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Scale economies, consistent conjectures and teams

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ABSTRACT

This paper models the behavior of team members in a consistent conjectures equilibrium. When subject to scale economies, team members produce more than Nash and when subject to scale diseconomies, they produce less than Nash. Moreover, even when effort levels of team members are perfect substitutes in production, they can be strategic complements in the face of scale economies. Finally, with sufficient scale economies, the complementarity eliminates free-riding and the team optimum is obtained.

1. Introduction

This paper isolates the critical role that scale economies play in the effort choice of team members who share their joint output. Scale economies determine the conjectures that one team member makes about how his effort influences the effort of his teammates. In turn, the equilibrium conjectures determine the effort choice of the team members. Adopting consistent conjectures equilibria, we show that the optimum can be reached and free-riding eliminated with sufficient scale economies. Moreover, even when assuming worker efforts are perfect substitutes in production, they will be strategic substitutes only with decreasing returns to scale and will be strategic complements with increasing returns to scale. Finally, the equilibrium effort levels will be less than Nash with diseconomies, equal to Nash with constant returns to scale and greater than Nash with scale economies.

We do not imagine a team member's utility function explicitly includes the behavior of his teammates toward him (Falk and Fischbacher, 2006; Cox et al., 2007). Instead, we retain the typical assumption of individual behavior but adopt consistent conjectures as the equilibrium concept. Conjectural variations implicitly model the belief formation process of each player about the conduct of others. In the equilibrium this belief matches the best response of other players, rather than the Nash belief of no response. This solution concept of conjectures that match other players' actual responses has been identified as more general and superior to Nash as it represents a "consistency of beliefs" absent in Nash where players stubbornly refuse to learn about the behavior of others (Aliprantis and Chakrabarti, 2000, p. 138).

This generalization has been criticized as it remains fundamentally one-shot with no explicit maximization of a stream of payoffs and because it can generate out of equilibrium behavior that can make little sense lacking normal stability properties (Friedman, 1983, pp. 109–110). Despite this criticism, it remains popular as a substitute for complete dynamic modeling. Indeed, a substantial literature has embedded static Nash behavior in a fully dynamic model and isolated the conditions under which the outcome matches that of consistent conjectures (see Cabral, 1995 for an early example). As Martin (2002, p. 51) emphasizes, "these results provide a formal justification for using the static conjectural variations model as a shortcut to analyze inherently dynamic models". As a shortcut to full dynamic modeling, the conjectural variations framework has been used in the literature on oligopolistic product markets (Bresnahan, 1981) and the private provision of public goods (Cornes and Sandler, 1985; Sugden, 1985). It has also found practical empirical applications both in bidding strategies for electric power (Song et al., 2003) and in estimating market power (Perloff et al., 2007).

2. The model

We examine a canonical two-worker team in which each worker puts forth an effort level, x_1 and x_2. These efforts are translated into output by a production function of the form:

$$Q = (x_1 + x_2)^{\varepsilon} \quad \text{where } \varepsilon > 0. \tag{1}$$

Note that this assumes the workers' efforts are perfectly substitutable in production. At the other extreme Leontief production

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with constant returns to scale can limit output to that of the lowest effort worker. More generally, strong production interdependences among worker effort may generate incentives for mutual monitoring that move output levels above that implied by Nash equilibrium. We assume perfect substitutes in order to focus on the unique role of scale economies.[1] The parameter ε is the degree of homogeneity and identifies the degree of scale economies: $\varepsilon < 1$ identifies the region of scale diseconomies, $\varepsilon = 1$ represents constant returns to scale and $\varepsilon > 1$ represents the region of scale economies. Adams (2006) examines team effort and output with a CES production function but only considers changes in substitutability in the realm of constant returns to scale. We focus on the consequences of different returns to scale.

The workers receive a payoff equal to the difference between their share of the revenue and the effort cost. We assume the price of the final product is one, that there are no material costs and that workers face convex effort cost. We adopt a simple quadratic effort cost:

$$\pi_i = \frac{Q}{2} - \frac{x_i^2}{2}. \tag{2}$$

The maximum remains defined (the objective function is concave) over the range $0 < \varepsilon < 2$.

The first step in establishing the equilibrium is determined by two first order conditions generated by the team members maximizing (2) given (1).

$$\begin{aligned} F_1 &= \frac{\partial \pi_1}{x_1} = \left(\frac{\varepsilon}{2}\right)(x_1+x_2)^{\varepsilon-1}\left(1+r_{1,2}\right) - x_1 = 0 \\ F_2 &= \frac{\partial \pi_2}{x_2} = \left(\frac{\varepsilon}{2}\right)(x_1+x_2)^{\varepsilon-1}\left(1+r_{2,1}\right) - x_2 = 0. \end{aligned} \tag{3}$$

The first order conditions include an explicit conjecture by each team member about how his output influences that of his teammate. Thus, $r_{i,j}$ is the conjecture by i about the output change of team member j: $\frac{\partial x_j}{\partial x_i}$. The solution concept defines the equilibrium as a consistent conjecture equilibrium if the conjecture by team member i, $r_{i,j}$, equals the actual best response of j along the reaction function of team member j. Symmetric team members allows imposing the structure that $x = x_1 = x_2$ and that $r = r_{2,1} = r_{1,2}$. Making these substitutions into either of the first order conditions provides an expression for the effort as a function of the conjecture itself.

$$x^* = \frac{1}{2}[\varepsilon(1+r)]^{\frac{1}{2-\varepsilon}}. \tag{4}$$

The Nash conjecture is $r = 0$ implying that $x^{ne} = \frac{(\varepsilon)^{\frac{1}{2-\varepsilon}}}{2}$. This allows a simple first comparison: when $r > 0, x^* > x^{ne}$ and when $r < 0, x^* < x^{ne}$. Obviously we have not yet solved for the consistent conjecture, we simply recognize that depending upon that conjecture, the equilibrium output can be either greater than, equal to, or less than the output associated with the Nash equilibrium.

This relationship is critical as it implies that the strategic view with which team members see their teammate's effort helps determine output. Moreover, that strategic view is directly a function of the degree of scale economies. This can be seen in either cross partial of (3):

$$\frac{\partial^2 \pi_1}{\partial x_1 x_2} = \left(1+r_{1,2}\right)\left(1+r_{2,1}\right)\left(\frac{\varepsilon}{2}\right)(\varepsilon-1)(x_1+x_2)^{\varepsilon-2}. \tag{5}$$

As is clear, when $\varepsilon > 1$, the value of (5) is positive (recognizing that the symmetry implies that the conjectures be the same) implying that the efforts of the team members are strategic complements. When $\varepsilon < 1$, the value of (5) is negative implying that the efforts of the team members are strategic substitutes. Thus scale economies determine whether a team member responds by increasing effort with a fellow team member's increase in effort or decreases effort in response to that same increase.

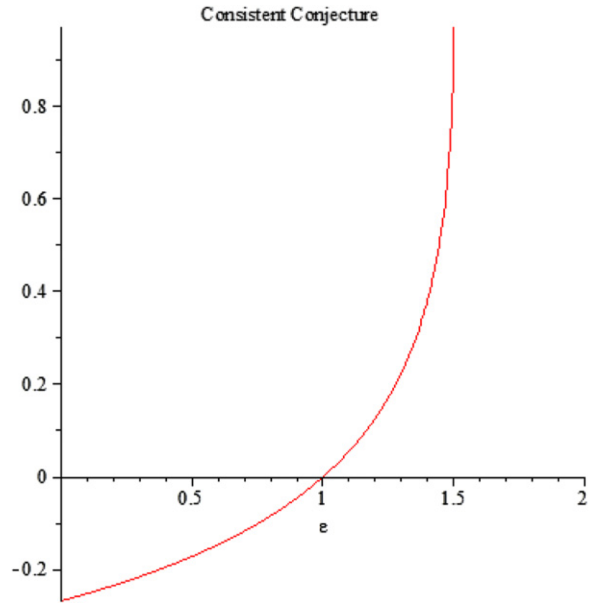

Fig. 1. Consistent conjectures as a function of the extent of scale economies.

3. Results

The conjectures become consistent when they equal the actual behavior along the best response of the teammate:

$$r_{1,2} = \frac{\partial x_2}{\partial x_1} \qquad r_{2,1} = \frac{\partial x_1}{\partial x_2}. \tag{6}$$

The best response function cannot be solved for explicitly but the Implicit Function Theorem allows identification of the relevant derivatives in (6). Concentrating on the first equality and recognizing the first of the two conditions in (3), F_1, yields:

$$\begin{aligned} \frac{\partial x_1}{\partial x_2} &= -\frac{\frac{\partial F_1}{\partial x_2}}{\frac{\partial F_1}{\partial x_1}} \\ &= \frac{-(\varepsilon-1)\left(\frac{\varepsilon}{2}\right)\left(1+r_{1,2}\right)\left(1+r_{2,1}\right)(x_1+x_2)^{\varepsilon-2}}{(\varepsilon-1)\left(\frac{\varepsilon}{2}\right)(1+r_{1,2})^2(x_1+x_2)^{\varepsilon-2}-1}. \end{aligned} \tag{7}$$

Recognizing that symmetry requires that $r = r_{2,1} = r_{1,2}$, the expression in (7) is a function of r and set equal to the conjecture r from (6). The resulting expression can be simplified to a quadratic in r: $r^2(\varepsilon-1) + 2r(\varepsilon-2) + \varepsilon - 1 = 0$. When $\varepsilon = 1$, $r = 0$ but otherwise two roots emerge. The root that adds the radical can be eliminated as it generates values of r which imply negative efforts by (4). Thus, the solution is

$$r = \frac{2-\varepsilon-\sqrt{3-2\varepsilon}}{\varepsilon-1} \quad \text{for } \varepsilon \neq 1 \text{ and } r = 0 \text{ for } \varepsilon = 1. \tag{8}$$

This root emerges as real when $\varepsilon \leq \frac{3}{2}$ and r is strictly increasing in ε. Within the real range, there can be negative consistent conjectures, offsetting behavior, and positive consistent conjectures, matching behavior. Specifically, (8) is such that as $\varepsilon \to 0$, $r \to -0.268$ and when $\varepsilon = \frac{3}{2}$, $r = 1$. This relationship is shown graphically in Fig. 1.

[1] We leave for future work circumstances in which both the number of workers and the degree of substitutability are generalized.

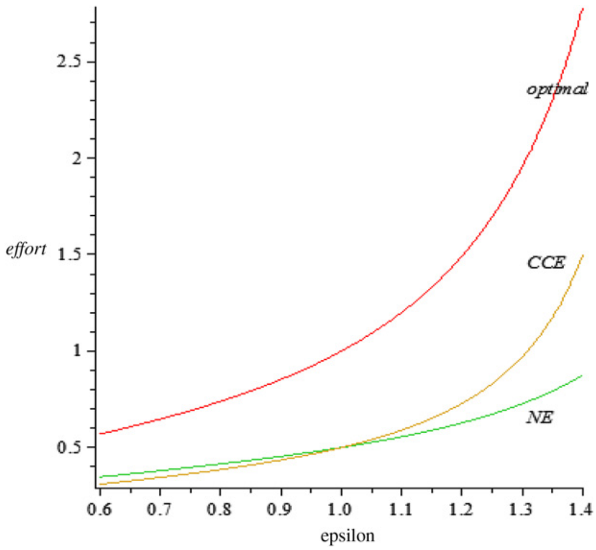

Fig. 2. Effort under consistent conjectures, Nash and optimality.

The consistent conjecture from (8) can be returned to (4) to calculate the effort levels associated with the consistent conjecture equilibrium (CCE) for the two teammates.

$$x^{CC} = \frac{1}{2}\left[\frac{\varepsilon(1-\sqrt{3-2\varepsilon})}{2\varepsilon-2}\right]^{\frac{1}{2-\varepsilon}}. \tag{9}$$

The CCE is graphed as a function of ε and shown as the yellow line in Fig. 2. The Nash prediction derived earlier is similarly graphed and shown as the green line in Fig. 2.

Several points emerge immediately from comparing the Nash and consistent conjectures equilibria. First, the effort level in the Nash is identical to that in the consistent conjectures equilibrium when there are constant returns to scale, $\varepsilon = 1$. Both the Nash and consistent conjectures equilibrium increase with extent of scale economies but with decreasing returns to scale, $\varepsilon < 1$, $x^{CC} < x^{NE}$ while with increasing returns to scale, $\varepsilon > 1$, $x^{CC} > x^{NE}$. Also note that the joint maximum, x^{o} (the effort level that maximizes the sum of payoffs for the team), is generally above both x^{CC} and x^{NE}. The optimal effort level is:

$$x^{o} = \left[\varepsilon 2^{\varepsilon-1}\right]^{\frac{1}{2-\varepsilon}}. \tag{10}$$

Yet, the effort associated with the consistent conjecture converges to the optimal joint effort as the extent of scale economics increase. Indeed, they exactly equal each other when $\varepsilon = 3/2$ (not shown in Fig. 2). Thus, scale economies and the increasing strategic complementarity of effort levels causes free-riding to diminish and moves effort levels closer to optimal than would be implied by Nash behavior.

4. Conclusion

When players adjust their conjectures to reflect their teammate's behavior, Nash should be abandoned in favor of consistent conjectures. We explore the implications of assuming such consistency in the face of various degrees of scale economies. First, we show that the deviation from the optimum is a function of the extent of scale economies. The optimum can be reached with sufficient scale economies. Second, even when assuming worker efforts are perfect substitutes in production, they will be strategic substitutes only with decreasing returns to scale and strategic complements with increasing returns to scale. Third, as a consequence of consistency, the predicted effort levels will be less than Nash with diseconomies, equal to Nash with constant returns to scale and greater than Nash with scale economies. This set of implications seem ripe for empirical and laboratory investigation.

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