent not only because of the interesting reversal in a long-standing paradox but also because of its relevance to actual markets. Thus, observers of the international airline industry have long felt that publicly owned "flagship" carriers have resulted in too many airlines, excess capacity and elevated costs. The result, often following deregulation, has been a combination of entry by low fare airlines and a rise in cross-border mergers (see Brueckner and Pels, 2005). Proponents point to the resulting efficiency gained from entry and consolidation. Indeed, European cross-border mergers include the SAS purchase of SpanAir, KLM and Air France combining, the former Sabina merging with Virgin and the attempt by Swiss Air to purchase a major interest in the private Portugalia Airline in 2000. Yet, these cross-border mergers and others influence markets in countries that often retain a public airline. Doganis (2001) identifies 85 airlines, including those in Europe, retaining a state owned controlling interest as of 2000. We emphasize that the issue is not limited to Europe as the attempted takeover of China Eastern by Singapore Airlines shows.

We stress that the issue is not limited to airlines. Recent years have also witnessed the advance of cross-border mergers in European banking starting with the contested case of Banco Santander from Spain attempting to acquire the Portugese bank Champalimaud in the late 1990s. Yet, even when the firms involved are private, as in this banking case, such mergers often involve one or more countries (including both Spain and Portugal) with major public banking firms (Barros and Modesto, 1999).[10] Earlier cross-border mergers of automobile manufactures in Europe have also been identified as occurring in markets with public firms (see the description of the acquisition of SEAT and of Dacia by Barcena-Ruiz and Garzon, 2005). Outside of Europe, recent changes in the Chinese regulation of cross-border mergers emphasize the authority of the government to intervene when the merger may have an impact on "traditional Chinese brands," many of which are associated with public companies (OECD, 2006). In sum, both the integration of developed economies and the transition in previously socialized countries provide many examples of cross-border mergers in a mixed oligopoly.

We recognize that our paper is firmly in the tradition of public firms maximizing welfare and that this assumption may not always be appropriate. To the extent that public firms are either given objective functions other than welfare (perhaps through delegation contracts) or have the latitude to pursue managerial or political objectives, our conclusions need not follow. Incorporating alternative objective functions stands as a subject for future research. A second interesting extension could be to allow for the partial privatization of the public firm as considered by Matsumura (1998). We've compared the case of a fully public firm to one with no public firms (fully private firms). Partial privatization may provide a middle ground between these extremes that yields greater domestic welfare.

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Appendix A. Required Convexity for a Profitable Merger

In a private oligopoly with convex costs and T number of firms, the critical locus for a two firm merger is:

$$T^*(k) = \frac{-k^2 + k + 4 + \sqrt{4k^4 + 32k^3 + 84k^2 + 112k + 32}}{3k + 4} \tag{A1}$$

as taken from Heywood and McGinty (2007 p. 345). A comparable locus is determined by setting $g(m = 1, n = T\text{-}2, k) = 0$ and solving for T. This generates two roots and the relevant one is:

$$T^{**}(k) = \frac{7k + 12 + \sqrt{9k^4 + 66k^3 + 148k^2 + 120k + 32}}{3k + 4} \tag{A2}$$

The two loci are show in Fig. 1. In each case, T and k combination below the locus are profitable while those above the locus are not profitable. It follows immediately that:

$$T^{**}(k) - T^*(k) = \frac{k^2 + 6k + 8 + \sqrt{9k^4 + 66k^3 + 148k^2 + 120k + 32} - \sqrt{4k^4 + 32k^3 + 84k^2 + 88k + 32}}{3k + 4} > 0 \ \forall k$$

The third locus for the merger of two foreign firms is determined by setting $h(m = 1, n = T\text{-}2, k) = 0$ and solving for T. This also generates two roots and the relevant one is:

$$T^F(k) = \frac{-k^4 + 2k^3 + 18k^2 + 27k + 12 + 2\sqrt{k^8 + 12k^7 + 59k^6 + 158k^5 + 255k^4 + 256k^3 + 157k^2 + 54k + 8}}{3k^3 + 10k^2 + 11k + 4}$$

This locus is also shown in Fig. 1. While complicated, the comparisons of the formulas reflect the Figure yielding $T^{**}(k) > T^F(k) > T^*(k)$ for all $k > 0$. The resulting comparisons are available from the authors.

[10] See Sensarma (2006) for an interesting study of the relative performance of public, domestic private and foreign private banks in the developing economy of India.

References

Anand, Jaideep, Delios, Andrew, 2002. Absolute and Relative Resources as Determinants of International Acquisitions. Strategic Management Journal 12, 119–134.

Arz, Benjamin, Heywood, John S., McGinty, Matthew, 2009. The Merger Paradox in a Mixed Oligopoly. Research in Economics 60, 1–10.

Barcena-Ruiz, Juan Carlos, Garzon, Maria Begona, 2005. International Trade and Strategic Privatization. Review of Development Economics 9, 502–513.

Barros, Fatima, Modesto, Leonor, 1999. Portuguese Banking Sector: A Mixed Oligopoly? International Journal of Industrial Organization 17, 869–886.

Bhaumik, S.K., Dimova, R., 2004. How Important is Ownership in a Market with a Level Playing Field: The Indian Banking Sector Revisited. Journal of Comparative Economics 32, 165–180.

Bjorvatn, Kjetil, 2004. Economic Integration and the Profitability of Cross-Border Mergers and Acquisitions. European Economic Review 48, 1211–1226.

Bortolotti, B., Fantini, M., Siniscalco, D., 2004. Privatization Around the World: Evidence from Panel Data. Journal of Public Economics 88, 305–322.

Brueckner, Jan K., Pels, Eric, 2005. European Airline Mergers, Alliance Consolidation and Consumer Welfare. Journal of Air Transport Management 11, 27–41.

Caves, Richard E., 1996. Multinational Enterprise and Economic Analysis, 2nd edition. Cambridge University Press, New York.

Chen, Chunlai, Findlay, Christopher, 2003. A Review of Cross-Border Mergers and Acquisitions in APEC. Asian-Pacific Economic Literature 17, 14–38.

Claude, D., Hindriks, J., 2005. Strategic Privatization and Regulation Policy in Mixed Markets. Discussion Paper, No. 2005-016. Universite catholique de Louvain.

Creane, Andrew, Davidson, Carl, 2004. Multidivisional Firms, Internal Competition and the Merger Paradox. Canadian Journal of Economics 37, 951–977.

Daughety, Andrew, 1990. Beneficial Concentration. American Economic Review 80, 1231–1237.

Davidson, D., Mukherjee, A., 2006. Horizontal Mergers with Free Entry. International Journal of Industrial Organization 25, 157–172.

DeFraja, G., Delbono, F., 1989. Alternative Strategies of a Public Enterprise in Oligopoly. Oxford Economic Papers 41, 302–311.

Doganis, R., 2001. The Airline Business in the 21st Century. Routledge, London.

Farrell, Joseph, Shapiro, Carl, 1990. Horizontal Merger: An Equilibrium Analysis. American Economic Review 80, 107–126.

Fjell, Kenneth, Pal, Debashis, 1996. Mixed Oligopoly in the Presence of Foreign Private Firms. Canadian Journal of Economics 29, 737–743.

Gonzalez-Maestre, Miguel, López-Cuñat, Javier, 2001. Delegation and Mergers in Oligopoly. International Journal of Industrial Organization 19, 1263–1279.

Head, Keith, Ries, John, 1997. International Mergers and Welfare under Decentralized Competition Policy. Canadian Journal of Economics 30, 1104–1123.

Heywood, John S., McGinty, Matthew, 2007. Convex Costs and the Merger Paradox Revisited. Economic Inquiry 45, 342–347.

Heywood, John S., McGinty, Matthew, 2008. Leading and Merging: Convex Costs, Stackelberg and the Merger Paradox. Southern Economic Journal 74, 879–893.

Hodge, Graeme, 2000. Privatization: An International Review of Performance. Westview Press, Boulder Colorado.

Horn, Hendrik, Persson, Lars, 2001. Equilibrium Ownership of an International Oligopoly. Journal of International Economics 53, 207–233.

Kang, N.H., Johansson, S., 2000. Cross-Border Mergers and Acquisitions: Their Role in Industrial Globalization. OECD STI Working Paper 2000/1. OECD.

Matsumura, T., 1998. Partial Privatization in Mixed Duopoly. Journal of Public Economics 70, 473–483.

Merrill, William C., Schneider, Norman, 1966. Government Firms in Oligopoly Industries: A Short Run Analysis. Quarterly Journal of Economics 80, 400–412.

Mialon, S.H., 2008. Efficient Horizontal Mergers: The Effects of Internal Capital Reallocation and Organizational Form. International Journal of Industrial Organization 26, 861–877.

Nakamura, Yasuhiko, Inoue, Tomohiro, 2007. Mixed Oligopoly and Productivity-Improving Mergers. Economics Bulletin 12 (20), 1–9.

Neary, J. Peter, 2007. Cross-Border Mergers as Instruments of Comparative Advantage. Review of Economic Studies 74, 1229–1257.

Nocke, Volker, Yeaple, Stephen, 2007. Cross-border Mergers and Acquisitions vs. Greenfield Foreign Direct Investment: The Role of Firm Heterogeneity. Journal of International Economics 72, 336–365.

OECD, 2003. OECD Economic Outlook, Volume 2003/1, No. 73. OECD, Paris.

OECD, 2006. Recent Developments in China's Policies toward Cross-Border Mergers and Acquisitions. OECD, Paris. December.

Perry, Martin, Porter, Robert H., 1985. Oligopoly and the Incentive for Horizontal Merger. American Economic Review 75, 219–227.

Rothschild, R., Heywood, John S., Monaco, Kristen, 2000. Spatial Price Discrimination and the Merger Paradox. Regional Science and Urban Economics 30, 491–506.

Salant, Stephen W., Switzer, Sheldon, Reynolds, Robert J., 1983. Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium. Quarterly Journal of Economics 98, 185–199.

Sensarma, Rudra, 2006. Are Foreign Banks Always the Best? Comparison of State-owned, Private and Foreign Banks in India. Economic Modelling 23, 717–735.

Zademach, Hans-Martin, Rodrigeuz-Pose, Andres, 2007. Cross-Border M&As and the Changing Economic Geography of Europe. Working Paper. Institute of Economic Geography, University of Munich.

Zhang, Anming, Zhang, Yimin, Zhao, Ronald, 2001. Impact of Ownership and Competition on the Productivity of Chinese Enterprises. Journal of Comparative Economics 29, 327–346.

Ziss, Steffen, 2001. Horizontal Mergers and Delegation. International Journal of Industrial Organization 19, 471–492.