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# UNIVERSITY OF CALIFORNIA SANTA CRUZ

## **Essays In International Environmental Economics**

A dissertation submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

INTERNATIONAL ECONOMICS

by

Matthew McGinty

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## **Abstract**

# Essays In International Environmental Economics Matthew McGinty

Chapter 1 is an international emissions agreement (IEA) between asymmetric countries. In order to achieve meaningful gains an IEA must be incentive compatible for a significant number of countries. It is shown that the gains to an IEA with full participation are potentially greater in the presence of asymmetry. The gains to an agreement with full participation are increasing in the variance of the benefit shares when abatement costs are symmetric. In general, the gains to an agreement are greater when the high benefit share countries are also high cost. The Nash and Stackelberg equilibria are derived for any arbitrary partition of countries into signatories and non-signatories to an agreement. An incentive compatibility constraint provides an upper bound on the required level of abatement under an agreement, as a function of any arbitrary coalition. Cost side asymmetries imply a role for pollution permit trading in implementing the efficient allocation of abatement. It is shown that a Pareto optimal IEA with full participation can be incentive compatible.

Chapter 2 is an evolutionary game trade model of goods that are environmentally differentiated. There is a single good that is differentiated as either of environmentally high or low quality. The short-run autarchy equilibria are Cournot-Nash, taking the proportion of high quality firms as given. The long-run evolutionary equilibrium is where the short-run Cournot-Nash equilibria are obtained and there is a zero-profit differential between high and low quality type firms. Under autarchy

there is a unique stable mix of both types of firms in each country, given a parameter restriction. Under free trade the autarchy equilibria are unstable and at least one country is completely specialized. There is a unique evolutionary equilibrium, whose basin of attraction is the entire state space. Production of environmentally low quality type is assumed to entail a negative production externality. A country that produces the low quality good has an incentive to impose a "lack-of-pollution content" tariff on imports of the high quality good. This type of tariff can lead to rent-capture and pollution-shifting effects, benefiting domestic firms and the domestic environment, as well as creating tariff revenue. Furthermore, since firms are imperfectly competitive the increase in domestic price is less than the tariff, so the tariff revenue can be redistributed to make consumers better off.

Chapter 3 is a public goods experiment that tests provision when agents have asymmetric wealth and benefit shares of the public good. The interior Nash and Pareto levels are identical across treatments to compare provision levels. Pilot results show that when endowments are asymmetric and benefit shares are symmetric that provision closely approaches the Pareto optimal level, in stark contrast to previous experiments. Furthermore, in both treatments with asymmetric benefit shares provision levels conform to the Nash equilibrium prediction, contrary to the typical finding of over-contribution. These results may be due to subjects perception of fairness, but additional runs of the experiment are needed to validate these preliminary findings.

## **Dedication**

For Anna, Lianne and Dennis

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## Chapter 1

## 1 An International Emissions Agreement

### 1.1 Introduction

Greenhouse gas abatement is one of the most important issues in managing the global environment. Abatement of greenhouse gases is a cost that is country specific, while the benefits are global in scope. This characteristic makes abatement a global public good, and as such, nations generally have an incentive to free-ride and let other countries undertake costly pollution abatement. The Kyoto Protocol is an effort to provide an international mechanism by which greenhouse gas emissions can be reduced. Is a voluntary international emissions agreement possible among self-interested nations?

Barrett (1994) addressed this question using a static framework with no uncertainty to focus on the incentives to cooperate. Barrett invoked the usual Cournot simplification that countries are identical on both the cost and benefit sides to determine the gains to cooperation from self-enforcing agreements. Simulations were conducted for a one-shot and an infinitely repeated game and he found that there is a trade-off between the number of signatories to an agreement and the gains from agreement. Barrett concludes that a self-enforcing international emissions agreement can not result in large gains relative to the non-cooperative outcome. A key element of Barrett's analysis is that no side payments are allowed in the model, because while they are optimal from the point of view of the signatories as a group, they are not individually credible at the Pareto optimal quantity due to a payoff advantage of being a non-signatory. Therefore, side payments to maintain

the agreement at the Pareto optimal level are not credible, and consequently the agreement is not self-enforcing for any significant number of signatories.

There are a wide variety of other approaches in the economic literature on international emissions agreements. The link between trans-boundary pollution and trade in goods in explored in general equilibrium trade models such as Copeland and Taylor (2000) and Caplan, et al.(1999). These models highlight the interaction between trade in goods and pollution policy. Copeland and Taylor, in a factor proportions model, show that trade in goods and pollution permits can make domestic and foreign abatement strategic complements rather than strategic substitutes, as is generally assumed in partial equilibrium models. Other models, such as Pizer (1999), focus on the role of uncertainty regarding abatement costs and benefits. Escarpa and Gutierrez (1997), Tahvonen (1994) and Chander and Tulkens (1992) look at the intertemporal aspect of emissions agreements to address the possibility that repeated interaction can support a cooperative solution. In general, the results of dynamic models are sensitive to the rate of time preference.

The model presented below is based on Barrett's work, but allows for countries to be asymmetric on both the benefit and the cost sides. Cost asymmetries provide a role for pollution permits in implementing the Pareto efficient amount of abatement at the country level. Permits imply that the required abatement under the agreement need not be the efficient level for each country. Permit revenue acts as a system of side payments which can be used to make the agreement incentive compatible. Theoretically, this is a single population game with each of the N players having an asymmetric payoff function. The payoffs are state-dependent, based on the number and characteristics, of the signatories and non-signatories.

Nash and Stackelberg equilibria are derived for any arbitrary partition of countries into signatories and non-signatories to an international emissions agreement (IEA). For all possible coalitions of signatories the Stackelberg equilibrium results in a *lower* level of abatement that the Nash equilibrium. This result is obtained due to a strategic effect by the signatories acting as Stackelberg leaders, increasing their payoff by reducing signatory abatement, inducing the non-signatories to increase their abatement relative to the Nash equilibrium. An incentive compatibility constraint provides an upper bound on the required level of abatement that is still consistent with a payoff advantage to being a signatory. Pollution permit trading allows for a divergence between the required and the Pareto efficient levels of abatement at the country level while maintaining economic efficiency.

This paper provides a theoretical framework for a self-enforcing international emissions agreement. There are interesting directions for future work. Using marginal abatement cost estimates from the MIT computable general equilibrium models, and using either population shares or GDP shares as an approximation for the share of global benefit that a country receives, is a possible parameterization of the model. This would provide insight into participation and incentive compatibility for particular countries and various forms of agreements.

The remainder of the paper is organized as follows. Section 2 derives the Nash equilibrium levels of abatement at the country and global level. Section 3 derives the Pareto optimal level of abatement and the efficient allocation of this level for each country. Section 4 investigates the implications of asymmetry. Section 5 derives the Nash and Stackelberg equilibria for any arbitrary coalition of signatories when there is less than full participation. Section 6 analyzes incentive

compatibility and coalition formation for the Nash and Stackelberg equilibria. Section 7 provides a numerical example of an incentive compatible international emissions agreement at the Pareto optimal level that is not incentive compatible under symmetry. Section 8 concludes, discusses policy implications and possible directions for future work.

## 1.2 Nash Equilibrium

Global benefit, B(Q), is assumed to be a quadratic function of the worldwide quantity of abatement undertaken in each of the N countries,  $Q = \sum_{i=1}^{N} q_i$ , as in Barrett (1994).

$$B(Q) = b\left(aQ - \frac{Q^2}{2}\right) \tag{1}$$

where a, b > 0. The global marginal benefit function is B'(Q) = b (a - Q), so the marginal benefit of the first unit of pollution abatement is ab, and the marginal benefit of the ath unit of abatement is zero. Country i's share of the global benefits of abatement is represented by the parameter  $\alpha_i$  where  $\alpha_i > 0$ , i = [1, ..., N] and  $\sum_{i=1}^{N} \alpha_i = 1$ . The atmosphere is considered a global public good in this specification and hence there is an implicit assumption that there are no other ancillary benefits that accrue to a nation from domestic abatement. The restriction on the benefit shares assumes that all countries receive some positive benefit from provision of the public good.

$$B_i(Q,\alpha_i) = b\left(aQ - \frac{Q^2}{2}\right)\alpha_i \tag{2}$$

The cost of abatement is assumed to depend only on the quantity of abatement that country i undertakes,  $q_i$ , and the slope of that countries marginal abatement

cost (MAC) curve.

$$C_i(q_i) = \frac{c_i(q_i)^2}{2} \tag{3}$$

The marginal cost function is  $C'_i(q_i) = c_i q_i$ , where  $c_i > 0$ . Each country's MAC curve is allowed to increase at a different rate. Given these functional forms the net benefit for country i is

$$\pi_i(\alpha_i, c_i, q_i, Q) = b \left( aQ - \frac{Q^2}{2} \right) \alpha_i - \frac{c_i(q_i)^2}{2} \tag{4}$$

In the Nash equilibrium solution each country i chooses  $q_i$  to maximize  $\pi_i$  taking the sum of the other countries abatement,  $Q_{-i} \equiv \sum_{j \neq i}^{N} q_j$ , as given. The first order condition is

$$b\left(a-q_{i}-Q_{-i}\right)\alpha_{i}-c_{i}q_{i}\leq0\tag{5}$$

The second order condition for a maximum is satisfied,  $\frac{\partial^2 \pi_i}{\partial q_i^2} = -b\alpha_i - c_i < 0$ . Solving equation (5) for  $q_i$ , conditional on an interior solution, yields the reaction function for country i.

$$q_i = \frac{b\left(a - Q_{-i}\right)\alpha_i}{c_i + b\alpha_i} \tag{6}$$

For any two countries i and j, equation (5) simplifies to

$$\frac{c_i q_i}{\alpha_i} = b (a - Q) 
\frac{c_j q_j}{\alpha_j} = b (a - Q)$$
(7)

Defining  $\theta_i \equiv \frac{\alpha_i}{c_i}$ , as the benefit share divided by the slope of the marginal abatement cost (MAC) curve for country i, (7) implies the relationship:

$$\frac{q_j}{\theta_j} = \frac{q_k}{\theta_k} \tag{8}$$

Either unequal benefit shares or unequal MAC curves will result in unequal abatement levels across countries. Given the linear MAC curves the marginal cost on the  $q_jth$  unit of abatement is  $q_jc_j$ . The cost minimizing allocation for a given level of abatement occurs where the MAC on the last unit of abatement are equalized across countries. Only if the benefit shares are equal for all countries,  $\alpha_i = \frac{1}{N}$  for i = [1, ..., N], are the MAC's equalized on the last unit of abatement, and is in an efficient allocation of abatement realized in the Nash equilibrium. Any asymmetry in the benefit shares results in an inefficient allocation of abatement levels across countries. If the MAC curves are identical then countries abate in proportion to their benefit shares. In the absence of either type of symmetry, high benefit share, low MAC countries abate by a larger amount.

The Nash equilibrium global level of abatement, denoted  $Q^* = \sum_{i=1}^{N} q_i^*$ , is found by solving (5) for  $q_i$  in terms of Q and summing across i.

$$Q^{\bullet} = \frac{ab \sum_{j=1}^{N} \theta_{j}}{1 + b \sum_{j=1}^{N} \theta_{j}} = \frac{a}{1 + \frac{1}{b \sum_{j=1}^{N} \theta_{j}}}$$

$$q_{i}^{\bullet} = \frac{ab\theta_{i}}{1 + b \sum_{j=1}^{N} \theta_{j}}$$
(9)

The Nash equilibrium level of abatement is increasing in a, b and  $\sum_{j=1}^{N} \theta_{j}$ . Identical countries implies that  $\theta_{i} = \frac{1}{Nc}$  for all countries, so the sum  $\sum_{j=1}^{N} \theta_{j} \equiv \sum_{i=1}^{N} \frac{\alpha_{i}}{c_{i}} = \frac{1}{c}$ . For this special case, equation (9) becomes  $Q^{\bullet} = \frac{ab(\frac{1}{c})}{1+(\frac{b}{c})} = \frac{a}{1+\gamma}$ , where  $\gamma \equiv \frac{c}{b}$  is the relative slopes of the marginal cost and global marginal benefit functions. This is the Nash equilibrium level of abatement in Barrett, given the functional forms in (4).

## 1.3 Pareto Optimality

Greenhouse gases tend to mix uniformly in the upper atmosphere, so there is equal global marginal benefit of a unit of abatement, regardless of where the abatement occurs. Abatement by one country results in a positive externality that accrues to all other N-1 countries. The socially optimal, or Pareto, level of abatement internalizes this externality and is set at a level of abatement such that the MAC in each country i is equal to the global marginal benefit. The Nash equilibrium is the result of countries acting only in their own self-interest and choosing an abatement level such that the MAC in country i is equal to the marginal benefit only for country i.

Efficiency here has two parts: (i) allocative efficiency, setting  $q_i$  in all countries so that the marginal abatement cost in each country is equal to global marginal benefit, and (ii) level efficiency, choosing a global level of abatement,  $Q^o$ , where the global marginal benefit curve is set equal to the global marginal abatement cost curve.

The Pareto optimal outcome is a set of  $q_i$  that maximizes the global net benefit function,  $\max_{q_1,\dots,q_N} [B(Q) - \sum_{i=1}^N C_i(q_i)]$ . The first order condition is the standard Samuelson condition for the efficient provision of a public good, the sum of the marginal benefits equals the marginal cost.

$$b(a - q_i - Q_{-i}) - c_i q_i \le 0 (10)$$

The Pareto optimal quantity of abatement in (10) is strictly greater than the Nash Equilibrium level of abatement in (5) for  $\alpha_i < 1$ , or equivalently, N > 1.

The second order condition for a maximum is satisfied,  $\frac{\partial^2 \pi_i}{\partial q_i^2} = -b - c_i < 0$ . The first order condition for a positive level of abatement simplifies to:

$$q_i = \frac{b(a-Q)}{c_i} \tag{11}$$

To determine the optimal level of abatement the global marginal abatement cost curve must be derived. The country specific marginal abatement cost curves are simply  $MAC_i = c_i q_i$ . The global marginal abatement cost curve is the horizontal summation of the individual countries MAC curves. Just as an efficient multiplant firm allocates production quantities where the marginal costs of production are equal for plants with different cost structures, an efficient agreement equates marginal abatement cost on the last unit of abatement in each country. Let p denote the common level of MAC on the last unit of abatement for each country. A system of pollution permits will equate the MAC in each country with the permit price.  $p = MAC_i = c_i q_i$ , or  $q_i = \frac{p}{c_i}$ . Summing across the individual countries quantities,  $Q = \sum_{j=1}^{N} q_j$  or  $Q = p \sum_{j=1}^{N} \frac{1}{c_j}$ . Solving for the marginal abatement cost. p, yields the global MAC curve.

$$p = MAC = \frac{Q^{o}}{\sum_{j=1}^{N} \frac{1}{c_{i}}}$$
 (12)

Being the horizontal summation of linear curves the global MAC is linear having constant slope  $\frac{1}{\sum_{i=1}^{N} \frac{1}{\epsilon_i}}$ .

The Pareto optimal quantity,  $Q^o$ , can be determined by using the condition that global MC = global MB. The global marginal benefit is b(a-Q), from equation (1). The Pareto optimal level of abatement,  $Q^o$ , is where  $\frac{Q^o}{\sum_{j=1}^N \frac{1}{c_j}} = b(a-Q^o)$ .

$$Q^{o} = \frac{ab\sum_{j=1}^{N} \frac{1}{c_{j}}}{1 + b\sum_{j=1}^{N} \frac{1}{c_{j}}} = \frac{a}{1 + \frac{1}{b\sum_{j=1}^{N} \frac{1}{c_{j}}}}$$
(13)

The Pareto optimal level of abatement<sup>1</sup> is increasing in a, b and  $\sum_{j=1}^{N} \frac{1}{c_j}$ . Using equation (11) the efficient allocation of the Pareto level for each country is

$$q_i^o = \frac{ab}{c_i \left[1 + b \sum_{j=1}^N \frac{1}{c_i}\right]} \tag{14}$$

Equation (14) implies that at  $Q^o$  the permit price is  $p = c_i q_i^o = \frac{ab}{\left[1 + b \sum_{j=1}^N \frac{1}{c_j}\right]}$ , the common level of MAC for all countries.

Proposition 1: The Pareto optimal level of abatement is strictly greater than the Nash equilibrium.

- (i)  $\sum_{j=1}^{N} \frac{1}{c_j} > \sum_{j=1}^{N} \theta_j$  for all  $0 < \alpha_i < 1$ ,  $\sum_{j=1}^{N} \alpha_j = 1$ .
- (ii) If either the benefit shares, the MAC curve slopes, or both are symmetric then:  $\sum_{j=1}^{N} \frac{1}{c_j} = N \sum_{j=1}^{N} \theta_j$ .

The gains to an agreement that implements the Pareto level arise from two sources: internalizing the global positive externality from country level abatement and efficiency gains from equating MAC across countries. In the Nash equilibrium asymmetric benefit share countries abate by different amounts and therefore the Nash results in an inefficient, or non cost-minimizing, allocation of abatement levels across countries.

The difference between the Pareto and Nash equilibrium global levels of abatement, and hence the gains to an agreement with full participation, is determined by the relationship between the sums  $S_c \equiv \sum_{j=1}^N \frac{1}{c_j}$  and  $S_\theta \equiv \sum_{j=1}^N \theta_i$ . Define the Pareto minus the Nash quantities as  $Q_D \equiv Q^o - Q^\bullet$ , and substituting in equations (13) and (9) yields

$$Q_D = \frac{ab\left(Sc - S_{\theta}\right)}{\left[1 + bS_c\right]\left[1 + bS_{\theta}\right]} \tag{15}$$

The Pareto level of abatement in Barrett is a special case of (13) when  $c_i = c$  and  $\alpha_i = \frac{1}{N}$  for i = [1, ..., N]. In this case the Pareto level of abatement is  $Q^o = \frac{a}{1 + \frac{c}{N}}$ , where  $\gamma \equiv \frac{c}{b}$ .

The gains to an agreement depend on the magnitudes of  $Q^{\circ}$  and  $Q^{\bullet}$ , as well as their difference  $Q_D$ , all of which are a function of the nature of the asymmetry.

## 1.4 Implications of Asymmetry

The gains to country i from an agreement with full participation is the difference in the net benefit function evaluated at the Pareto and Nash levels. In addition, given the abatement requirement negotiated under the agreement, country i will have permit revenue equal to  $p(q_i^o - q_i^r)$ . The assumption of linear MAC curves implies that the country specific MAC on the last units of abatement are equalized across countries and equal to the permit price  $p = c_i q_i^o$  at the Pareto level, so that country i will undertake domestic abatement equal to  $q_i^o$ . For the agreement as a whole the permit revenue is a zero-sum transfer between countries, given the condition that the required abatement under the agreement be set equal to the Pareto level.  $\sum_{j=1}^N p(q_j^o - q_j^r) = p\left(\sum_{j=1}^N q_j^o - \sum_{j=1}^N q_j^r\right) = p(Q^o - Q^o) = 0$ . To analyze the implications of asymmetry the permit revenue term for country i is suppressed.

The country i payoff differential,  $\pi_{Di} = \pi(Q^o) - \pi(Q^*)$ , from an agreement with full participation is found by substituting the Pareto and Nash quantities into the payoff function in equation (4). The following notation conserves on space:  $S_c \equiv \sum_{j=1}^N \frac{1}{c_j}$ , and  $S_\theta \equiv \sum_{j=1}^N \theta_i$ .

$$\pi_{Di} = b \left( a \left[ \frac{abS_c}{1 + bS_c} \right] - \frac{\left[ \frac{abS_c}{1 + bS_c} \right]^2}{2} \right) \alpha_i - \frac{c_i \left[ \frac{ab}{c_i [1 + bS_c]} \right]^2}{2} - \left\{ b \left( a \left[ \frac{abS_{\theta}}{1 + bS_{\theta}} \right] - \frac{\left[ \frac{abS_{\theta}}{1 + bS_{\theta}} \right]^2}{2} \right) \alpha_i - \frac{c_i \left[ \frac{ab\theta_i}{1 + bS_{\theta}} \right]^2}{2} \right\}$$
(16)

The payoff differential for country i simplifies to:

$$\pi_{Di} = \frac{a^2 b^2 \left\{ (\alpha_i)^2 \left[ 1 + bS_c \right]^2 + \alpha_i c_i \left( S_c - S_\theta \right) \left( bS_c + bS_\theta + 2 \right) - \left[ 1 + bS_\theta \right]^2 \right\}}{2c_i \left[ 1 + bS_c \right]^2 \left[ 1 + bS_\theta \right]^2} \tag{17}$$

The global net benefit of the Pareto level of abatement relative to the Nash level is found by summing equation (17) across all N countries,  $\Pi_D \equiv \sum_{j=1}^N \pi_{Dj}$ .

$$\Pi_{D} = \frac{a^{2}b^{2} \left\{ (S_{c} - S_{\theta}) \left( bS_{c} + bS_{\theta} + 2 \right) - \left[ 1 + bS_{\theta} \right]^{2} S_{c} + \left[ 1 + bS_{c} \right]^{2} \sum_{j=1}^{N} \frac{(\alpha_{i})^{2}}{c_{i}} \right\}}{2 \left[ 1 + bS_{c} \right]^{2} \left[ 1 + bS_{\theta} \right]^{2}}$$
(18)

#### 1.4.1 Complete Symmetry

The country and global gains are evaluated for three possible degrees of symmetry: complete symmetry, symmetric benefits only, and symmetric rates of MAC increase. MAC symmetry implies  $S_c \equiv \sum_{j=1}^N \frac{1}{c} = \frac{N}{c}$  and  $S_\theta \equiv \sum_{j=1}^N \theta_j = \sum_{j=1}^N \frac{\alpha_i}{c} = \frac{1}{c}$ . Benefit share symmetry implies  $S_\theta = \frac{1}{N} \sum_{j=1}^N \frac{1}{c_j}$ . All types of symmetry imply  $S_c = NS_\theta$ , a relationship that is useful in simplifying the expressions. For identical countries (complete symmetry) we have  $S_c \equiv \sum_{j=1}^N \frac{1}{c} = \frac{N}{c}$  and  $S_\theta \equiv \sum_{j=1}^N \theta_j = \frac{1}{c}$ . Defining the parameter  $\gamma \equiv \frac{c}{b}$ , as the relative slopes of the country MAC curves and the global marginal benefit curves, equation (17) simplifies to

$$\pi_{Di} = \frac{a^2 \gamma^2 (c + bN) (N - 1)^2}{2N^2 [1 + \gamma]^2 [N + \gamma]^2}$$
(19)

The global net benefits are obtained by multiplying equation (19) by N.

$$\Pi_{D} = \frac{a^{2} \gamma^{2} (c + bN) (N - 1)^{2}}{2N [1 + \gamma]^{2} [N + \gamma]^{2}}$$
(20)

### 1.4.2 Asymmetric Benefits, Symmetric MAC

For the case of asymmetric benefit shares, but symmetric MAC curves the payoff advantage of the Pareto solution over the Nash for country i is

$$\pi_{Di} = \frac{a^2 c \left\{ (\alpha_i)^2 \left[ N + \gamma \right]^2 + \alpha_i \gamma \left[ 2\gamma \left( N - 1 \right) + N^2 - 1 \right] - \left[ 1 + \gamma \right]^2 \right\}}{2 \left[ 1 + \gamma \right]^2 \left[ N + \gamma \right]^2} \tag{21}$$

The global net benefit is the summation of (21) across all N countries.

$$\Pi_{D} = \frac{a^{2}c\left[\gamma\left[2\gamma\left(N-1\right)+N^{2}-1\right]-N(1+\gamma)^{2}+\left(N+\gamma\right)^{2}\sum_{j=1}^{N}\left(\alpha_{i}\right)^{2}\right]}{2\left[1+\gamma\right]^{2}\left[N+\gamma\right]^{2}}$$
(22)

Lemma 1: The sum  $\sum_{j=1}^{N} (\alpha_j)^2$  is minimized for benefit share symmetry,  $\alpha_j = \frac{1}{N}$  for j = [1, ..., N], and the variance of the benefit shares is strictly increasing in  $\sum_{j=1}^{N} (\alpha_j)^2$ .

Proposition 2: For the case of symmetric marginal abatement cost curve slopes, the global net benefit of the Pareto optimal solution over the Nash equilibrium is increasing in the variance of the benefit shares.

The intuition behind Proposition 2 lies in equation (8), which states that the relationship between any two countries abatement at the Nash equilibrium is  $\frac{q_1}{\theta_j} = \frac{q_k}{\theta_k}$ , where  $\theta_j \equiv \frac{\alpha_j}{c_j}$ . When the MAC curves are symmetric and benefit shares asymmetric, then countries abate in proportion to their benefit share. Allocative efficiency, for any given level of abatement, is obtained when the MAC on the last units are equated, implying equal abatement levels for MAC curve symmetry. Therefore, countries internalizing different shares of the benefits of the public good leads to a Nash equilibrium that does not exhibit allocative efficiency when abatement costs increase at the same rate. The Pareto solution, with permit trading, achieves both

allocative and level efficiency, achieving the cost minimizing allocation of the level of abatement that maximizes global welfare.

Proposition 2 is similar to results in the industrial organization literature relating the Herfindahl index of market shares to industry profitability, as a result of asymmetric marginal costs<sup>2</sup>. For the case of constant, but asymmetric marginal costs, and linear demand curves, Cournot competition implies a higher Herfindahl index of market shares, defined as  $H \equiv \sum_{i=1}^{N} \left(\frac{q_i}{Q}\right)^2$ , which results in greater industry profit. Cost asymmetries imply output asymmetries, which in turn lead to higher industry profit.

However, Proposition 2 is different than the Herfindahl index for a variety of reasons. Proposition 2 is the result of marginal abatement costs that are increasing linearly, rather than asymmetric but constant. The Herfindahl index measures concentration through market shares which arise as a result of cost asymmetries. The benefit shares in Proposition 2 are independent of the cost curves. The public goods nature of this paper implies that countries benefit from the overall level of abatement, which is a positive externality. In standard Cournot competition an increase in the quantity produced by one firm results in a negative externality imposed on the other firms as the market price falls.

Lemma 2 implies that the lower bound on the distribution of  $\sum_{j=1}^{N} (\alpha_j)^2$  is the point of symmetry,  $\alpha_j = \frac{1}{N}$ , j = [1, ..., N]. In the numerical example below this implies that the distribution of feasible  $\sum_{j=1}^{N} (\alpha_j)^2$  is truncated from below at  $\frac{1}{N} = 0.2$ , the point of intersection. We can conclude that in the case of MAC symmetry that the global gains to cooperation are monotonically increasing in the

<sup>&</sup>lt;sup>2</sup>See Tirole (1990), Demsetz (1973) or Schmalensee (1987)

variance of the benefit shares by Proposition 2. The global gains of the Pareto solution, relative to the Nash are given in Figure 1.1 below.

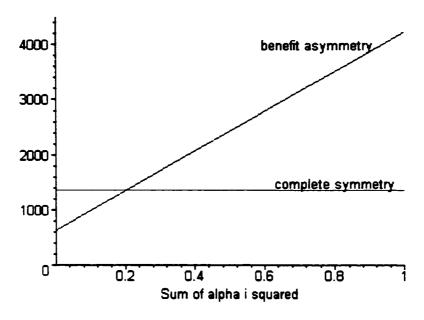


Figure 1.1: Global gains for complete symmetry, relative to benefit share asymmetry for a = 100, b = 3, c = 2, N = 5.

## 1.4.3 Symmetric Benefits, Asymmetric MAC

The opposite case is when the benefit shares are symmetric and the MAC curves are asymmetric. Equation (17) simplifies to

$$\pi_{Di} = \frac{a^2 b^2 (N-1) \left[ S_c^2 c_i b \left(N+1\right) + 2 S_c N (c_i - b) - N \left(N+1\right) \right]}{2 N c_i \left[1 + b S_c\right]^2 \left[N + b S_c\right]^2} \tag{23}$$

The global net benefit,  $\Pi_D \equiv \sum_{j=1}^N \pi_{Dj}$ , can be found by summing (23) again using the relationship  $S_c = NS_\theta$  to write the result only in terms of  $S_c$ .

$$\Pi_D = \frac{a^2 b^2 (N-1)^2 S_c}{2 \left[1 + b S_c\right] \left[N + b S_c\right]^2} \tag{24}$$

Lemma 2: For all mean preserving distributions of the slopes of the MAC curves,

 $c_i$ , the sum  $S_c \equiv \sum_{j=1}^N \frac{1}{c_j}$  is minimized for identical  $c_i$ .

Lemma 2 implies that we can truncate the distribution of the sum  $S_c = \sum_{j=1}^{N} \frac{1}{c_j}$  from below at the point of symmetry, for the example below at  $\frac{N}{c} = 2.5$ , where the curves intersect.

The magnitude of the difference between the Pareto and the Nash levels of abatement was given in equation (15). For any degree of symmetry the relation  $S_c = NS_\theta$  holds, so for any type of symmetry the difference between the Pareto and Nash levels of abatement simplifies to

$$Q_D = \frac{abS_c(N-1)}{(1+bS_c)(n+bS_c)}$$
 (25)

The partial derivative of the symmetric  $Q_D$  with respect to  $S_c$  is

$$\frac{\partial Q_D}{\partial S_c} = \frac{ba(N-1)\left[N - b^2 (S_c)^2\right]}{(1 + bS_c)^2 (N + bS_c)^2}$$
(26)

Therefore, if  $N > b^2 (S_c)^2$  then difference between the Pareto and Nash quantities is increasing in  $S_c$ , and when  $N < b^2 (S_c)^2$  the difference is decreasing in  $S_c$ . The latter case is depicted in Figure 2 below. With complete symmetry the sum  $S_c = \frac{N}{c}$ , and with c = 2, b = 3 and n = 5, so this condition is:  $5 < 9(2.5)^2 = 56.25$  for the numerical example. Therefore the difference between the Pareto and Nash quantities is smaller. In addition, the Nash achieves allocative efficiency in the asymmetric MAC case so there are no allocative gains to an agreement, as with benefit share asymmetry.

An increase in  $S_c$  increases the quantity differential of an agreement with full participation if  $N > b^2 (S_c)^2$ . Lemma 2 establishes that the sum  $\sum_{j=1}^N \frac{1}{c_j}$  is minimized for  $c_i$  symmetry, and in this case the value of  $S_c = \frac{N}{c}$ . So, the lower bound

on the right hand side of the condition  $N > b^2 (S_c)^2$  is  $N > \frac{b^2 N^2}{c^2}$ . Therefore, if  $c < b\sqrt{N}$  then we can conclude that increasing  $S_c$  will decrease the difference between the Pareto and Nash levels of abatement and lead to smaller gains to an agreement. Figure 2 shows the global gains to cooperation, for benefit share symmetry and MAC asymmetry, when  $S_c > \frac{\sqrt{N}}{b}$ , so increasing  $S_c$  decreases the difference between the Pareto and Nash levels of abatement, by equation (26).

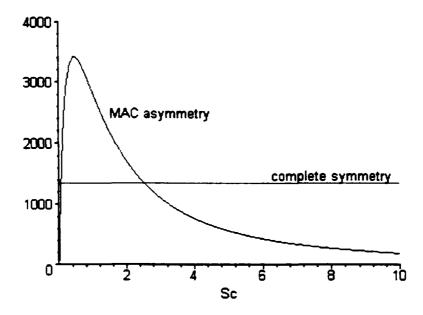


Figure 1.2: Global gains for complete symmetry, relative to MAC asymmetry for a = 100, b = 3, c = 2, N = 5.

The sum  $\sum_{j=1}^{N} \frac{1}{c_j}$  is a normalization of the harmonic mean of the distribution of the  $c_i$ , defined as  $H(c) \equiv \frac{N}{\sum_{j=1}^{N} \frac{1}{c_j}}$ . In the context of comparing two distributions that have the same number of elements, N, the harmonic mean is determined entirely by the sum  $\sum_{j=1}^{N} \frac{1}{c_j}$ . The further restriction that the two distributions be arithmetic mean preserving implies that the second moment is a relevant metric. However, there is not a monotonic relationship between the variance and harmonic

mean of such distributions. It is possible to find two distributions that have the same variance, but different harmonic means. For this reason it is not possible to make general statements regarding the sum  $\sum_{j=1}^{N} \frac{1}{c_j}$ , based on the typical definition of variance. Therefore, there is not a monotonic relation between the global gains to full cooperation and variance of the  $c_i$ , when benefit shares are symmetric and MAC curves are asymmetric.

Overall, we can conclude that when MAC curves are symmetric that the global gains are monotonically increasing in the variance of the benefit shares. Alternatively, when the benefit shares are symmetric, we can conclude that the difference between the Pareto and Nash quantities is increasing in the sum  $\sum_{j=1}^{N} \frac{1}{c_j}$  (or decreasing in the harmonic mean) if  $N > b^2 (S_c)^2$ , and decreasing if  $N < b^2 (S_c)^2$ . However, there is not a monotonic relation between the sum  $S_c \equiv \sum_{j=1}^{N} \frac{1}{c_j}$  and the variance of the  $c_j$ .

All three types of symmetry imply  $S_c = NS_{\theta}$ . The difference between the Pareto and the Nash equilibrium quantities, from equation (15), is  $Q_D = \frac{ab(Sc - S_{\theta})}{[1+bS_c][1+bS_{\theta}]}$ . Assuming symmetry implies the numerator is constant, however in general the difference between the sums  $S_c \equiv \sum_{j=1}^N \frac{1}{c_j}$  and  $S_{\theta} \equiv \sum_{j=1}^N \theta_j = \sum_{j=1}^N \frac{\alpha_j}{c_j}$  depends on the nature of the asymmetry. Specifically, for a given distribution of  $c_j$  the sum  $S_{\theta}$  will be smaller when there are larger weights  $(\alpha_j)$  on the smaller terms  $(\frac{1}{c_j})$ . Therefore, when the high benefit share countries are also the high MAC countries the sum  $S_{\theta}$  will be smaller. For a given distribution of the  $c_j$ , and either type of symmetry, the partial derivative of the Pareto minus the Nash quantity simplifies to

$$\frac{\partial Q_D}{\partial S_{\theta}} = -\frac{ab}{\left[1 + bS_{\theta}\right]^2} \tag{27}$$

The quantity differential is decreasing in  $S_{\theta}$ . We would expect that the gains to an agreement with full participation would be greatest when  $S_{\theta}$  is small, which occurs when the high benefit share countries are also the high MAC countries, and visa-versa.

Lemmas 1 and 2 established that the sums  $\sum_{j=1}^{N} (\alpha_j)^2$  and  $\sum_{j=1}^{N} \frac{1}{c_j}$ , respectively, were minimized for symmetry. However, symmetry is not the lower bound on the value of the sum  $S_{\theta} \equiv \sum_{j=1}^{N} \theta_j$ . A N=3 example illustrates this point. Let the arithmetic mean  $\bar{c}=2$ . Complete symmetry implies  $S_{\theta}=\frac{1}{\bar{c}}=0.5$ . Consider a distribution of MAC slopes  $[c_1=1,\,c_2=2,\,c_3=3]$ , and a distribution of benefit shares  $[\alpha_1=\frac{1}{12},\,\alpha_2=\frac{4}{12},\,\alpha_3=\frac{7}{12}]$ , so that high cost country, 3, is also the high benefit share country. The sum  $\sum_{j=1}^{N} \theta_j = 0.\bar{4}$ , which is lower than complete symmetry. Now switch the benefit shares of countries 1 and 3, so the distributions are  $[c_1=1,\,c_2=2,\,c_3=3]$  and  $[\alpha_1=\frac{7}{12},\,\alpha_2=\frac{4}{12},\,\alpha_3=\frac{1}{12}]$ . The value of the sum  $\sum_{j=1}^{N} \theta_j = 0.\bar{7}$  when the high cost country is also the low benefit share country.

The previous discussion has related the global quantities of abatement to these sums. The global gains to an agreement, in the absence of symmetry, is equation (18). For a given distribution of  $c_j$  the global gains are increasing in the sum  $\sum_{j=1}^{N} \frac{(\alpha_i)^2}{c_i}$ .

Lemma 3: For all mean preserving distributions of the slopes of the MAC curves,  $c_j$ , the sum  $\sum_{j=1}^{N} \frac{(\alpha_j)^2}{c_j}$  is minimized for complete symmetry;  $c_j = \bar{c}$  for j = [1, ..., N] and  $\alpha_j = \frac{1}{N}$  for j = [1, ..., N].

Proof of Lemma 3 is in the Appendix.

For the N=3 example, complete symmetry implies the sum  $\sum_{j=1}^{N} \frac{(\alpha_i)^2}{c_i} = \frac{1}{N\bar{c}} = 0.1\bar{6}$ . For the distribution  $[c_1=1, c_2=2, c_3=3]$  and  $[\alpha_1=\frac{1}{12}, \alpha_2=\frac{4}{12}, \alpha_3=\frac{7}{12}]$ 

the value of the sum  $\sum_{j=1}^{N} \frac{(\alpha_i)^2}{c_i} = 0.252$ , and for the distribution  $[c_1 = 1, c_2 = 2, c_3 = 3]$  and  $[\alpha_1 = \frac{7}{12}, \alpha_2 = \frac{4}{12}, \alpha_3 = \frac{1}{12}]$ , the value of the sum  $\sum_{j=1}^{N} \frac{(\alpha_i)^2}{c_i} = 0.398$ . The example shows that the distribution that places a higher weight  $(\alpha_j)$  on the higher value of  $\theta$  has a greater sum. So for a given distribution of the  $c_j$  the global gains of the Pareto over the Nash equilibrium will be greater if the high benefit share countries are also the high MAC countries. The assumption of symmetry has the potential to greatly understate the gains to an agreement in such a situation.

## 1.5 An International Emissions Agreement

The previous sections compared the levels of abatement when all countries act as Nash players relative to the outcome when all countries choose the socially optimal level. An international emissions agreement must allow for the possibility that not all countries are signatories. Furthermore, since the countries are asymmetric the design must allow for endogenous interaction based on any arbitrary partition of countries into signatories and non-signatories, and those countries characteristics, described by  $\alpha_i$  and  $c_i$ .

It must be mutually beneficial for both the coalition and a non-signatory for a coalition of signatories to increase. From the coalitions perspective the additional signatory must provide additional coalition wide benefits that are greater than the additional costs associated with inclusion. This condition will be referred to as the coalition formation constraint. In addition, there is the requirement of incentive compatibility. Ultimately it must be individually rational for a country to either become, or remain, a signatory to any viable agreement. The required levels of abatement under the terms of the agreement must incorporate an incentive

compatibility constraint. The incentive compatibility (IC) constraint is determined by restricting the payoff differential of being a signatory, relative to a non-signatory, to be positive. The IC constraint determines an upper bound on the required level of abatement for country i, denoted  $q_i^r$ , for any arbitrary coalition. The signatory level of abatement under the terms of the agreement must also be a function of this endogenous partition of countries. This implies that the signatory and non-signatory aggregate and country level quantities of abatement must be determined for any arbitrary partition of countries. The permit price will then be a function of the signatory quantity of abatement and the MAC curves of the signatory countries.

Pollution permits are the mechanism that will allow the implementation of the signatory level of abatement at the lowest possible cost. With participation of all countries an efficient agreement requires that the sum of the required abatement be set equal to the Pareto level,  $\sum_{j=1}^{N} q_j^r = Q^o$ . The required amount of abatement is equivalent to an initial allocation of pollution permits since a fully functioning<sup>3</sup> system would equate the marginal abatement cost of the last unit of abatement undertaken in any country with the permit price,  $MAC_i = p$ . Countries whose abatement requirement is below the Pareto level would then be suppliers of permits and those countries whose required abatement is above the Pareto level would purchase a quantity of permits equal to the difference between the required and

<sup>&</sup>lt;sup>3</sup>Fully functioning means that there are no restrictions, such as cartelization of the supply of permits by the low cost countries, or legislated barriers to permit trading. The latter is relevent because of concerns that "rich" countries will purchase all of their required abatement from "poor" countries. There have been suggestions that countries be required to undertake a given proportion of their mandated abatement domestically, limiting permit trading and thus efficiency.

Pareto levels. Any agreement containing permits, where  $q_i^r \neq q_i^o$  for at least two countries, will achieve the Pareto level of abatement at a lower cost than an agreement without permits.

Permits then have multiple purposes in this model. First, they act as a mechanism which allows each country to meet their required abatement at the lowest possible cost. Second, they act as a transfer scheme between countries via the choice of abatement requirements for each country under the terms of the agreement. The required level of abatement is then a required contribution by country i to the global total cost of abatement, not a requirement of domestic abatement.

Assumption: only countries participating in the agreement may buy or sell permits.

The purpose of this assumption is two-fold. First, gains from trade in pollution permits provide a possible incentive to join the agreement. Second, it simplifies the derivation of the signatory abatement level.

The signatory and non-signatory quantities are derived for both Nash and Stackelberg type interaction between the two groups. If the signatories take the reaction of the non-signatories as given, then interaction is Nash. However, if the agreement itself is viewed as a credible commitment device then the signatories can act as a Stackelberg leader, incorporating the non-signatories reaction directly into the signatory objective function.

#### 1.5.1 Non-Signatory Reaction Function

The reaction function for any non-signatory i is given in equation (6). These reaction functions can be aggregated to form a group non-signatory reaction function

using the relationship  $\frac{q_i}{\theta_j} = \frac{q_k}{\theta_k}$ , in equation (8). The sum of the other N-1 countries abatement,  $Q_{-i}$ , in the reaction function has been decomposed into abatement by signatories  $Q^s \equiv \sum_{j=1}^M q_j^s$ , and other non-signatories  $Q_{-i}^n \equiv -q_i^n + \sum_{j=M+1}^N q_j^n$ .

$$q_i^n = \left\{ \frac{b\alpha_i \left( a - Q^s - Q_{-i}^n \right)}{c_i + b\alpha_i} \right\},\,$$

resulting in the reaction function for non-signatory i to abatement by the signatories.

$$q_i^{n*} = \frac{b\theta_i (a - Q^s)}{1 + b\sum_{i=M+1}^{N} \theta_i^n}$$
 (28)

With respect to country i, the denominator and all terms in the numerator except  $\theta_i$  are constant, for a given number of non-signatories. The aggregate non-signatory reaction function is solved by summing (28) across countries M+1 to N, using the definitions  $Q^n \equiv \sum_{j=M+1}^N q_j^n$  and  $Q^s \equiv \sum_{j=1}^M q_j^s$ . The aggregate reaction function of N-M non-signatories to abatement undertaken by M signatories is

$$Q^{n} = \frac{b(a - Q^{s}) \sum_{j=M+1}^{N} \theta_{j}^{n}}{1 + b \sum_{j=M+1}^{N} \theta_{j}^{n}} = \frac{(a - Q^{s})}{1 + \frac{1}{b \sum_{j=M+1}^{N} \theta_{j}^{n}}}$$
(29)

The aggregate non-signatory reaction function to signatory abatement is downward sloping, were the absolute value of the slope is less than one. The non-signatories partially offset an increase in signatory abatement by reducing their abatement, but by a smaller amount.

#### 1.5.2 Nash Solution

The Nash solution is the result of determining the aggregate signatory abatement level  $Q^s$  taking the non-signatory quantity  $Q^n$  as given. The solution method is to

<sup>&</sup>lt;sup>4</sup>There has been no a priori ordering of countries based on any criteria until this point.

maximize with respect to  $Q^s$  taking  $Q^n$  as given, then allow permits to efficiently allocate  $Q^s$  among countries [1, ..., M] implying a signatory level of abatement,  $q_i^s$ , in each country. In an agreement with full participation the efficient signatory level is equal to the Pareto level,  $q_i^s = q_i^o$ .

The assumption that only signatories may trade in pollution permits provides an additional incentive to join (remain in) the agreement. It is also consistent with an even stronger statement: when choosing the aggregate quantity of signatory abatement, signatories care only about the welfare of themselves and other signatories. The distinction is that signatories choose quantities of abatement where the sum of the marginal benefit to the signatories is equal to the marginal cost in each signatory country, a modified Samuelson rule. Signatories respond to a defector from the agreement by reducing abatement, providing an endogenous defense against free-riding.

The signatory MAC function is analogous to the MAC function in the Pareto optimal solution. An efficient agreement equates MAC on the last unit of abatement in each country that participates in the agreement with the permit price,  $p = c_i q_i^s$  for i = [1, ..., M]. Aggregating across the M signatories yields the aggregate signatory MAC curve:  $Q^s = \sum_{j=1}^M q_j^s = \sum_{j=1}^M \frac{p}{c_j}$ 

$$p = MAC^{s} = \frac{Q^{s}}{\sum_{j=1}^{M} \frac{1}{c_{j}}}$$
 (30)

The signatories' aggregate MAC curve becomes more elastic as the number of signatories increases. Signatory efficiency, as a function of M signatories, is determined where the signatories set  $\sum_{j=1}^{M} MB_{j}^{s} = MAC_{j}^{s}$ , ignoring the external benefits that accrue to non-signatories. Permits enter the payoff differential between being

a signatory and a non-signatory, but not the efficient level of abatement. The signatories choose a set of  $q_i^s$  that maximizes the sum of the signatories net benefit. The first order condition for an interior solution is

$$b(a - Q^{n} - Q^{s}) \sum_{j=1}^{M} \alpha_{j}^{s} = \frac{Q^{s}}{\sum_{j=1}^{M} \frac{1}{c_{j}}}$$
(31)

Solving for  $Q^n$ , equating with the non-signatory reaction function in (29) and then solving for  $Q^s$  yields the optimal quantity of signatory abatement.

$$Q^{s} = \frac{ab \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}}{1 + b \left( \sum_{j=M+1}^{N} \theta_{j}^{n} + \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s} \right)}$$

$$q_{i}^{s} = \frac{ab \sum_{j=1}^{M} \alpha_{j}^{s}}{c_{i} \left[ 1 + b \left( \sum_{j=M+1}^{N} \theta_{j}^{n} + \sum_{j=1}^{M} \frac{1}{c_{i}} \sum_{j=1}^{M} \alpha_{j}^{s} \right) \right]}$$
(32)

From equation (32)  $Q^n$  can be determined using the reaction function for the N-M non-signatories given in equation (29).

$$Q^{n*} = \frac{ab \sum_{j=M+1}^{N} \theta_{j}^{n}}{\left[1 + b \left(\sum_{j=M+1}^{N} \theta_{j}^{n} + \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}\right)\right]}$$

$$q_{i}^{n*} = \frac{ab\alpha_{i}}{c_{i} \left[1 + b \left(\sum_{j=M+1}^{N} \theta_{j}^{n} + \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}\right)\right]}$$
(33)

Combining the signatory,  $Q^s$ , and non-signatory,  $Q^{ns}$ , abatement yields the global abatement for any arbitrary partition of signatories and non-signatories.

$$Q = \frac{ab\left(\sum_{j=M+1}^{N} \theta_{j}^{n} + \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}\right)}{1 + b\left(\sum_{j=M+1}^{N} \theta_{j}^{n} + \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}\right)}$$
(34)

In the limit as all countries become signatories the solution in (34) collapses to the Pareto optimal level<sup>5</sup> of abatement in equation (13),  $Q^o = \frac{ab\sum_{j=1}^{N}\frac{1}{c_j}}{1+b\sum_{j=1}^{N}\frac{1}{c_j}}$ . In the limit as all countries become non-signatories the solution in (34) becomes the Nash equilibrium level of abatement with no signatories in equation (9),  $Q^o = \frac{ab\sum_{j=1}^{N}\theta_j}{1+b\sum_{j=1}^{N}\theta_j}$ .

<sup>&</sup>lt;sup>5</sup>In the limit as all countries become signatories,  $M \to N$ ,  $\sum_{j=M+1}^N \theta_j^n \to 0$ ,  $\sum_{j=1}^M \alpha_j^s \to 1$ ,

### 1.5.3 Stackelberg Solution

If the agreement is viewed as a credible commitment device then the signatories can act as a Stackelberg leader. If the signatories as a group anticipate the reaction of the non-signatories they can use this information to increase the aggregate signatory payoff. The signatory objective function is the same, but the Stackelberg equilibrium is the result of the signatories substituting the non-signatory reaction function directly into the signatory payoff function and then maximizing. The signatory solution in the previous section assumed that the signatories and non-signatories interacted as standard Nash equilibrium players, each taking the reaction of the other as given.

The first-order condition for an interior Stackelberg solution is

$$b\sum_{j=1}^{M}\alpha_{j}^{s}\left(a\left[\frac{1}{1+b\sum_{j=M+1}^{N}\theta_{j}^{n}}\right]-\left[\frac{Q^{s}+ab\sum_{j=M+1}^{N}\theta_{j}^{n}}{\left(1+b\sum_{j=M+1}^{N}\theta_{j}^{n}\right)^{2}}\right]\right)=c_{i}q_{i}^{s}$$

Simplification yields the Stackelberg equilibrium signatory level of abatement

$$Q^{s} = \frac{ab \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}}{\left[1 + b \sum_{j=M+1}^{N} \theta_{j}^{n}\right]^{2} + b \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}}$$

$$q_{i}^{s} = \frac{ab \sum_{j=1}^{M} \alpha_{j}^{s}}{c_{i} \left[\left(1 + b \sum_{j=M+1}^{N} \theta_{j}^{n}\right)^{2} + b \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}\right]}$$
(35)

From the reaction function (29) the non-signatory level of abatement is

$$Q^{n} = \frac{ab \sum_{j=M+1}^{N} \theta_{j}^{n} \left[ 1 + b \sum_{j=M+1}^{N} \theta_{j}^{n} \right]}{\left[ 1 + b \sum_{j=M+1}^{N} \theta_{j}^{n} \right]^{2} + b \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}}$$

and  $\sum_{j=1}^{M} \frac{1}{c_{j}} \to \sum_{j=1}^{N} \frac{1}{c_{j}}$  the solution in (34) collapses to the Pareto optimal level of abatement in equation (13)  $Q^{o} = \frac{ab\sum_{j=1}^{N} \frac{1}{c_{j}}}{1+b\sum_{j=1}^{N} \frac{1}{c_{j}}}$ . In the limit as all countries become non-signatories,  $M \to 0$ ,  $\sum_{j=1}^{M} \frac{1}{c_{j}} \to 0$ ,  $\sum_{j=1}^{M} \alpha_{j}^{s} \to 0$  and  $\sum_{j=M+1}^{N} \theta_{j}^{n} \to \sum_{j=1}^{N} \theta_{j}^{n}$  the solution in (34) becomes the Nash equilibrium level of abatement with no signatories in equation (9)  $Q^{\bullet} = \frac{ab\sum_{j=1}^{N} \theta_{j}}{1+b\sum_{j=1}^{N} \theta_{j}}$ 

$$q_{i}^{n} = \frac{ab\alpha_{i} \left[1 + b\sum_{j=M+1}^{N} \theta_{j}^{n}\right]}{c_{i} \left[\left(1 + b\sum_{j=M+1}^{N} \theta_{j}^{n}\right)^{2} + b\sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}\right]}$$
(36)

The global quantity of Stackelberg abatement is equation (35) plus equation (36).

$$Q = \frac{ab \left[ \sum_{j=M+1}^{N} \theta_{j}^{n} \left[ 1 + b \sum_{j=M+1}^{N} \theta_{j}^{n} \right] + \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s} \right]}{\left[ 1 + b \sum_{j=M+1}^{N} \theta_{j}^{n} \right]^{2} + b \sum_{j=1}^{M} \frac{1}{c_{j}} \sum_{j=1}^{M} \alpha_{j}^{s}}$$
(37)

As shown in the Appendix, the Stackelberg level of signatory abatement, for any given coalition of signatories, is strictly less than the Nash level of abatement. The numerators are identical while the Stackelberg has a strictly larger denominator for at least one non-signatory. However, for the non-signatories the opposite is true, the Stackelberg level of abatement is strictly greater than the Nash. Dividing by the  $\left[1+b\sum_{j=M+1}^N\theta_j^n\right]$  term shows that the numerators are identical while the Nash has a strictly greater denominator. Taken together, the global level of abatement is strictly greater in the Nash equilibrium for at least one non-signatory. Furthermore, the Pareto optimal level of abatement, in equation (13), is strictly greater than the Nash when there is at least one non-signatory. Of course, both equilibrium concepts result in the Pareto level when all countries are signatories and the no-agreement Nash level of abatement when all countries are non-signatories.

Proposition 3: For all possible coalitions with at least one non-signatory, global abatement in the Stackelberg equilibrium is strictly less than that of the Nash equilibrium which is strictly less than Pareto optimal.

Proof of Proposition 3 is in the Appendix.

It may come as a surprise that the Nash quantity is strictly greater than the Stackelberg equilibrium. In traditional Cournot-Stackelberg models the leaders increase their profits by increasing their quantity, leaving a smaller market share for the followers. The standard result is that Stackelberg models lead to a larger market quantity than the Nash equilibrium. When one firm increases quantity it imposes a negative externality on the other N-1 firms through a reduction in market price.

The public goods nature of abatement in this model is the reason that the Nash equilibrium is greater than the Stackelberg quantity. The benefit functions are increasing in the global quantity of abatement, not just the quantity of abatement in a particular country. An external benefit accrues to all other N-1 nations from a unit of domestic abatement. The Stackelberg signatories increase their net payoff by choosing a place on the non-signatories reaction function that induces the non-signatories to increase their abatement, exploiting these positive spillovers.

# 1.6 Incentive Compatibility

This section shows the requirements for a set of international emissions agreements that are self-enforcing. A coalition formation requirement shows the condition on benefit share asymmetry that ensures that the coalition desires accession by any non-signatory, for any arbitrary coalition, when signatories and non-signatories behave as Nash players. An incentive compatibility (IC) constraint derives the upper bound on level of required abatement under the agreement for any country and any arbitrary coalition. This constraint is then summed across all signatories to show that a coalition will result from countries whose inclusion results in additional coalition wide benefits that are sufficient to compensate the additional abatement costs from inclusion. A sufficient condition is derived for the Nash

solution concept which shows that for all  $\alpha_i < \frac{1}{2}$ , i = [1, ..., N], the coalition desires full-participation. However, the Stackelberg equilibrium is not so clear. Since the signatories have a strategic incentive to reduce their abatement inducing the non-signatories to increase their abatement (relative to the Nash level) accession by a non-signatory could imply a reduction in abatement. In this case it may be advantageous from the coalitions' point of view to have such a country outside the agreement.

The first task is to derive an expression that determines the possible ranges of required abatement that are incentive compatible under an agreement for any possible coalition of signatories and non-signatories. With permit trading the required level of abatement is a commitment to contribute a portion of the total cost of signatory abatement, not a commitment to a level of domestic abatement.

Proposition 4: There is an incentive compatible set of abatement requirements for all coalitions of M signatories whenever the sum of the gains is less than the sum of the losses from leaving the agreement,  $\sum_{j=1}^{M} C_i(q_i^{s,M}) - C_i(q_i^{n,M \cup \{i\}}) < \sum_{j=1}^{M} B_i(Q^M) - B_i(Q^{M \cup \{i\}})$ .

The gains to leaving the agreement is the reduction in abatement cost as country i becomes a non-signatory. The loss is the reduction in benefits that country i internalizes from a lower level of abatement with less signatories. If the sum of the gains to leaving are less than the sum of the losses then a credible system of side-payments exists that makes the agreement incentive compatible. This system of side-payments can be implemented through the choice of abatement requirements and permit trading. Proposition 4 states the necessary condition for simultaneously satisfying the incentive compatibility requirement for each member of the

coalition. There is a subtle, yet powerful distinction between Proposition 4 and the coalition formation requirement. The coalition desires accession of a country as long as inclusion of that country adds more to the coalitions benefits than it does to the coalitions costs. It entirely feasible that the coalition will desire accession, yet inclusion will not satisfy the incentive compatibility constraint in Proposition 4. The numerical example at the end of the paper helps illustrate this point.

The coalition will desire accession by a non-signatory as long as the additional coalition wide benefit is greater than the additional costs. The incentive compatibility constraint for each i is that the signatory payoff exceeds the non-signatory payoff. This constraint will be used to bound the required level of abatement for any country,  $q_i^{r,M}$  given a coalition of M signatories, to determine when the coalition will desire accession by non-signatory i. The superscripts M and  $M \cup \{i\}$  denote the quantities of abatement for any arbitrary coalition of M signatories and that coalition plus additional signatory i. The payoff advantage to being a signatory is positive if:

$$\left[B_{i}(Q^{M\cup\{i\}},\alpha_{i}) - C_{i}(q_{i}^{s,M\cup\{i\}})\right] + p\left(q_{i}^{s,M\cup\{i\}} - q_{i}^{r,M\cup\{i\}}\right)$$

$$> \left[B_{i}(Q^{M},\alpha_{i}) - C_{i}(q_{i}^{n,M})\right] \tag{38}$$

If country i is required to abate beyond the efficient level then the  $p\left(q_i^{s,M\cup\{i\}}-q_i^{r,M\cup\{i\}}\right)$  term is negative and country i is a purchaser of permits. Substituting in the functional forms for benefits and costs:

$$b\alpha_{i} \left\{ \left( aQ^{M \cup \{i\}} - \frac{(Q^{M \cup \{i\}})^{2}}{2} \right) - \left( aQ^{M} - \frac{(Q^{M})^{2}}{2} \right) \right\} + p \left( q_{i}^{s,M+\{i\}} - q_{i}^{r,M+\{i\}} \right) > \frac{c_{i} \left\{ (q_{i}^{s,M+\{i\}})^{2} - (q_{i}^{n,M})^{2} \right\}}{2}$$

$$(39)$$

Solving for  $q_i^{r,M\cup\{i\}}$  gives the upper bound for the required level of abatement, that is consistent with incentive compatibility.

$$q_{i}^{r,M\cup\{i\}} < q_{i}^{s,M\cup\{i\}} + \frac{b\alpha_{i}}{p} \left\{ \left( aQ^{M\cup\{i\}} - \frac{(Q^{M\cup\{i\}})^{2}}{2} \right) - \left( aQ^{M} - \frac{(Q^{M})^{2}}{2} \right) \right\} - \frac{c_{i}}{2p} \left\{ (q_{i}^{s,M\cup\{i\}})^{2} - (q_{i}^{n,M})^{2} \right\}$$

$$(40)$$

The larger the  $\alpha_i$ , the larger the reduction in benefit that country i internalizes from leaving the agreement. The incentive compatibility constraint for any country in (40) can be summed across all  $M \cup \{i\}$  signatories.

$$\sum_{j=1}^{M \cup \{i\}} q_{j}^{r,M \cup \{i\}} < \sum_{j=1}^{M \cup \{i\}} q_{j}^{s,M \cup \{i\}} 
+ \sum_{j=1}^{M \cup \{i\}} \frac{b\alpha_{j}}{P} \left\{ \left( aQ^{M \cup \{i\}} - \frac{(Q^{M \cup \{i\}})^{2}}{2} \right) - \left( aQ^{M} - \frac{(Q^{M})^{2}}{2} \right) \right\} 
- \sum_{j=1}^{M+\{i\}} \frac{c_{i}}{2p} \left\{ (q_{i}^{s,M \cup \{i\}})^{2} - (q_{i}^{n,M})^{2} \right\}$$
(41)

Signatory efficiency implies that the sum of the required level of abatement is equal to the efficient level for that coalition,  $\sum_{j=1}^{M\cup\{i\}} q_j^{r,M\cup\{i\}} = \sum_{j=1}^{M\cup\{i\}} q_j^{s,M\cup\{i\}}$ . After multiplying by the permit price the constraint reduces to

$$\sum_{j=1}^{M \cup \{i\}} b\alpha_{j} \left\{ \left( aQ^{M \cup \{i\}} - \frac{(Q^{M \cup \{i\}})^{2}}{2} \right) - \left( aQ^{M} - \frac{(Q^{M})^{2}}{2} \right) \right\}$$

$$> \sum_{i=1}^{M \cup \{i\}} \frac{c_{j}}{2} \left\{ (q_{j}^{s,M \cup \{i\}})^{2} - (q_{j}^{n,M})^{2} \right\}$$

$$(42)$$

This condition shows when it is advantageous for the coalition to include additional member i. If inclusion implies that the additional coalition wide benefits are greater than the additional coalition wide costs then inclusion of country i is desirable. An alternative interpretation is that a coalition consists of members

whose inclusion generates a surplus to the coalition that is sufficient to cover the additional costs.

Country *i* signing the agreement results in an endogenous response by all other N-1 countries. Both the Nash and Stackelberg equilibrium concepts lead to an increase in the abatement of the M original signatories when non-signatory i accedes to the agreement, since the sums  $\sum_{j=1}^{M} \frac{1}{c_j}$  and  $\sum_{j=1}^{M} \alpha_j^s$  both increase and the sum  $\sum_{j=M+1}^{N} \theta_j^n$  decreases in equations (32) and (35). The partial derivative of the non-signatory reaction function with respect to signatory abatement, from equation (29) is

$$\frac{\partial}{\partial Q^s} \left[ Q^{ns} \right] = \frac{\partial}{\partial Q^s} \left[ \frac{b \sum_{j=M+1}^N \theta_j^n \left( a - Q^s \right)}{1 + b \sum_{j=M+1}^N \theta_j^n} \right] = \frac{-1}{1 + \frac{1}{b \sum_{j=M+1}^N \theta_j^n}} \tag{43}$$

The partial derivative is a negative fraction for at least one non-signatory, so the reduction in non-signatory abatement is strictly less than the increased abatement by the signatories. Therefore, it is sufficient to show that if country i increases abatement when it accedes to the agreement that the global level of abatement increases. It remains to show that this is indeed the case for the Nash but not necessarily the Stackelberg equilibrium.

#### 1.6.1 Nash Coalition Formation

From any arbitrary coalition of M signatories, non-signatory i joining the agreement results in an increase in i's abatement if  $q_i^{s,M+\{i\}} - q_i^{n,M} > 0$ . The Nash equilibrium country levels of signatory and non-signatory abatement for any arbitrary coalition of [1, ..., M] signatories and [M+1, ..., N] non-signatories are given

by equations (32) and (33).

$$q_{i}^{s,M+\{i\}} = \frac{ab\left(\sum_{j=1}^{M} \alpha_{j}^{s} + \alpha_{i}\right)}{c_{i}\left[1 + b\left[\left(\sum_{j=M+1}^{N} \theta_{j}^{n} - \theta_{i}\right) + \left(\sum_{j=1}^{M} \frac{1}{c_{j}} + \frac{1}{c_{i}}\right)\left(\sum_{j=1}^{M} \alpha_{j}^{s} + \alpha_{i}\right)\right]\right]}$$

$$q_{i}^{n*,M} = \frac{ab\alpha_{i}}{c_{i}\left[1 + b\left(\sum_{j=M+1}^{N} + \sum_{j=1}^{M} \frac{1}{c_{i}}\sum_{j=1}^{M} \alpha_{j}^{s}\right)\right]}$$
(44)

Becoming a signatory implies an increase in abatement if  $q_i^{s,M+\{i\}} - q_i^{ns,M} > 0$ . The sign of this condition reduces to<sup>6</sup>

$$\sum_{j=1}^{M} \alpha_{j}^{s} + b \sum_{j=1}^{M} \frac{1}{c_{j}} \left[ \left( \sum_{j=1}^{M} \alpha_{j}^{s} \right)^{2} - (\alpha_{i})^{2} \right] + b \sum_{j=1}^{M} \alpha_{j}^{s} \left[ \sum_{j=M+1}^{N} \theta_{j}^{n} - \theta_{i} \right] > 0$$
 (45)

For all coalitions of more than one non-signatory  $\sum_{j=M+1}^{N} \theta_{j}^{n} - \theta_{i} > 0$  since  $\sum_{j=M+1}^{N} \theta_{j}^{n}$  contains  $\theta_{i}$ . If country i is the only non-signatory then  $\sum_{j=M+1}^{N} \theta_{j}^{n} = \theta_{i}$  and the  $\left[\sum_{j=M+1}^{N} \theta_{j}^{n} - \theta_{i}\right]$  term is zero. Equation (45) shows that the Nash equilibrium will eventually include all countries given the sufficient, but not necessary, condition that  $\alpha_{i} < \frac{1}{2}$  for i = [1, ..., N].

Proposition 5: If no country has  $\alpha_i > \frac{1}{2}$  then the Nash equilibrium concept implies that all sub-coalitions will desire full participation, and hence the Pareto level of abatement.

The Nash equilibrium results in a situation where from any arbitrary coalition of signatories there exists an abatement level for each non-signatory that is greater than the Nash level while providing gains to the coalition.

#### 1.6.2 Stackelberg Coalition Formation

Since the Stackelberg level of abatement is strictly below the Pareto level the inclusion of an additional member would seem to imply an abatement level closer

<sup>&</sup>lt;sup>6</sup>Some intermediate steps are in the appendix.

to the Pareto optimum. However, this is not necessarily the case. The signatory level of abatement is not monotonically increasing in the number of signatories in the Stackelberg equilibrium for all possible coalitions. Recall that the Stackelberg signatories reduce their abatement inducing the non-signatories to increase their abatement, relative to the Nash. This means that it is possible that any arbitrary non-signatory may actually reduce abatement if it accedes to the agreement. The summation of the IC constraint across all members of the coalition in equation (42) shows that accession is desirable if it increases global abatement. The Stackelberg equilibrium is more complicated than the Nash since there is a strategic effect by the signatories. The Stackelberg quantities, from equations (35) and (36) are

$$q_{i}^{s,M+\{i\}} = \frac{ab\left(\sum_{j=1}^{M} \alpha_{j}^{s} + \alpha_{i}\right)}{c_{i} \left[\left(1 + b\left(\sum_{j=M+1}^{N} \theta_{j}^{n} - \theta_{i}\right)\right)^{2} + b\left(\sum_{j=1}^{M} \frac{1}{c_{j}} + \frac{1}{c_{i}}\right)\left(\sum_{j=1}^{M} \alpha_{j}^{s} + \alpha_{i}\right)\right]}$$

$$q_{i}^{n,M} = \frac{ab\alpha_{i}}{c_{i} \left[\left(1 + b\sum_{j=M+1}^{N}\right)^{2} + b\sum_{j=1}^{M} \frac{1}{c_{j}}\sum_{j=1}^{M} \alpha_{j}^{s}\right]}$$
(46)

Unfortunately, the condition which determines whether accession results in an increase in abatement has no simple interpretation as in the Nash equilibrium<sup>7</sup>. It is not possible to sign the Stackelberg increase in abatement as a signatory for any arbitrary coalition. The Stackelberg equilibrium presents the possibility that there are coalitions that would prefer to have a given country outside the agreement since inclusion would imply a reduction in that countries abatement. There are potential coalitions that will result in less than full participation and global abatement levels below the Pareto optimum.

If the goal of the agreement is obtaining an agreement with full participation then the agreement should specify that the signatories, even while acting as

<sup>&</sup>lt;sup>7</sup>The result is in the Appendix.

Stackelberg leaders, choose the higher Nash level of abatement rather than the Stackelberg level. The Stackelberg equilibrium is a credible threat for any arbitrary coalition of countries, however, in terms of coalition formation, it might be in the coalitions interest to achieve the full participation Pareto outcome. Hence, such a coalition should choose the higher Nash level of coalition abatement in an effort to stimulate coalition formation and ultimately full participation, even though that particular coalition might not appear to have an incentive to do. For full participation to be ensured an agreement should specify the Nash level of signatory abatement, not the lower Stackelberg level of abatement.

## 1.7 Numerical Example

A numerical example provides insight as to how the model works. Proposition 2 states that the gains to cooperation are increasing in the variance of the squares of the benefit shares for the case of symmetric MAC. Equation (26) shows that the difference between the Pareto and the Nash equilibrium quantities of abatement is increasing in the sum  $\sum_{j=1}^{N} \frac{1}{c_j}$ , given the condition  $\sum_{j=1}^{N} \frac{1}{c_j} < \frac{\sqrt{N}}{b}$  holds. The sum  $\sum_{j=1}^{N} \theta_j$  is smaller when the high cost countries are also the high benefit share countries. The coalition formation requirement was that no single country have a benefit share greater than  $\frac{1}{2}$ . Incentive compatibility requires that the sum of the gains from deviating must be less than the sum of the losses. The parameters for the numerical example are chosen to illustrate this scenario. The distribution of the MAC parameters is  $c_i = [1, 2, 3, 4, 5]$  and the benefit share distribution is  $\alpha_i = [\frac{1}{25}, \frac{3}{25}, \frac{5}{25}, \frac{7}{25}, \frac{9}{25}]$ . The Nash and Pareto levels are denoted \* and o, respectively. The totals are given in the bottom row of the table.

Table	Table 1.1: Nash and Pareto Payoffs							
i	$\alpha_i$	$c_i$	$\theta_i$	q* Nash	$q_i^o$ Pareto	$\pi_i^*$ Nash	$\pi_i^o$ Pareto	
1	$\frac{1}{25}$	1	$\frac{1}{25}$	2.57	28.30	50.51	-661.02	
2	3 25	2	<u>3</u> 50	3.85	14.15	141.65	19.43	
3	<u>5</u> 25	3	$\frac{1}{15}$	4.28	9.43	230.60	432.88	
4	$\frac{7}{25}$	4	7 100	4.49	7.08	318.99	779.58	
5	9 25	5	9 125	4.62	5.66	407.17	1099.58	
$\sum_{j=1}^{N}$	1	$S_c = 2.28$	$S_{\theta} = 0.31$	$Q^* = 19.80$	$Q^o = 64.62$	$\Pi^* = 1149$	$\Pi^o = 1670$	

Table 1.2: Nash and Pareto Payoffs for Symmetry									
i	$i$ $\alpha_i$ $c_i$ $\theta_i$ $q_i^*$ Nash $q_i^o$ Pareto $\pi_i^*$ Nash $\pi_i^o$ Pareto								
all	$\frac{1}{5}$	3	15	4.21	11.43	274.79	457.14		
$\sum_{j=1}^{N}$	1	$S_c = 1.67$	$S_{\theta} = 0.33$	$Q^* = 21.05$	$Q^o = 57.4$	$\Pi^* = 1374$	$\Pi^o = 2286$		

Clearly the two lowest cost countries will require compensation to implement the Pareto level of abatement, as required by efficiency. Complete symmetry implies that  $c_i = \bar{c} = 3$ , the arithmetic mean of the distribution  $c_i = [1, 2, 3, 4, 5]$ , and  $\alpha_i = \frac{1}{5}$ . Note that the sum  $\sum_{j=1}^{N} \frac{1}{c_j}$  is greater and the sum  $\sum_{j=1}^{N} \theta_j$  is less than the symmetric case, which implies that the Pareto level of abatement is greater, and the Nash level lower, than the symmetric case.

The gain that a country receives from leaving the agreement is the reduction in abatement cost. Evaluating incentive compatibility at the Pareto level, the gain is the cost difference between the Pareto level of abatement,  $q_i^a$ , and the quantity that country i would undertake as the single non-signatory,  $q_i^n$ . The loss from deviating from the Pareto level is the reduction in benefit. If country i deviates then they internalize  $\alpha_i$  of the reduction in global benefit as a result of their own quantity reduction, as well as the reduction in  $Q_{-i}$  from the endogenous response of the remaining signatories. The remaining signatories decrease their abatement, given a defector, an automatic punishment mechanism that is credible since it is the signatory abatement level that maximizes the remaining signatories collective

net benefits.

The numerical example is given under the assumption that the signatories and non-signatories behave as Nash players, that is, taking the actions of the other as given. If the agreement is viewed as a credible commitment device, then the interaction between the two groups could be modeled as a Stackelberg equilibrium. Proposition 3 states that for at least one non-signatory that the Stackelberg global quantity is less than the Nash, making the losses from leaving larger. Furthermore, the Stackelberg non-signatory quantity is strictly greater than the Nash non-signatory quantity, for coalitions of signatories with at least one non-signatory. Therefore the gains due to cost reduction from leaving the agreement are smaller for the Stackelberg equilibrium concept. The Stackelberg equilibrium has strictly greater incentive compatibility properties than the Nash, a fact that was not utilized in the numerical example. If country i leaves the agreement the global level of abatement is denoted  $Q^{i}$  nonsig.

Table	Table 1.3: Incentive Compatibility							
i	$\alpha_i$	Ci	$\theta_i$	$q_i^n$	$Q^{i \text{ nonsig}}$	$gain_i$	$loss_i$	
1	1 25	1	$\frac{1}{25}$	1.59	50.44	399.24	135.15	
2	3 25	2	<u>3</u> 50	2.08	56.59	195.91	373.18	
3	5 25	3	15	2.31	56.55	125.44	570.09	
4	$\frac{7}{25}$	4	7 100	2.51	55.10	87.48	729.45	
5	$\frac{7}{25}$	5	9 125	2.71	52.92	61.72	856.26	
$\sum_{j=1}^{N}$	1	$S_c = 2.28$	$S_{\theta} = 0.31$			Sum = 870	Sum = 2664	

A set of abatement requirements exist at the Pareto optimal level that is incentive compatible since  $\sum_{j=1}^{N} gain_i < \sum_{j=1}^{N} loss_i$ . In fact the magnitude of the difference is quite large. The set of abatement requirements, with permit trading, need only compensate the lowest cost country. For example an agreement where

 $q_2^r = q_2^o$ ,  $q_3^r = q_3^o$ ,  $q_4^r = q_4^o$ , and countries 1 and 5 having abatement requirements of 18.30 and 15.66, respectively, is incentive compatible. This implies that country 1 sells 10 permits at the permit price of  $p = \frac{Q^o}{S_c} = 28.34$  resulting in an effective transfer of 283.40 from country 5 to country 1, and achieving incentive compatibility for all countries at the Pareto level. This is just one possibility from the set of incentive compatible requirements, the determination of which will be subject to negotiation. Having the large benefit share country being the high cost countries implies that  $Q^{i \text{ nonsig}}$  is relatively similar across countries, and thus the loss from leaving the agreement varies more strongly with  $\alpha_i$  than if the high benefit share countries are also low cost. If we have symmetry then incentive compatibility requires that gain for each country is less than the loss from deviating from the Pareto level.

Table 1.4: Incentive Compatibility for Symmetry								
i	$i$ $\alpha_i$ $c_i$ $\theta_i$ $q_i^n$ $Q^{i \text{ nonsig}}$ $gain_i$ $loss_i$							
all	<u>1</u>	3	15	2.80	47.55	184.18	73.12	
$\sum_{j=1}^{N}$	1	$S_c = 1.67$	$S_{\theta} = 0.33$			Sum = 921	Sum = 366	

The symmetric case is not incentive compatible. Each (symmetric) country has an incentive to deviate from the Pareto level and the agreement is not sustainable.

### 1.8 Conclusion

This paper has provided a theoretical framework for analyzing self-enforcing international emissions agreements (IEA) when countries are asymmetric. The main point of inquiry is the incentives of asymmetric countries to join, and subsequently honor. an agreement based on their benefit shares, marginal abatement costs (MAC), and required abatement under the terms of an agreement. The signatory and non-signatory levels of abatement are derived for any arbitrary partition

of countries, when signatories as a group act as a Nash player and when the signatories act as a Stackelberg leader. The pollution permit price is determined for any arbitrary level of abatement by signatories with any arbitrary MAC characteristics. An upper bound on the required level of abatement for any country is derived stating when an IEA is incentive compatible. The Nash equilibrium can result in a IEA that is incentive compatible for all countries, achieving full participation from any initial coalition of signatories, as long as no single country has a benefit share greater than  $\frac{1}{2}$ . The Stackelberg equilibrium raises the possibility of less than full participation and sub-Pareto abatement levels, due to a signatory strategic incentive to reduce abatement.

There are many policy implications of this model. First and foremost, the Kyoto Protocol requires essentially equi-proportional abatement, irrespective of incentive compatibility. This means that a country such as the United States, which accounts for roughly  $\frac{1}{5}$  of the world emissions of greenhouse gases, will have a required level of abatement that is  $\frac{1}{5}$  of the signatory abatement. There is a wide range of parameter values for which this requirement will violate the incentive compatibility constraint, and such a country would have an incentive to remain outside the agreement.

Second, it is crucial that free-trade in pollution permits be allowed. Pollution permits allow a divergence between the required level of abatement and the efficient level for each country. Limiting the gains from permit trading through requirements of domestic abatement reduces the potential gains to cooperation and thus undermines the effectiveness of the agreement. Furthermore, it is essential

that the permit market is competitive. Cartelization of the permit supply will similarly undermine the effectiveness of the agreement.

The third main implication is that the agreement should implement the Nash level of abatement for any coalition of signatories. Allowing the signatories to behave as Stackelberg leaders, reducing their abatement to induce the non-signatories to increase their abatement could be detrimental to achieving full participation.

There are many possible extensions of this framework for future work. The first are empirical investigations of the model. The MIT Joint Program on the Science and Policy of Global Climate Change (1999) estimated MAC curves for a world divided into 12 regions. Their estimation results in an extremely wide range of MAC curves across regions. Given these estimates the incentive compatibility constraint will determine which countries will find it rational to accede to an agreement. The required levels of abatement under the Kyoto protocol, nearly equi-proportional, will show which countries will find the Kyoto Protocol incentive compatible.

The underlying justification for benefit share asymmetry has not been addressed in this paper. If each person in the world receives an equal share of the benefit then the share for country i could be interpreted as  $\alpha_i = \frac{N_i}{N}$ , where  $N_i$  is the population in country i and N is the global population. Under this assumption, the MIT (1999) estimates imply that certain high benefit share countries, such as China. India and Brazil are also low MAC countries. The results of the model suggest that these countries will find it rational to accede to an agreement, given that their required levels of abatement are not too high.

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If the benefits of abatement are taken to be avoided costs, then a possible interpretation is that benefits are greater in richer countries. For example, The Kyoto Protocol and the President's Policies to Address Climate Change: Administration Economic Analysis (1998) predicted that a 20 inch rise in sea level "could inundate approximately 7,000 square miles of US territory." The equivalent amount of land in, say, Bangladesh has a much lower monetary value. So, it the benefits of abatement are taken to be avoided costs then the benefit share of country i could be proxied by the GDP share of country i,  $\alpha_i = \frac{GDP_i}{GDP}$ . Finally, the bargaining process that determines the required levels of abatement from the set of IC agreements could be modeled, particularly when not all countries are present at the bargaining table.

# Chapter 2

# 2 An Evolutionary Game Environmental Trade Model

#### 2.1 Introduction

Recently there has been a great deal of interest in achieving trade and environmental goals simultaneously. Free-trade has received criticism from groups that traditionally have had divergent interests. Much of the trade and environment literature models interaction between these groups in terms of rent-seeking where, say, producers gain from a tariff at consumers expense. This chapter shows how producers, consumers, environmentalists and government can all gain from a tariff, in a model characterized by imperfect competition and pollution that is local in nature. This result is obtained in an evolutionary game model that is more complicated than standard bi-matrix evolutionary games. This model allows for both non-linearities and own-population effects, which preclude the use of a simple bi-matrix, and allows for a higher degree of endogeneity.

There has been a wide range of literature on trade and the environment mainly focused on optimal taxation of domestic pollution in an open economy. The implications of these models are mainly driven by the assumed market structure and whether pollution is modeled as trans-boundary, or is contained within the country that produces the good. Brian Copeland and Scott Taylor (1994) incorporate pollution into a dynamic Ricardian model (Dornbusch, Fischer and Samuelson 1977) in which a continuum of goods are produced, each with a different pollution intensity. In their two country general equilibrium model, trade is driven

by income induced differences in environmental policy, in which the richer North has a higher domestic tax rate on pollution because environmental quality is considered a normal good. Since pollution is modeled as a factor of production, in equilibrium, the North imports goods which contain a relatively higher pollution content. They decompose the impact of trade on pollution into three components: a scale effect, a composition effect and technique effect. The scale effect leads to an increase in pollution since free-trade increases world output. The composition of output in any one country may change under free-trade, which can lower pollution if a country imports relatively pollution intensive goods. Finally, they point to a technique effect, which is negative. As countries trade they become richer and demand higher environmental standards, raising domestic tax rates on pollution, inducing cleaner techniques of production. They find that free-trade leads to a decrease in pollution in the North, an increase in the South, and an overall increase in worldwide pollution as the scale effect dominates the technique effect. There is no trans-boundary pollution in the model, so each country regulates domestic pollution optimally in the form of pollution taxes, which are set equal to marginal pollution damage.

Copeland and Taylor (1995) incorporates trans-boundary pollution and terms-of-trade effects of domestic policies into their earlier paper. Governments choose domestic pollution targets to maximize national welfare and then auction off that quantity of pollution permits. As in their previous paper, trade is driven by differences in income levels, and subsequently the domestic price of the services of pollution as a productive factor. They find that free-trade leads to an increase in pollution if incomes between the North and the South are sufficiently different.

Copeland (1996) shows that the Home country has an incentive to impose a pollution content tariff on a neighboring country that produces a good that generates trans-boundary pollution, and subsequently exports that good to the home country. Furthermore, foreign domestic regulation of pollution can increase the home countries incentive to impose a pollution content tariff in an attempt capture foreign rents. Taxing the distortion directly, in the form of a tariff, is generally the first-best policy. However, Copeland notes that "The standard objection to using trade policy to target foreign pollution is that it provides another thinly disguised source of protectionism. This paper shows that this problem may be even more serious than previously thought."

Rent-shifting arguments dominate the trade and environment literature using models of imperfect competition. Kennedy (1994) shows that countries have a strategic incentive to distort domestic pollution taxes under free-trade in a model that allows for trans-boundary pollution. This incentive is decomposed into a rent-capture effect and a pollution-shifting effect. The rent-capture effect tends to reduce the domestic taxation of pollution in an attempt to improve the terms of trade. The pollution shifting effect works in the opposite direction, tending to increase the domestic taxation of pollution in an attempt to shift pollution to the other country. In Kennedy's model the rent-capture effect dominates the pollution-shifting effect leaving domestic taxation of pollution below the optimal level. except in the case of perfect competition and no trans-boundary pollution. Since producers are imperfectly competitive, price is greater than marginal cost, and output is below the efficient level. Since a polluting imperfectly competitive firm incorporates two distortions, Kennedy notes that taxing pollution will reduce

output further, which implies that an efficient tax must be set at a level that is less than marginal damage. In other words, imperfect competition implies an optimal tax that is set below the Pigouvian level. Kennedy asserts that the first-best outcome requires two instruments, a production subsidy and a pollution tax.

Brander and Krugman (1983) develop an imperfectly competitive trade model between asymmetric countries, in which demand elasticities differ, so firms sell in both countries. In a Cournot model with free entry, they show that trade is welfare improving due to increased competitiveness, even in the presence of transport costs. When there is product differentiation they obtain intra-industry trade, that is, two-way trade in the same type of goods. Pollution is not addressed in their model.

The analytical framework for this model is adapted from Friedman and Fung (1996). in which they analyze the different modes of production that evolve from trade in differentiated products. They conduct simulations of a Cournot based trade model between the United States and Japan in which there is a state dependant cost-side externality. They find that corner equilibria dominate, and that interior equilibria are inherently unstable.

This chapter incorporates elements from all of the models mentioned above. A single product is differentiated by the environmental impact of its production, clean or dirty. Firms behave in a Cournot setting, choosing quantity to maximize profits, taking the other firms' output as given. The two countries are asymmetric on both the demand and the cost sides. Pollution is not trans-boundary, but rather is contained to the country in which the dirty good is produced. However, in contrast to the previous literature on trade and the environment, this paper

models the dynamic interaction of profit maximizing firms in an evolutionary game setting. Both the short-run quantity decision and long-run decision of production method, clean or dirty, are endogenous following a free-trade agreement when it is possible to distinguish a good based on the method of production.

Due to evolutionary dynamics the result of a free-trade agreement is that any interior initial condition, given by interior autarchy equilibria, is unstable. The evolutionary equilibrium (EE) results in at least one country being completely specialized as intra-industry trade in both types of the good occurs only along the adjustment path to the EE. Furthermore, at least one country will have an incentive to deviate from the free-trade EE. When this deviation takes the form of a "lack-of-pollution" content tariff, that is a tariff on clean imports, national welfare can be raised through rent-capture, pollution shifting and tariff revenue effects.

The remained of the paper is organized as follows. The short-run autarchy equilibria are determined as a Cournot game in which the two types of firms, clean or dirty, choose quantity to maximize their profits. In autarchy the long-run, or evolutionary, equilibria are determined when there is a zero profit differential between the two types. Presumably the clean type faces a higher marginal cost, but can charge a price premium<sup>8</sup>. The autarchy equilibrium is obtained at that proportion of firms where these two effects exactly offset. The autarchy equilibrium is used as the initial condition for the free-trade agreement between two asymmetric countries. The good is assumed to be differentiated only by its type, clean or dirty, not by the country of origin. With trade, in the short run, each of the four

<sup>&</sup>lt;sup>8</sup>The results are derived in general form to allow for any marginal cost, price premium combination.

types of firms chooses quantities, for sale in the domestic and foreign markets, to maximize profits. The advantage of using an evolutionary game theory is that it analyzes firms' dynamic response to endogenous profit differentials. Firms tend to abandon the less profitable type and adopt the more profitable type over time. The short-run equilibrium is only stable in a dynamic setting when there is no profit advantage to switching type. Thus, the proportion of clean firms is endogenous in the long-run, and taken as given in the short-run. The free trade (EE) is obtained, and the implications for trade policy for a given EE are investigated. Finally, the welfare and distributional aspects of the autarchy, free-trade and pollution content tariff equilibria are examined for a numerical example. After the conclusion, the Appendix derives general parameter restrictions as an alternative to simulations.

## 2.2 Autarchy

## 2.2.1 Home Country

The clean and dirty goods are close, but imperfect substitutes, so there is a reduction in price when the quantity of the substitute increases. In the home market the following linear demand curves are assumed.

$$P_{c} = \alpha_{c} - \beta Q_{c} - \delta Q_{d}$$

$$P_{d} = \alpha_{d} - \beta Q_{d} - \delta Q_{c}$$

$$(47)$$

 $P_i$  and  $Q_i$  denote the market price and quantity of good i, and the subscripts c and d denote clean and dirty respectively. The own effect of quantity on price is assumed to dominate the cross effect, implying  $\beta > \delta$ . If  $\alpha_c$  is greater than  $\alpha_d$ , there is a price premium paid for the clean good at equal quantities of each type.

There are N firms, proportion s of which are of the clean type, and proportion (1-s) of which are dirty. It is assumed that all firms have equal access to either the clean or dirty technology and inputs, so that all firms of any given type are of identical size. The firms are assumed to have constant, yet asymmetric marginal costs,  $c_c$  and  $c_d$ . Fixed costs are assumed to be zero for simplicity<sup>9</sup>. In the short-run the total cost for a firm is

$$TC_c = c_c q_c \tag{48}$$

$$TC_d = c_d q_d$$

where  $q_i$  is the quantity produced by a type *i* firm. Firms choose their quantities to maximize their profit functions,  $\pi_c$  for clean, and  $\pi_d$  for dirty, taking all other firms output as given.

$$\pi_c = q_c(P_c - c_c) \tag{49}$$

$$\pi_d = q_d(P_d - c_d)$$

The assumption of identical sized firms means that the market quantity of each type is equal to the number of firms that are that type, multiplied by the quantity for that type,  $Q_c = sNq_c$  and  $Q_d = (1-s)Nq_d$ . Substituting in the firm quantities into the demand curves, and then the demand curves into the profit functions, yields profit as a function the firms quantities and the parameters.

$$\pi_c = q_c \left[ \alpha_c - \beta(q_c + \hat{Q}_c) - \delta(1 - s) N q_d - c_c \right]$$

$$\pi_d = q_d \left[ \alpha_d - \beta(q_d + \hat{Q}_d) - \delta s N q_c - c_c \right]$$
(50)

<sup>&</sup>lt;sup>9</sup>The results are qualitatively the same with fixed costs. See de Vries (2001) for a similar model including fixed costs.

Where  $\hat{Q}_c$  is the sum of the other clean firms output and  $\hat{Q}_d$  is the sum of the other dirty firms output. The first order condition for profit maximization of a clean firm is

$$\alpha_c - \beta(q_c + \hat{Q}_c) - \delta(1 - s)Nq_d - c_c - \beta q_c \leq 0$$

$$q_c \geq 0 \tag{51}$$

Given equation (51) the first-order-condition for profit maximization is of the form  $P_i - c_i - \beta q_i \leq 0$ , where i = c, d. For strictly positive quantities,  $P_i = \beta q_i + c_i$ . In this case,  $\pi_i = (P_i - c_i)q_i = \beta(q_i)^2$ . The first order conditions for profit maximization lead to the following downward sloping reaction functions, indicative of strategic substitutes.

$$q_{c} = \frac{\alpha_{c} - c_{c} - \delta(1 - s)Nq_{d}}{\beta(1 + sN)}$$

$$q_{d} = \frac{\alpha_{d} - c_{d} - \delta sNq_{c}}{\beta(1 + (1 - s)N)}$$
(52)

Figure 1 below is a graph of the reaction functions for specified values of the parameters<sup>10</sup> and a given value of the state variable. Now we verify that the reaction functions have a unique crossing for each value of the state variable  $s \in [0,1]$ . The slope of the clean reaction function is  $\frac{-\beta(1-sN)}{\delta(1-s)N}$ , and the slope of the dirty reaction function is  $\frac{-\delta sN}{\beta(1+(1-s)N)}$ . The clean reaction function is steeper for all  $s \in [0,1]$ . At low values of s, where the dirty type is more prevalent, the reaction functions intersect at quantities where  $q_c > q_d$ , and at values of s which are higher

<sup>&</sup>lt;sup>10</sup>The parameters used throughout this paper are:  $\alpha_c = 200$ ,  $\alpha_d = 190$ ,  $c_c = 10$ ,  $c_d = 8$ , N = 7.  $\alpha_c^* = 150$ ,  $\alpha_d^* = 145$ ,  $c_c^* = 9$ ,  $c_d^* = 6$ ,  $N^* = 7$ ,  $\beta = 1$  and  $\delta = 0.9$ , where the \* denotes a foreign variable. General parameter restrictions for interior equilibria under autarchy and trade are given in the Appendix.

than the long-run Nash equilibrium the reaction functions intersect at quantities where  $q_d > q_c$ . The long-run equilibrium is that particular value of s where the reaction functions intersect at equal profits, and hence, equal quantities given the discussion after equation (51).

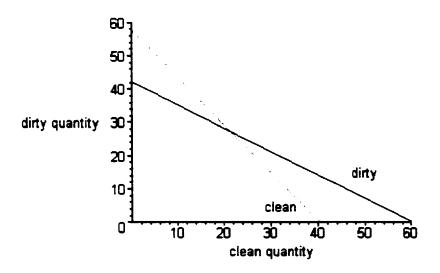


Figure 2.1: Reaction functions for s = 0.5.

The unique Nash equilibrium quantities are the simultaneous solution to the reaction functions given in (52)

$$q_c^{NE} = \frac{\beta(1 + (1 - s)N)\theta_c - \delta(1 - s)N\theta_d}{\Delta}$$

$$q_d^{NE} = \frac{\beta(1 + sN)\theta_d - \delta sN\theta_c}{\Delta}$$
(53)

where  $\Delta \equiv \beta^2 (1+sN)(1+(1-s)N) - \delta^2 (s(1-s)N^2) > 0$  for  $\beta \geq \delta$ , and  $\theta_c \equiv \alpha_c - c_c$ ,  $\theta_d \equiv \alpha_d - c_d$ . The Nash equilibrium quantities in (53) are quadratic in s. Figure 2 shows the Nash equilibrium quantities as a function of s, for given values of the parameters.

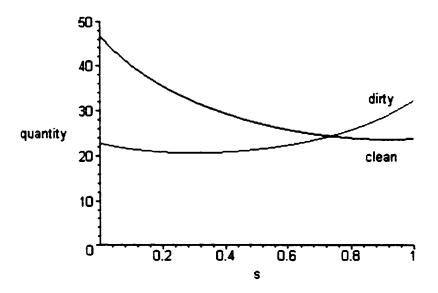


Figure 2.2: Nash equilibrium firm quantities.

The graphs of the prices at the Nash equilibrium, as a function of s, have the same shape as their respective quantities.

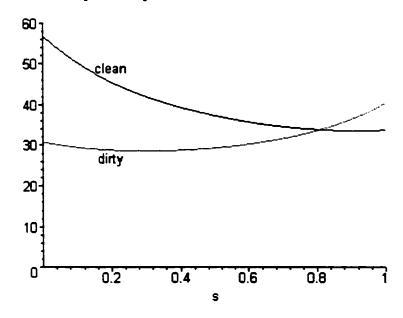


Figure 2.3: Nash equilibrium prices.

For positive quantities the short-run Cournot-Nash equilibrium profits are equal to  $\pi_c = \beta(q_c)^2$  and  $\pi_d = \beta(q_d)^2$ . If the profits of the clean firms are greater, then we would expect some of the dirty firms will switch their type. The function  $\Pi_D \equiv \pi_c - \pi_d$  is the profit advantage for a clean firm, where  $\pi_i$  satisfies the short-run Cournot-Nash equilibrium for type i. The dynamics are a function of the parameters and the state variable s, the proportion of clean firms. The profit differential is

$$\Pi_D = \beta \left[ (q_c)^2 - (q_d)^2 \right] \tag{54}$$

Equation (54) at the Nash equilibrium simplifies to

$$\Pi_{D}^{NE} = \frac{\begin{bmatrix} \theta_{c}^{2} \left[ \beta^{3} \left[ 1 + (1 - s)N \right]^{2} - \beta \delta s^{2}N^{2} \right] \\ -\theta_{d}^{2} \left[ \beta^{3} (1 + sN)^{2} - \beta \delta^{2}N^{2} (1 - s)^{2} \right] - 2\theta_{c}\theta_{d}\beta^{2}\delta \left[ (1 - 2s)N(N + 1) \right] \end{bmatrix}}{\Delta^{2}}$$
(55)

where  $\Delta$  is given after equation (53). The numerator of  $\Pi_D^{NE}$  is quadratic in s, and the denominator is a 4th degree polynomial in s. The numerator is positive at s=0 and negative at s=1 for the parameter values used in the numerical example<sup>11</sup>. The long-run autarchy equilibrium has the following characterization.

<sup>&</sup>lt;sup>11</sup>The general parameter restriction for this condition is in the Appendix.

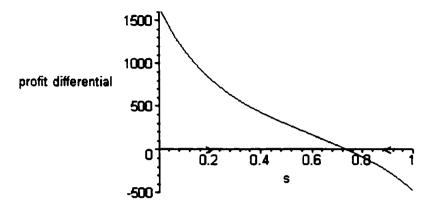


Figure 2.4: Home autarchy EE.

The home market autarchy equilibrium is unique and stable at s=0.735. This is the proportion of clean firms where both the short-run quantity decision maximizes firms profits and firms have no incentive to switch their type in the long-run. It is stable due to the fact that the profit differential is a decreasing function of s. In the neighborhood of the equilibrium, as s increases the optimal quantity of a clean firm decreases, and the optimal quantity of a dirty firm increases. The overall effect of this is to make the clean good relatively less profitable. Alternatively, since profits are strictly increasing in the Nash equilibrium quantities the profit advantage is negative for values of s greater than the equilibrium, and positive for s less than the equilibrium. This makes  $\Pi_D = 0$  the unique evolutionary equilibrium (EE). Any initial proportion of clean firms,  $s \in [0,1]$ , will converge to the EE, and the EE is robust to perturbations<sup>12</sup>. This stable, unique, interior equilibrium is the s that is, a firm choosing a type, clean or dirty, that

analogue of a single population, non-linear, Hawk-Dove game in the evolutionary game literature, where as one type becomes more prevalent, its payoff advantage decreases<sup>13</sup>. The adjustment dynamics to the long-run equilibrium have only been required to be sign-preserving, that is, the proportion of clean firms increases when the profit advantage to being clean is positive

$$\dot{s} = A(s)\Pi_D(s) \tag{56}$$

where  $\frac{\partial A}{\partial s} > 0$ , and the dot denotes the time derivative. Sign preserving dynamics are more general than, say, replicator dynamics, in which strategies that have a relatively higher payoff advantage increase at a higher rate<sup>14</sup>. The fixed point of (56) is stable and unique, implying that both types of firms will exist in the long-run under autarchy. The intuitive explanation for this is that the clean type can charge a price premium, but faces a higher marginal cost. The equilibrium is where these two effects offset each other, as well as the degree of substitutability. If both types have the same marginal costs and demand intercepts then unique EE would occur at s = 0.5, for positive values of  $\beta$  and  $\delta$ . The illustrative parameter values chosen result in an EE at s = 0.735.

#### 2.2.2 Foreign Country

This paper models asymmetric integration, to analyze the case of a smaller, less developed country integrating with a larger, more developed country. The foreign

was not optimal. This type of evolutionary equilibrium is also robust to equilbrium entrants, although the number of firms, N, is held fixed in this model. See the Appendix for a discussion of why holding N fixed is not critical in obtaining the results of the model.

<sup>&</sup>lt;sup>13</sup>See Weibull (1995) for a discription linear Hawk-Dove evolutionary games.

<sup>&</sup>lt;sup>14</sup>Strictly speaking replicator dynamics for this model are  $\dot{s} = \left[\pi_c(s) - \frac{s\pi_c(s) + (1-s)\pi_d(s)}{2}\right]s$ . This implies that the growth rate of the population share of clean is increasing in the profit advantage of clean, which is true for the dynamics in (56). Replicator dynamics are therefore a special case of the dynamics in (56).

market is qualitatively the same, but the magnitudes of the parameters are assumed to be different. Specifically, the intercepts of the demand curves and the marginal costs are assumed to retain the same ordinal rankings, but each corresponding parameter is assumed to have a lower value in the foreign country. The degree of substitutability in preferences between the clean and dirty types,  $\beta$  and  $\delta$ , are assumed to be the same in each country so that the trade equilibrium is driven by the price premium, the differences in the relative costs of production, and the number of firms, rather than the degree of substitutability. The foreign market has qualitatively the same reaction functions as in equation (52). The foreign market autarchy Nash equilibrium quantities are

$$q_c^{*NE} = \frac{\beta(1 + (1 - s^*)N^*)\theta_c^* - \delta(1 - s^*)N^*\theta_d^*}{\Delta^*}$$

$$q_d^{*NE} = \frac{\beta(1 + s^*N^*)\theta_d^* - \delta s^*N^*\theta_c^*}{\Delta^*}$$
(57)

Where  $\Delta^* = \beta^2 (1 + s^* N^*)(1 + (1 - s^*)N^*) - \delta^2 (s^*(1 - s^*)N^{*2})$ , which is the same as  $\Delta$  given after equation (53) with  $s^*$  and  $N^*$  replacing s and N. The foreign market equilibrium is essentially the same as the home market in terms of a unique, stable equilibrium, however the equilibrium value of  $s^*$  is typically different than s unless all of the parameters are identical across countries. The Nash equilibrium quantities for the foreign country under autarchy are given in Figure 5.

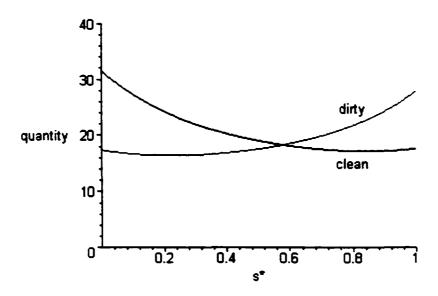


Figure 2.5: Nash equilibrium foreign firm quantities.

Again, the short-run profits of the firm are  $\pi_c^* = \beta(q_c^*)^2$  and  $\pi_d^* = \beta(q_d^*)^2$ . The dynamics are

$$\dot{s}^{\bullet} = B\Pi_D^{\bullet}(s) \tag{58}$$

where B>0. The foreign autarchy equilibrium is fundamentally the same as the home country, unique and stable with  $s^*=0.578$ . The long-run autarchy equilibrium has the following characterization

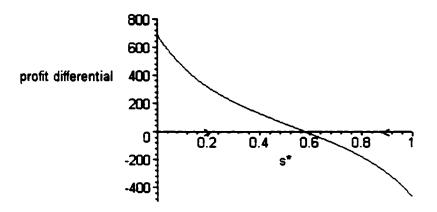


Figure 2.6: Foreign autarchy EE.

## 2.2.3 Autarchy Evolutionary Equilibria

Proposition 1: There is a unique, stable, interior evolutionary equilibrium for all parameter values that satisfy the restriction  $\frac{\beta}{\delta} > \frac{\theta_d}{\theta_c}, \frac{\theta_c}{\theta_d}$  in the home country and  $\frac{\beta}{\delta} > \frac{\theta_d^*}{\theta_c^*}, \frac{\theta_c^*}{\theta_d^*}$  in the foreign country.

Proof in Appendix 1.

There is a stable unique interior equilibrium in each country, meaning that both goods are produced in each country for all parameter values satisfying the restriction in Proposition 1, which is given in the Appendix. In general, the restriction ensures that:

- (i)  $q_i > 0$ , for  $i = c, d, \forall s \in (0, 1)$
- (ii)  $\Pi_D > 0$  at s = 0,  $\Pi_D < 0$  at s = 1

The equilibrium values for the parameters used in the numerical example (

 $\alpha_c = 200, \ \alpha_d = 190, \ c_c = 10, \ c_d = 8, \ N = 7, \ \alpha_c^* = 150, \ \alpha_d^* = 145, \ c_c^* = 9, \ c_d^* = 6,$   $N^* = 7, \ \beta = 1 \ \text{and} \ \delta = 0.9) \ \text{are}$ :

Table 2.1: Autarchy EE								
	Home		Foreign					
	Clean	Dirty	Clean	Dirty				
Firm Quantity	$q_c^{EE} = 24.31$	$q_d^{EE} = 24.31$	$q_c^{*EE} = 18.30$	$q_d^{*EE} = 18.30$				
Firm Profit	$\pi_c = 591.16$	$\pi_d = 591.16$	$\pi_c^* = 334.91$	$\pi_d^* = 334.91$				
Market Quantity	$Q_c = 125.10$	$Q_d = 45.10$	$Q_c^* = 74.05$	$Q_d^* = 54.05$				
Market Price	$P_c = 34.31$	$P_d = 32.31$	$P_c^* = 27.30$	$P_d^* = 24.30$				
EE	s = 0.735		$s^* = 0.578$					

In each market clean and dirty firms produce the same quantity, and therefore have equal profits. The home country produces the clean good relatively intensively. The home firms produce more and therefore have a higher profit than the foreign firms of the same type. In each market the clean good sells for a higher price than the dirty good and this premium is exactly equal to the additional marginal cost of producing the clean type.

The values for prices, quantities and profits are parameter specific and are meant to illustrate the features of the model, and as such should be interpreted in an ordinal, rather than cardinal sense. The main point is that there is a stable mix of both types of firms in autarchy. This is the evolutionary game analogue to two separate single population Hawk-Dove games. In this situation when one strategy type becomes too prevalent there is a payoff advantage to being the other type<sup>15</sup>. The populations are separated because under autarchy neither trade in

 $<sup>^{15}</sup>$ In general a Hawk-Dove game is linear in its payoff differentials. Since the firms payoffs (profits) are non-linear in the state space s or  $s^{\bullet}$  it is impossible to reduce the game to a bimatrix. It should be noted that the firm has two separate strategies depending on the time frame. The firms short-run strategy is the quantity choice, which is a continuum, while their long-run choice is binary, clean or dirty.

goods, nor trade in factors is allowed. In the next section we only allow for trade in goods, while not allowing firms to locate in the other country. With barrier-free trade in goods and factors the model reverts to a single, larger, population and the autarchy result is obtained as marginal costs of each type are equalized across countries.

#### 2.3 Trade

#### 2.3.1 Home Market

The home and foreign autarchy evolutionary equilibria serve as the initial condition when the model is opened up to trade. In the short-run each firm maximizes profits by simultaneously choosing quantities for sale in both the home and foreign markets. Products are assumed to be differentiated only by the method of production and not by the country of origin. The Nash equilibrium now consists of four quantities in each market. Initially free-trade is investigated and then the implications of allowing each country to impose tariffs which are differentiated by the type of the good.

As under autarchy, in the short-run firms simultaneously choose quantities to sell in the home and foreign market taking their type, and the output of other firms as given. For example, the short-run profit function for a home clean firm is is  $\pi_c = \beta \left[ (q_c^d)^2 + (q_c^e)^2 \right]$ , and the short-run profit function for a foreign clean firm is  $\pi_c^* = \beta \left[ (q_c^{d*})^2 + (q_c^{e*})^2 \right]$ , where the superscript d denotes for sale in the domestic market, and the superscript e denotes for export to the foreign market. The \* denotes foreign values, so  $q_c^{d*}$  is the quantity of a foreign clean firm for sale in the

<sup>&</sup>lt;sup>16</sup>See the discussion after equation (51) for why profits take this form.

foreign market, and  $q_c^{e\bullet}$  is the quantity of a foreign clean firm that is exported to the home market. Since a home firm of a similar type faces different cost conditions than a foreign firm, and each market is separated, there are four quantities sold in each market. The Kuhn-Tucker first order conditions for profit maximization for a home clean firm are

$$\alpha_{c} - \beta(q_{c}^{d} + \hat{Q}_{c}^{d} + Q_{c}^{e*}) - \delta(Q_{d}^{d} + Q_{d}^{e*}) - \beta q_{c}^{d} - c_{c} \leq 0$$

$$q_{c}^{d} \geq 0$$

$$\alpha_{c}^{*} - \beta(q_{c}^{e} + \hat{Q}_{c}^{e} + Q_{c}^{d*}) - \delta(Q_{d}^{e} + Q_{d}^{d*}) - \beta q_{c}^{e} - c_{c} - t_{c}^{*} \leq 0$$

$$q_{c}^{e} \geq 0$$
 (59)

Where  $t_c^*$  is a possible tariff on imports of the clean good imposed by the foreign country. The second order conditions for profit maximization are satisfied,  $\frac{\partial^2 \pi_c}{\partial (q_c^d)^2} = -2\beta = \frac{\partial^2 \pi_c}{\partial (q_c^e)^2} < 0$ . The symmetry assumption implies that  $\hat{Q}_c^d = (sN-1) q_c^d$ ,  $Q_c^{e*} = s^* N^* q_c^{e*}$ ,  $Q_d^d = (1-s) N q_d^d$ , and  $Q_d^{e*} = (1-s^*) N^* q_d^{e*}$ . This allows the first line of (59) to be written as the following reaction function, conditional on  $q_c^d > 0$ .

$$q_c^d = \frac{\alpha_c - c_c - \beta s^* N^* q_c^{e^*} - \delta (1 - s) N q_d^d - \delta (1 - s^*) N^* q_d^{e^*}}{\beta (sN + 1)}$$
(60)

The reaction functions for the other three types sold in the home market are found similarly. The entire system of reaction functions, in matrix form, is

$$AQ = X \tag{61}$$

Where

$$A = \begin{bmatrix} \beta(sN+1) & \beta s^*N^* & \delta(1-s)N & \delta(1-s^*)N^* \\ \beta sN & \beta(s^*N^*+1) & \delta(1-s)N & \delta(1-s^*)N^* \\ \delta sN & \delta s^*N^* & \beta\left[(1-s)N+1\right] & \beta\left(1-s^*\right)N^* \\ \delta sN & \delta s^*N^* & \beta\left(1-s\right)N & \beta\left[(1-s^*)N^*+1\right] \end{bmatrix}$$

$$Q = \begin{bmatrix} q_c^d \\ q_c^{e*} \\ q_d^d \\ q_d^{e*} \end{bmatrix} \text{ and } X = \begin{bmatrix} \alpha_c - c_c \\ \alpha_c - c_c^* - t_c \\ \alpha_d - c_d \\ \alpha_d - c_d^* - t_d \end{bmatrix}$$

and  $t_c$  and  $t_d$  are potential tariffs imposed by the home country on clean and dirty imports, respectively. The Nash equilibrium is the simultaneous solution to (61), taking the state variables s and  $s^*$  as given. The inverse of the coefficient matrix, A, is given in the Appendix. The Nash equilibrium is  $Q = A^{-1}X$ .

Using the same parameter values as previously<sup>17</sup>, we can get a 3-dimensional representation of the Nash equilibrium quantities for all possible values of s and  $s^*$ . The short-run Nash equilibrium for the four quantities sold in the home market are given in figures 7-10 below. The parameters used were chosen based on the fact that the correspond to the situation described in the introduction (clean can charge a price premium but faces a higher marginal cost) and the fact that the Kuhn-Tucker conditions are not binding in the entire state space  $s, s^* \in [0, 1]^2$ . The general parameter restriction for an interior solution, derived from this inverse, is in the Appendix.

 $<sup>\</sup>frac{17\alpha_c = 200, \ \alpha_d = 190, \ c_c = 10, \ c_d = 8, \ N = 7, \ \alpha_c^* = 150, \ \alpha_d^* = 145, \ c_c^* = 9, \ c_d^* = 6, \ N^* = 7, \ \beta = 1 \ \text{and} \ \delta = 0.9$ 

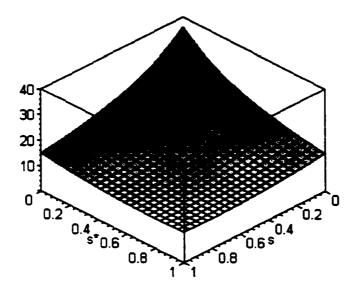


Figure 2.7: Nash equilibrium quantity of a home clean firm.

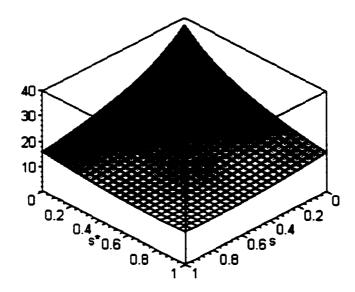


Figure 2.8: Nash equilibrium quantity of a foreign clean firm.

The first of the 3-dimensional Nash equilibrium quantities, figure 7, shows the optimal quantity of a (symmetric) home firm as a function of the entire state

space  $s, s^* \in [0, 1]^2$ . The home and foreign clean firms Nash quantities are nearly identical since the goods are distinguished only by clean or dirty type, not by country of origin. With trade the foreign firms have a slightly higher quantity due to their lower marginal cost of production.

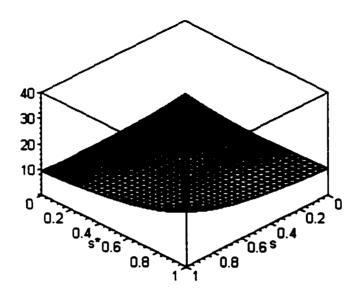


Figure 2.9: Nash equilibrium quantity of a home dirty firm.

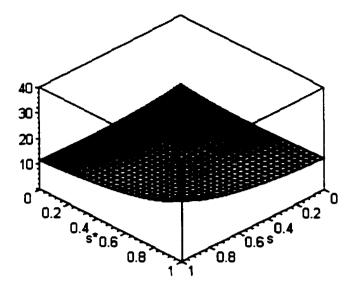


Figure 2.10: Nash equilibrium quantity of a foreign dirty firm.

The shapes of the home and foreign dirty firms are nearly identical to each other, and are almost a mirror image of the clean reaction functions. There is, however, an increase in the Nash equilibrium quantity for the dirty firms near the [0,0] corner of the box, that is absent for the clean firms near the [1,1] corner. Near the [0,0] corner, as all of the firms are dirty, there is an increase in the quantity reaction of a dirty firm to capture some of the premium in the clean demand curve shown by the sharp increase in the price of the clean good as s approaches 0 in figure 3. This effect is less pronounced at the [1,1] corner since there is not the same rapid increase in the price of dirty if all firms converge to the clean type. When the clean and dirty demand curves have equal intercepts,  $\alpha_c = \alpha_d$ ,  $\alpha_c^* = \alpha_d^*$ , both clean and dirty have increases in quantity near the [0,0] and [1,1] corners respectively. There is also a tilt in the Nash quantities of the foreign firms relative to the home firms. This is due to the slight difference in marginal costs. The home

firms face a relatively higher marginal cost at the asymmetric corners [0,1] and [1,0], since  $c_c = 10$ ,  $c_d = 8$ ,  $c_c^* = 9$  and  $c_d^* = 6$ .

# 2.3.2 Foreign Market

The home and foreign markets are separated, yet firms from each country sell in both markets. The demand parameters  $\beta$  and  $\delta$  were assumed to be identical across countries so that the implications of the model would not be driven by differences in relative preferences, but rather by different market sizes, cost parameters, and potentially environmental policy. Due to the fact that  $\beta$  and  $\delta$  are identical in both countries the matrix of coefficients, A, in equations (61) and (62) are identical. The foreign market matrix of reaction functions is

$$AQ^* = X^* \tag{62}$$

where

$$Q^* = \begin{bmatrix} q_c^e \\ q_c^{d*} \\ q_d^e \\ q_d^{d*} \end{bmatrix} \text{ and } X^* = \begin{bmatrix} \alpha_c^* - c_c - t_c^* \\ \alpha_c^* - c_c \\ \alpha_d^* - c_d - t_d^* \\ \alpha_d^* - c_d^* \end{bmatrix}$$

The solution for the foreign market differs only by the vector of constants on the right hand side of (62), and the location at which the good is sold. The quantity vectors in (61) and (62) are ordered so that each row in both vectors is the quantity produced by one firm. For example, the first row in the home market quantity vector, Q, is the quantity produced by a home clean firm and sold in the home market, and the first row in the foreign market quantity vector,  $Q^*$ , is the quantity produced by a home clean firm and sold in the foreign market. Due to this symmetry the Nash equilibrium quantities are qualitatively the same as in

figures 7-10, just shifted down, since the foreign market is assumed to be smaller, as indicated by the demand curve intercepts  $\alpha_c = 200$ ,  $\alpha_d = 190$ ,  $\alpha_c^* = 150$ ,  $\alpha_d^* = 145$ . The foreign market solution is  $Q^* = A^{-1}X^*$ , where the elements of  $A^{-1}$  are given in the Appendix.

# 2.3.3 Free Trade Evolutionary Equilibrium

The evolution of the state variables s and  $s^{\bullet}$ , the proportion of clean firms in the home and foreign countries, respectively, are mutually dependent since firms are selling in both markets. In autarchy the profits of a home clean firm are  $\pi_c = \beta(q_c)^2$ , while under trade the profits of a home clean firm are  $\pi_c = \beta\left[(q_c^d)^2 + (q_c^e)^2\right]$ . Again, the dynamics are driven by profit differentials between the dirty and clean firms. The two profit differential functions,  $\Pi_D$  and  $\Pi_D^{\bullet}$  are

$$\Pi_{D} = \beta \left( \left[ (q_{c}^{d})^{2} + (q_{c}^{e})^{2} \right] - \left[ (q_{d}^{d})^{2} + (q_{d}^{e})^{2} \right] \right)$$

$$\Pi_{D}^{*} = \beta \left( \left[ (q_{c}^{d*})^{2} + (q_{c}^{e*})^{2} \right] - \left[ (q_{d}^{d*})^{2} + (q_{d}^{e*})^{2} \right] \right)$$
(63)

As in autarchy the dynamics are only assumed to be sign preserving<sup>19</sup>, that is C, D > 0.

$$\dot{\Pi}_D = C\Pi_D \tag{64}$$

$$\dot{\Pi}_D^* = D\Pi_D^*$$

<sup>&</sup>lt;sup>18</sup>Firms choose their type in the long run, but it is assumed that factors of production are not mobile. That is, home firms can not locate in the foreign country to take advantage of the lower marginal costs. If home firms were allowed to locate in the foreign country then we would be endogenizing N and  $N^*$ , the number of firms in each country. Here the process is of switching type, not of entry and exit.

 $<sup>^{19}\</sup>mbox{Replicator}$  dynamics for the model under trade would be:

 $<sup>\</sup>dot{s} = \left[\pi_c(s, s^*) - \frac{s\pi_c(s, s^*) + (1-s)\pi_d(s, s^*)}{2}\right]s. \qquad \dot{s}^* = \left[\pi_c^*(s, s^*) - \frac{s^*\pi_c^*(s, s^*) + (1-s^*)\pi_d^*(s, s^*)}{2}\right]s^*$ 

This implies that the growth rate of the population share of clean is increasing in the profit advantage of clean within that population, but the profitability of all four types of firms depends on the state  $(s, s^*)$ .

The two profit differential functions can be plotted in  $(\Pi, s, s^*)$  space. There is no incentive for firms to switch type along the zero profit differential loci. The zero profit differential loci are horizontal slices through the surfaces in figures 11 and 12, that is where  $\Pi_D = \Pi_D^* = 0$ .

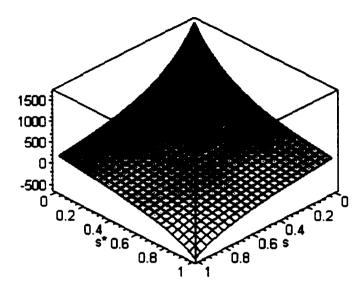


Figure 2.11: Home profit differential.

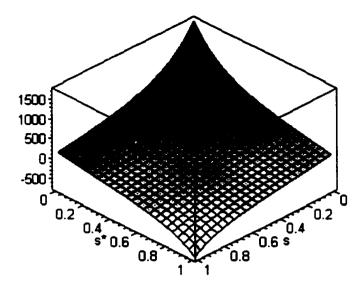


Figure 2.12: Foreign profit differential.

The zero-profit differential loci are essentially linear in the relevant ranges despite the fact that the profits for each type of firm are a ratio of third and fourth degree polynomials in the state variables  $(s, s^*)$ . They have nearly the same slope since the coefficient matrices in the reaction functions, equations (61) and (62), are identical.

Each zero profit differential locus is the sum of four different squared quantity terms, given in equation (63). Each of these quantities is the corresponding row of the solutions  $Q = A^{-1}X$  and  $Q^* = A^{-1}X^*$ , for  $\Pi_D$  and  $\Pi_D^*$  respectively. However complicated the loci are they may be plotted as a phase diagram. Since the zero-profit differential loci do not cross in the unit box the EE is complete specialization in one country and incomplete specialization in the other country.

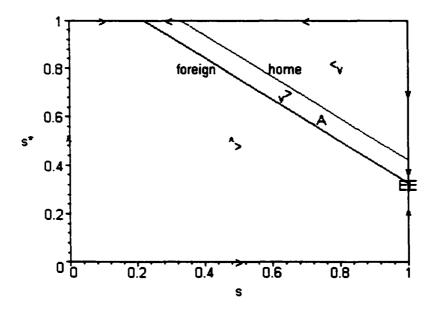


Figure 2.13: Free trade evolutionary equilibrium,  $(s = 1, s^* = 0.325)$ .

The loci have nearly identical, but slightly different slopes, and do not cross in the relevant range. The reason for this is the degree of symmetry between the two countries. Specifically, this result is obtained since  $\beta$  and  $\delta$  are assumed to be the same across countries and the good is assumed to be distinguished only by type, not by the country of origin. This leaves us with only three quadrants in the phase diagram. This is a key difference between this model and Friedman and Fung (1996). In the Friedman and Fung model the zero profit loci crossed in the unit box primarily due to the presence of a state dependent cost externality, an effect which is absent in this model.

The foreign locus lies below the home locus since there is a relatively larger cost advantage to being the dirty type in the foreign country. The initial condition from autarchy lies in the zone between the two loci, point A in figure 13. The evolutionary equilibrium of this system is at the intersection of the foreign loci and

the right edge of the box, that is  $s = 1, s^* = 0.325$ . The pattern of specialization that emerges from free-trade in goods is complete specialization of the clean good in the home country and incomplete specialization in the foreign country. This result could be interpreted as an explanation for the pollution-haven hypothesis in which the dirty firms locate in the country with the lower marginal cost, potentially due to lower pollution standards. However, the evolution need not be driven by differences in pollution standards across countries, but rather is a natural evolution of the system under profit-maximizing behavior by firms. The differences in country size and marginal costs of production, while holding relative preferences constant, are sufficient to obtain this result in this model<sup>20</sup>.

Intra-industry trade in both types of the good is a phenomenon that occurs only along the adjustment path to the EE. In this example the foreign country has an absolute advantage in both goods and a comparative advantage in the dirty good, since  $c_c = 10$ ,  $c_d = 8$ ,  $c_c^* = 9$ , and  $c_d^* = 6$ . A perfect competition Ricardian model would predict that the home country would produce the clean good, the foreign country the dirty good, with relative world demands determining complete or incomplete specialization. This imperfectly competitive trade model has the same prediction at the EE, however allows for the possibility of intra-industry trade in both types along the adjustment path to the EE.

The autarchy equilibrium was characterized as a non-linear version of a single population Hawk-Dove game. The trade equilibrium analogue is a two population non-linear Hawk-Dove game. A standard result from evolutionary game theory

<sup>&</sup>lt;sup>20</sup>This result is not invariant to the parameters chosen. Parameters that do not satisfy the conditions in Appendix 2 may lead to different patterns of specialization.

is that the single population game has a stable interior equilibrium with a positive proportion of both hawks and doves. However, Hawk-Dove for more than one population has an unstable interior equilibrium (Weibull 1995). The EE is generally at one of the asymmetric corners, meaning one population is completely hawk and the other completely dove. In the setting of this model this would be interpreted as one country being completely clean and the other completely dirty. The reason that this result is not obtained in this model in general is that a firm of one type competes with firms from both populations, rather than a firm from one country only competing against a "mixed strategy" firm from the other country. The mixed strategy firm represents the probability of being matched against either a hawk or a dove from the other population, and therefore represents the other populations state. The standard result is obtained by assuming members of one population are matched only against members of the other population, an effect that destabilizes the single population interior equilibrium. As the prevalence of hawks in one population increases there is a payoff advantage to being a dove in the other population, leading to complete polarization of the populations.

For this example, the trade equilibrium is characterized by complete specialization of the clean good in the home country. This means that there is no longer an interior solution at the evolutionary equilibrium. For the short-run quantity decision by a firm, this is equivalent to the Kuhn-Tucker constraint binding for the home dirty production. That is,  $q_d^d = 0$ , in the home market, and  $q_d^e = 0$ , in the foreign market. The solution for the goods sold in the home market is found by deleting the third row and third column of the reaction function coefficient matrix

in equation (61). The Nash equilibrium is the solution to

$$\begin{bmatrix} \beta(sN+1) & \beta s^*N^* & \delta(1-s^*)N^* \\ \beta sN & \beta\left(s^*N^*+1\right) & \delta(1-s^*)N^* \\ \delta sN & \delta s^*N^* & \beta\left[\left(1-s^*\right)N^*+1\right] \end{bmatrix} \begin{bmatrix} q_c^d \\ q_c^{e*} \\ q_d^{e*} \end{bmatrix} = \begin{bmatrix} \alpha_c - c_c \\ \alpha_c - c_c^* - t_c \\ \alpha_d - c_d^* - t_d \end{bmatrix}$$
(65)

Inverting the coefficient matrix and evaluating at the s=1 edge of the unit box vields

$$\begin{bmatrix} \frac{\beta^{2}(s^{*}N^{*}+1[(1-s^{*})N^{*}+1]-\delta^{2}s^{*}N^{*}(1-s^{*})N^{*}]}{\Delta} & \frac{-\beta^{2}s^{*}N^{*}[(1-s^{*})N^{*}+1]+\delta^{2}s^{*}N^{*}(1-s^{*})N^{*}}{\Delta} & \frac{-\beta\delta(1-s^{*})N^{*}}{\Delta} \\ \frac{-\beta^{2}N[(1-s^{*})N^{*}+1]+\delta^{2}N(1-s^{*})N^{*}}{\Delta} & \frac{\beta^{2}(N+1)[(1-s^{*})N^{*}+1]-\delta^{2}N(1-s^{*})N^{*}}{\Delta} & \frac{-\beta\delta(1-s^{*})N^{*}}{\Delta} \\ \frac{-\beta\delta s^{*}N^{*}}{\Delta} & \frac{-\beta\delta s^{*}N^{*}}{\Delta} & \frac{\beta^{2}(s^{*}N^{*}+N+1)}{\Delta} \end{bmatrix}$$
(66)

Where  $\Delta = \beta^3[N + s^*N^* + 1][(1 - s^*)N^* + 1] - \beta\delta^2[N + s^*N^*](1 - s^*)N^*$ , which is exactly the same as  $\Delta$  given after equation (61) evaluated at s = 1. The inverse of (65) is exactly the same as the inverse of the coefficient matrix when all four quantities are sold, given in (61), evaluated at s = 1. This result is not immediately obvious, but is important to ensure that there are no discontinuities along the zero profit loci as the model collapses from four quantities to three. This leads us to the second proposition, familiar from perfect competition trade models.

Proposition 2: Any Laissez-faire evolutionary equilibrium must result in at least one country being completely specialized.

Proof follows from the lack of a stable interior evolutionary equilibrium.

The free-trade evolutionary equilibrium, for the numerical example, is summarized below<sup>21</sup>. The first table is for goods sold in the home market the second for the foreign market.

<sup>&</sup>lt;sup>21</sup>A general parameter restriction for when the Kuhn-Tucker non-negativity constraints are slack is given in appendix 2.

Table 2.2 Free Trade EE					
Home Market					
	Home Clean	Foreign Clean	Foreign Dirty		
Firm Quantity	$q_c^d = 12.90$	$q_c^{e*}=13.90$	$q_d^{e*} = 12.97$		
Market Quantity	$Q_c = 121.94 \ Q_d = 61.29$				
Market Price	$P_c = 22.90  P_d = 18.97$				
Foreign Market					
	Home Clean	Foreign Clean	Foreign Dirty		
Firm Quantity	$q_c^e = 8.84$	$q_c^{d*} = 9.84$	$q_d^{d\bullet} = 11.04$		
Market Quantity	$Q_c^* = 84.21 \ Q_d^* = 52.17$				
Market Price	$P_c^* = 18.84 \ P_d^* = 17.04$				
Profits		$\pi_c^* = 290.03$	$\pi_d^* = 290.03$		
Trade Balance	TB = -721.71	$TB^* = 721.71$			
EE	s = 1	$s^{\bullet} = 0.325$			

The free-trade evolutionary equilibrium (EE) is characterized by complete specialization of the clean good in home and incomplete specialization in foreign,  $s = 1, s^* = 0.325$ . In home the amount of the clean good sold is less than under autarchy and the amount of the dirty good is greater. In foreign the opposite is true. Overall, free-trade has increased the scale of production of both goods. Furthermore, the composition of output has become dirtier. Under autarchy the ratio of global output of clean to dirty was  $\frac{Q_c + Q_c^*}{Q_d + Q_d^*} = 2.01$ . Free-trade has lowered the ratio to 1.82, reflecting a dirtier composition of output. The movement from autarchy to free-trade has led to an increase in global production of the dirty good, all of which is produced in foreign. These results are similar to the scale and composition effects in Copeland and Taylor (1994). Free trade, in an evolutionary environment, has shifted all the pollution to the foreign country, and increased global pollution, without the use of policy instruments. When the home country is completely specialized in the clean good, the trade balance, from the home

countries perspective, is:  $TB = P_c^* q_c^e s N - P_c s^* N^* q_c^{e*} - P_d (1 - s^*) N^* q_d^{d*}$ . At the free trade EE,  $TB = -721.71 = -TB^*$ .

The profits for a home clean firm are  $\pi_c = 224.55$ , which is less than half of autarchy profits, even though they have sales in both the domestic and foreign markets. The profits for both types of foreign firms are lower,  $\pi_c^* = \pi_d^* = 290.03$ , relative to 334.92 under autarchy. Free-trade has decreased the profits of all types of firms, due to increased competition, relative to autarchy. Similarly, prices have fallen substantially in both markets due to the increased number of firms competing in each market. Under autarchy, the price differentials between the clean and dirty goods in each market were exactly equal to the country specific marginal cost differentials. Under trade, this is no longer true. In the home market  $P_c = 22.90$ , and  $P_d = 18.97$ , which is greater than the autarchy price premium for the clean good. In the foreign market  $P_c^{\bullet} = 18.84$ , and  $P_d^{\bullet} = 17.04$ , which is smaller than the autarchy price premium for the clean good. Free-trade has led to an increase in the terms of trade for the clean good in the home country and a decrease in the terms of trade for the clean good in the foreign country. In terms of the good produced relatively intensively domestically, there is an improvement in the terms of trade for both countries. This result can be attributed to the assumption of imperfect competition. In a perfectly competitive Ricardian trade model, free-trade only leads to an improvement in the terms of trade of the good produced relatively intensively domestically if the country is completely specialized.

#### 2.3.4 Trade Policy

Since the pollution is assumed to be point-source, there is a negative production externality associated with having production of the dirty good within a nations

borders. The negative production externality raises the possibility that free-trade is not optimal for a country producing the dirty good. In this case a welfare improving policy could be a "lack-of-pollution" content tariff, that is, a tariff imposed on clean imports. The effect of this policy, for a tariff sufficient to reverse the comparative advantage, is to shift the zero profit loci, reversing the pattern of specialization. Since solutions along edges of the unit box,  $(s, s^*) \in [0, 1]^2$ , are a special case of solutions in the interior, we can analyze the case beginning at the free-trade EE, moving to the interior of the  $s, s^*$  unit box, and ultimately reaching the new EE along a different edge<sup>22</sup>.

Initially we analyze the effect of the foreign country imposing a tariff on clean imports from the home country. A foreign tariff on the clean good makes  $t_c^* > 0$  in equation (62), and leaves the home market solution, equation (61), unchanged. However, the home market is indirectly affected by the negative impact of the tariff on a home clean firm's profits. As clean becomes relatively more profitable in the foreign country the zero-profit locus for the foreign firms shifts up. The zero-profit locus for the home firms shifts down, as clean becomes relatively less profitable in the foreign market. The X vector in equation (61) shows that an ad valorem tariff is equivalent to an increase in marginal cost, in effect reversing the comparative advantage. In the long-run the state variables s and  $s^*$  adjust from the free-trade EE to the  $t_c^* = 4$  EE, ultimately reversing the pattern of specialization, while intra-industry trade in both types of the good is contained to the adjustment path.

<sup>&</sup>lt;sup>22</sup>See the discussion after equation (66).

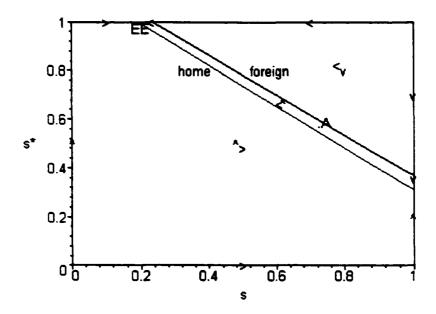


Figure 2.14:  $t_c^* = 4$  evolutionary equilibrium,  $(s = 0.19, s^* = 1)$ .

A lack-of-pollution content tariff results in pollution shifting from foreign to home. This type of tariff can be set below the level of a standard Pigouvian tax on domestic pollution without exacerbating the quantity distortion due to imperfect competition. A lack-of-pollution content tariff also results in tariff revenue and a rent-capture effect, both of which will be discussed in greater detail in the next section on the distributional effects of trade policy. The lack-of-pollution content tariff  $t_c^* = 4$  EE results in the following values for the numerical example.

Table 2.3 $t_c^* = 4$ EE					
Home Market					
	Home Clean	Foreign Clean	Home Dirty		
Firm Quantity	$q_c^d = 13.51$	$q_c^{e*} = 14.51$	$q_d^d = 11.16$		
Market Quantity	$Q_c = 119.64 \ Q_d = 63.17$				
Market Price	$P_c = 23.51$ $P_d = 19.16$				
Foreign Market					
	Home Clean	Foreign Clean	Home Dirty		
Firm Quantity	$q_c^e = 5.64$	$q_c^{d*} = 10.64$	$q_d^e = 9.48$		
Market Quantity	$Q_c^* = 82.05 \ Q_d^* = 53.68$				
Market Price	$P_c^* = 19.64 \ P_d^* = 17.48$				
Profits	$\pi_c = 214.34$	$\pi_d = 214.34$	$\pi_c^* = 323.78$		
Trade Balance	TB = -1301.13	$TB^* = 1301.13$			
EE	s = 0.191	$s^* = 1$			

The pattern of specialization has been reversed. The  $t_c^*=4$  EE results in complete specialization of the clean good in the foreign country and incomplete specialization in the home country. Foreign's imposition of a tariff on the clean good has resulted in foreign importing all of its dirty good consumption from home. The  $t_c^*=4$  EE has reduced global output of clean and increased output of dirty, so the ratio of global output of clean to dirty is  $\frac{Q_c+Q_c^*}{Q_d+Q_d^*}=1.73$ , lower than both free-trade and autarchy.

Profits for the home firms are lower, and profits for the foreign firms are higher, relative to the free-trade EE, as foreign's tariff captures rents from the home country. The prices of both goods in both markets have risen due to the single tariff. The value of the trade flows at the  $t_c^* = 4$  EE, in which home is incompletely specialized and foreign is completely specialized in the clean good, is:  $TB = (P_c^* - t_c^*)sNq_c^e + P_d^*(1-s)Nq_d^e - P_cN^*q_c^{e*}$ . At the  $t_c^* = 4$  EE the value of the trade balance is  $TB = -1301.13 = -TB^*$ . The foreign tariff revenue is

$$TR = t_c^* s N q_c^e = 30.2.$$

Relative to the free-trade EE, by imposing a lack-of-pollution content tariff foreign has increased firm's profits, raised tariff revenue, eliminated domestic pollution, and has improved the trade balance. Domestic prices have risen, so national welfare will be higher in foreign if the reduction in consumers surplus is not large enough to dominate welfare improvements due to pollution-shifting, rent-capture and tariff revenue.

In home the profitability of firms has decreased, prices have risen, the trade balance has worsened, and Home bears the additional cost of domestic pollution. Therefore, home welfare is unambiguously lower at the  $t_c^* = 4$  EE than at the free trade EE. Indeed, home has an incentive to retaliate and impose an import tariff on the clean good as well. The Nash equilibrium of the tariff game between the governments is when either  $\frac{\partial NW}{\partial t_c} \leq 0$  or  $\frac{\partial NW^*}{\partial t_c^*} \leq 0$ . In general, the two countries may impose tariffs on both the dirty and clean goods, until one country no longer finds it optimal to do so, an issue left for future work. We now turn to the welfare analysis.

# 2.4 Welfare Analysis

### 2.4.1 Autarchy

The national welfare (NW) of a country is assumed to be the sum of firm profits and consumer surplus minus the negative production externality associated with production of the dirty good within the nations borders. Since the pollution is assumed to be point-source there are no international spillovers from foreign production of the dirty type. Although we have previously identified the short-run

and long-run equilibria under autarchy and trade, the welfare measures are presented for all possible interior values of the state variables  $s, s^*$  and then evaluated at each of the equilibria.

The firms in this model operate in a constant marginal cost setting so industry profits are simply the price minus marginal cost times the number of units sold. Consumers are assumed to have linear demand curves and so the consumer surplus is the triangle formed by the demand curve intercept, equilibrium price and the quantity. The per-unit negative production externality, E, times the dirty output is the aggregate impact of the pollution. Again, for simplicity, the marginal damage of a unit of pollution is assumed to be constant. For home under autarchy we have

$$NW = \Pi + CS - Pollution$$

$$NW = [(P_c - c_c)sNq_c + (P_d - c_d)(1 - s)Nq_d] + \frac{1}{2}[(\alpha_c - P_c)sNq_c + (\alpha_d - P_d)(1 - s)Nq_d] - E(1 - s)Nq_d$$
(67)

The first term is the aggregate profits of both types of firms, the second term is consumers surplus, and the third term is the pollution damage from dirty production. Given the simplifications of the model the changes in national welfare are more reliable than the levels, so that the measures of national welfare presented should be interpreted in an ordinal, rather than a cardinal sense. Substituting in the demand curves (67) simplifies to

$$NW = sNq_{c}\left\{\alpha_{c} - c_{c} + \frac{1}{2}[-\beta sNq_{c} - \delta(1-s)Nq_{d}] + (68)\right\}$$

$$(1-s)Nq_{d}\left\{\alpha_{d} - c_{d} - E + \frac{1}{2}[-\beta(1-s)Nq_{d} - \delta sNq_{c}]\right\}$$

National welfare can be decomposed into its parts to identify the distributional effects of different EE.

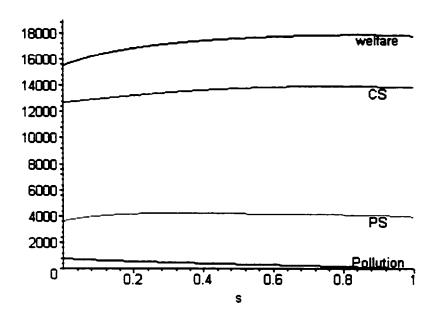


Figure 2.15: Home autarchy welfare by component.

The consumers' surplus has a maximum in the neighborhood of the home autarchy EE, s=0.735. The intuition behind this result is that at the EE all profitable deviations of type have been exploited, so the EE most closely approaches a competitive equilibrium. This means that the combined prices are near a minimum at the EE, combined quantity is near a maximum, making consumers surplus near a maximum. The producers surplus at the EE is below the maximum. Producers surplus is at its highest when the dirty good is more prevalent, due to the lower marginal cost of producing the dirty good and the fact that the lower the clean quantity, the greater the markup that a clean firm can charge for a given quantity of dirty. Home autarchy welfare is plotted for a negative production externality of 5 per unit of dirty output. National welfare is near the maximum level at the EE. Under autarchy it is welfare reducing to have complete specialization of either good due to losses of both consumers and producers surplus.

The foreign country is qualitatively the same as the home country, but the consumers and producers surplus graphs are shifted slightly to the left of the home diagrams, due to the proportionally lower price premium and higher relative marginal cost of clean in foreign. With the same negative production externality of 5 per unit, foreign national welfare is similar to the home diagram.

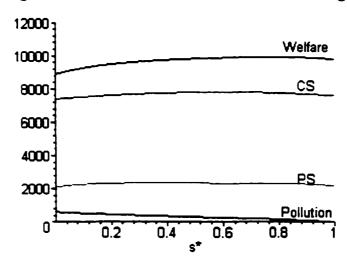


Figure 2.16: Foreign autarchy welfare by component.

Foreign national welfare is below the maximum at the autarchy EE, so the foreign country would find it welfare improving to subsidize the domestic clean good, or tax the dirty good. For the numerical example the autarchy EE levels of national welfare are NW = 17,832 and  $NW^* = 9,879$ , making aggregate welfare (AW) = 27,711.

#### 2.4.2 Trade

With trade the home firms profits depend on domestic and foreign sales. Consumers surplus depends on consumption of domestically produced and imported goods. Since the pollution externality is assumed to have no international spillovers

the damage from pollution depends only on domestic production of the dirty good.

Under free-trade the national welfare of the home country is:

$$NW = \Pi + CS - Pollution$$

$$NW = \begin{bmatrix} (P_c - c_c)sNq_c^d + (P_d - c_d)(1 - s)Nq_d^d \\ + (P_c^* - c_c)sNq_c^e + (P_d^* - c_d)(1 - s)Nq_d^e \end{bmatrix}$$

$$+ \frac{1}{2} \begin{bmatrix} (\alpha_c - P_c)(sNq_c^d + s^*N^*q_c^{e*}) \\ + (\alpha_d - P_d)(q_d^d(1 - s)N + q_d^{e*}(1 - s^*)N^*) \end{bmatrix}$$

$$-E[(1 - s)Nq_d^d + (1 - s)Nq_d^e]$$
(69)

The home consumers surplus is unambiguously higher than the consumers surplus in autarchy due to decreased prices and increased quantities in the domestic market. The home consumers surplus is lowest at complete global specialization in the dirty good, the s = 0,  $s^* = 0$  corner.

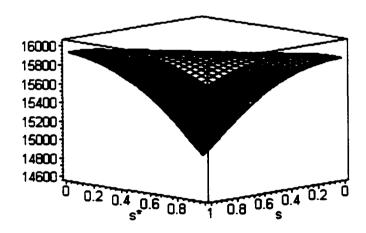


Figure 2.17: Home consumers surplus.

The producers surplus is unambiguously lower than under autarchy. Domestic firms are hurt by competition with imports that are produced at a lower marginal

cost, as would be expected. The home producers surplus is highest when foreign is completely specialized in the dirty good, and home is incompletely specialized. At the free-trade EE ( $s = 1, s^* = 0.325$ ), home producers surplus would be increased by either a reduction in the foreign proportion of clean, or by a reduction in the domestic proportion of clean.

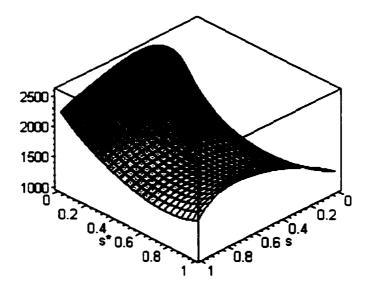


Figure 2.18: Home producers surplus.

The home countries welfare is lower than the autarchy EE welfare level unless the home country is completely specialized in the clean good and the foreign country is completely specialized in the dirty good. The exception to this is near the  $(s=1,s^*=0)$  corner. The autarchy EE are:  $s=0.735, s^*=0.578$ , so when going from autarchy to free-trade the welfare in home initially falls and rises along the adjustment path to the free-trade EE  $(s=1,s^*=0.325)$ , which is very close to the autarchy welfare level. The overall effect of the free-trade agreement in home is for consumers surplus to rise, producers surplus to fall, pollution to drop to zero, and the overall level of welfare to fall.

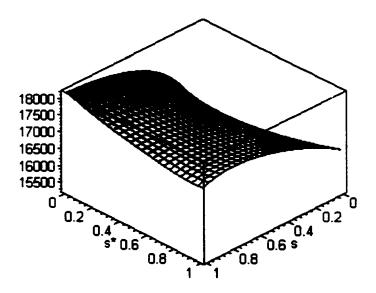


Figure 2.19: Home welfare.

Foreign consumers surplus is also unambiguously higher than under autarchy. It has the exact same shape as the home consumers surplus in Figure 17. At the free-trade EE,  $(s=1,s^*=0325)$ , foreign producers surplus is lower relative to autarchy. If the pattern of specialization is reversed, as in the  $t_c^*=4$  EE,  $(s=0.191,s^*=1)$ , then foreign producers surplus may approach the autarchy level.

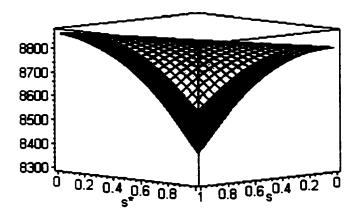


Figure 2.20: Foreign consumers surplus.

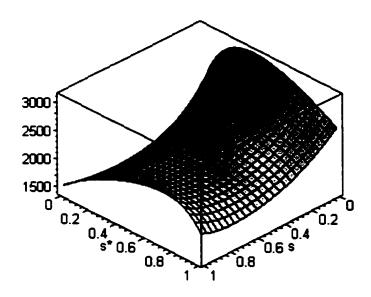


Figure 2.21: Foreign producers surplus.

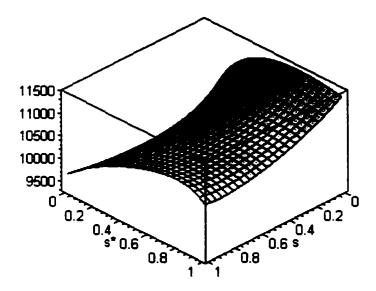


Figure 2.22: Foreign welfare.

Foreign welfare is unambiguously higher than under autarchy except for the  $(s=1,\ s^*=0)$  corner. Also, it is welfare improving to reverse the free-trade pattern of specialization. The welfare levels in Figures 19 and 22 are not valid once a tariff is imposed. The reason for this is that tariff revenue is not included, as well as the endogenous quantity responses from a single tariff on all seven other quantities. The foreign countries welfare is higher than the autarchy EE welfare level unless home is completely specialized in the clean good and foreign is completely specialized in the dirty good. The autarchy EE are s=0.735, and  $s^*=0.578$ , so when going from autarchy to free-trade the welfare in foreign initially jumps and continues to rise along the adjustment path to the free-trade EE  $(s=1,s^*=0.325)$ . Figure 23 shows the aggregate welfare which is  $AW \equiv NW + NW^*$ .

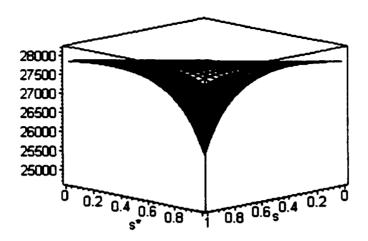


Figure 2.23: Aggregate welfare.

# 2.4.3 Evolutionary Equilibrium Welfare Levels

Evaluating the autarchy, free-trade, and  $t_c^{\star}=4$  EE yields the following welfare levels:

Table 2.4 Welfare Levels				
Autarchy	Home	Foreign		
CS	13,919	7,805		
PS	4, 138	2,344		
Pollution	-225	-270		
NW	17,832	9,879		
	s = 0.735	$s^* = 0.578$		
Free Trade				
CS	16,038	8,861		
PS	1,712	2,030		
Pollution	0	-567		
NW	17,750	10, 324		
	s = 1	$s^* = 0.325$		
$t_c^* = 4 \text{ EE}$				
CS	15,954	8,770		
PS	1,500	2, 266		
Pollution	-584	0		
Tariff Revenue	0	30		
NW	16,870	11,067		
	s = 0.191	$s^* = 1$		

The movement from autarchy to free-trade has reduced producers surplus and increased consumers surplus in both countries. Free-trade has increased global pollution due to both scale and composition effects, however all of the pollution is located in the foreign country. Home welfare is lower than in autarchy due to the reduction in producers surplus as a result of the increased competitiveness under trade. Even with the benefits from shifting pollution to the foreign country Home's welfare is *lower* under free trade than autarchy, a result in stark contrast to perfect competition models. This result is driven by the rent-shifting from producers to consumers, and from home to foreign, which dominates the pollution-shifting from home to foreign. Foreign welfare is higher than autarchy as the gains in consumers surplus dominate the pollution damage and the reduction in producers surplus. Free-trade aggregate welfare is 28,074 which is higher than autarchy.

The effect of the single tariff, relative to free-trade, has been to reduce consumers surplus in both countries, but is still greater than autarchy consumers surplus. Home producers surplus has fallen by a smaller amount than the rise in foreign producers surplus. The tariff has resulted in rent-shifting from home to foreign, and from consumers to producers, as well as transferring all of the pollution to the home country. The tariff revenue is less than the reduction in foreign consumers surplus. However, if it is consumers who benefit from the reduction in pollution, then consumers are better off under the tariff. The lack-of-pollution content tariff has appropriated rents from home producers, generated tariff revenue, and shifted pollution to the home country by an amount sufficient to compensate consumers. A single instrument achieved all of these benefits with no domestic group being made worse off. From a political economy perspective it would be difficult to maintain a free-trade agreement in such a situation. The  $t_c^*=4~\mathrm{EE}$ aggregate welfare is 27,937, which is higher than autarchy, but below the freetrade EE. Thus, in situations characterized by imperfect competition and negative production externalities that are contained within the nations borders we may find some interesting results; free-trade may be inferior to autarchy for certain countries, and it is feasible that each interest groups in a country may have an incentive to lobby for a lack-of-pollution content tariff.

Standard political economy models stress the importance of tariffs as different interest groups attempt to capture rents from each other. Generally, a tariff benefits domestic producers and harms domestic consumers. The positive optimal tariff for a large country usually rests on the assumption that the tariff revenue is redistributed to consumers to compensate for higher domestic prices, an action

that may not materialize. This example has shown a situation where the tariff revenue need not be redistributed to make consumers better off, given the reduction in domestic pollution. From a political economy perspective, it would appear that a lack-of-pollution content tariff may face less resistance than a standard non-zero optimal tariff that relies on credible redistribution by the government.

# 2.5 Conclusion

In situations of imperfect competition and a negative production externality, the theory of the second best asserts that the optimal policy is a combination of a pollution tax and a production subsidy. This paper has shown how a single policy instrument, a lack-of-pollution content tariff, can simultaneously achieve goals appealing to all of the domestic interest groups. With imperfect competition and a negative production externality a free-trade agreement may be confronted with two fatal flaws: a country may find autarchy superior and even when a country has higher welfare with free-trade, they may have an incentive to impose an import tariff on clean goods.

The results of this model are not to be interpreted as a general indictment of free-trade, but rather shows a plausible example where standard policy prescriptions are misguided, and shows how groups previously considered disparate in their interests (environmentalists and union workers, for example) may share common policy goals. The numerical example arose since it is not possible to get a closed form analytic solution to this model. Previous work with Cournot models of this type have dealt with this problem by using simulations to gain insight into which conclusions are obtained for different parameter sets. For this reason the

results of simulations are always qualified. This paper has derived general parameter restrictions for where interior solutions exist in a short-run Cournot setting for both trade and autarchy. A parameter restriction is also derived for autarchy that shows what range of parameters can sustain an interior EE under autarchy. A parameter set was then chosen that satisfied the criteria given in the restrictions, as well as relating to a particular scenario. This allowed the construction of 3-dimensional graphs to visually examine some comparative statics, and implied incentives, for those particular parameters. The parameter specific results of this paper are therefore also qualified.

The case analyzed in this paper is that of the environmentally clean good selling for a price premium, but with a higher marginal cost. The Home market was assumed to be larger, and the Foreign market was assumed to face a relatively lower cost of producing the dirty good. The pollution was assumed to be point-source, specifically of a form that has no international spillovers. Pollution was assumed to impose a negative externality of 5 per unit of dirty output. The autarchy evolutionary equilibria (EE) results are obtained for two asymmetric countries in which both types of the good are produced in both countries. This is analogous to two separate single population Hawk-Dove games, each with a stable unique interior EE.

The autarchy equilibria are the initial state of a free-trade agreement. Initially, the story is reminiscent of a reciprocal dumping scenario, such as Brander and Krugman (1983), in which both countries export both goods to the other country. However, the initial state from autarchy is unstable in a two population evolutionary setting. As firms respond to the dynamic profit differential one country

becomes completely specialized, while the other country is incompletely specialized. In the example, the overall effect of the free-trade agreement, in Home, is for consumers surplus to rise, producers surplus to fall, pollution to drop to zero, and the overall level of welfare to fall. All of the dirty good production takes place in Foreign. This pollution-haven result is not driven by policy, but rather is a result of profit maximizing behavior by firms in asymmetric countries in a free trade setting.

For Foreign, the overall effect of the free-trade agreement is for consumers surplus to rise, producers surplus to rise, pollution to increase, and the overall level of welfare to increase. Although the foreign country is better off than under autarchy, there exists the incentive for the foreign country to impose an import tariff on the clean good to increase national welfare. For the chosen parameters a clean import tariff that is less than the marginal damage of a unit of pollution can reverse the pattern of specialization, so Foreign is completely specialized in the clean good and Home is incompletely specialized. The home country has a lower welfare level at this new equilibrium and therefore has an incentive to respond with a similar tariff. The Nash equilibrium of this tariff game is when neither country finds the welfare gain of having the clean good produced in their country is worth the welfare loss from further trade restrictions. A surprising result is that the  $t_c^* = 4$  EE aggregate welfare is higher than both the autarchy and the free-trade EE. This is because the loss of consumers surplus from increased prices is more than offset by the gains in producers surplus in the foreign country, which is not a zero-sum due to imperfect competition, and tariff revenue. The main technical contributions of this paper are the inverses of the reaction function matrices, the parameter restrictions in the Appendix, and showing that there are no discontinuities along the zero profit loci as they intersect an edge of the unit box. Further work could investigate the different implications of equivalent tariff and subsidy policies and derive the Nash equilibrium of the tariff game between the governments.

# Chapter 3

# 3 A Public Goods Experiment by Asymmetric Agents

## 3.1 Introduction

The concept of fairness is crucial in understanding pure public goods provision. How individuals react to different levels of wealth, as well as different shares of the benefit derived from public good provision provides insight into how the free-rider problem may be overcome. This experiment investigates if differences in endowments and benefit shares leads to levels of public good provision that are closer to the Nash equilibrium prediction or Pareto optimal level. Will high endowment subjects contribute more to the public good, or will this happen only if they also receive a high share of the benefits of the public good? Will low benefit share subjects free ride on the contributions of high benefit share subjects, supporting the Nash prediction, or will they over-contribute? The Nash and Pareto levels are identical across treatments, so differences in provision may be attributed to perceptions of fairness, since provision is voluntary.

In the Voluntary Contribution Mechanism (VCM) each individual makes a voluntary division between a public and a private good. This mechanism does not suffer from the revelation problems inherent in simple message and tax implementation of public goods. In general, all VCM experiments can be nested in the following payoff function.

$$\pi_i = P(e_i, q_i) + G(\sum_{j=1}^{N} q_j)$$
 (70)

In this specification the function  $P(\cdot)$  is the return from the private good, which is a function of the endowment,  $e_i$ , and the contribution,  $q_i$ , to the public good for player i. In the majority of experiments<sup>23</sup>, such as Marwell and Ames (1979), Isaac. Walker and Thomas (1984), Isaac, McCue and Plott (1985) and Sefton and Steinberg (1996), the return from the private good is constant for all units, while the return from the public good, G, depends on the contributions of all the players. The marginal rate of substitution of the private for the public good, generally called the marginal per capita return (MPCR), is defined as:  $MPCR \equiv -\frac{\left(\frac{\partial G}{\partial q_i}\right)}{\left(\frac{\partial P}{\partial q_i}\right)}$ . In addition to the returns to the public and private goods, defining the MPCR, all VCM experiments are described by a vector of parameters consisting of the total and the distribution of endowments,  $E \equiv \sum_{i=1}^{N} e_i$ , the group size, N, and the distribution of the shares of the benefit of the public good.

The simplest specification is linear benefits from the public good, where the values of the marginal contributions to both the private and public goods are constant and symmetric across players. For this specification equation (70) becomes:

$$\pi_i = p(e_i - q_i) + b(\sum_{j=1}^{N} q_j) \left[ \frac{1}{N} \right]$$
 (71)

Equation (71) implies the private value of each unit of the endowment that is contributed to the public good is -p, the private value to player i of her own contribution to the public good is  $\frac{b}{N}$ , and the value to the entire group is b. For this specification the  $MPCR = \frac{b}{pN}$ , since player i's share of the public good is equal to  $\frac{1}{N}$  for i = [1, ..., N].

<sup>&</sup>lt;sup>23</sup>Exceptions with declining marginal benefit of the private good are Kesar (1996), Palfrey and Prisbrev (1997)

Early VCM experiments were designed so that MPCR is a positive fraction, implying the dominant strategy in a 1-shot game is to contribute zero to the public good, while the Pareto optimality dictates each player contribute their entire endowment. From these early experiments a set of stylized facts emerged (see Ledyard (1995) for a survey). First, subjects consistently contribute positive amounts to the public good in violation of the dominant strategy. This contribution usually varies between 20% and 70% of the Pareto optimal level (Ledyard 1995). Isaac, Walker and Thomas (1984) show that MPCR is a critical design variable even when the value of MPCR does not change the Nash and Pareto optimal equilibria. They tested MPCR = 0.3 and MPCR = 0.75, both of which imply a unique dominant strategy equilibrium of zero-contribution, with full contribution being Pareto optimal. Subjects were informed that there would be 10 decision periods and that the members of their group would remain the same for all 10 rounds. They found with the high MPCR that mean contributions were initially about 60% of optimal and decayed to less than 10% by the tenth period. However, for the low MPCR of 0.3 the mean contribution was about 40% of optimal for the first period and steadily decayed to approximately 20% of optimal by the tenth period. An inherent limitation of this type of design is that it is only possible to over-contribute relative to the Nash equilibrium quantity and to under-contribute relative to the Pareto optimal quantity, and hence are commonly referred to as corner equilibria experiments.

Andreoni (1993), Ledyard (1995), Kesar (1996) and others have questioned the robustness of this systematic over-contribution in environments where both the Nash and Pareto optimal levels imply partial contribution of the subjects' endowment. Incorporating either declining marginal benefits to the private good or declining marginal benefits to the public good (or both) generates an interior equilibrium. Kesar (1996), Sefton and Steinberg (1996) and Andreoni (1993) all employ declining marginal benefits to the private good,  $\frac{\partial P(e_i,q_i)}{\partial q_i} < 0$  in equation (70). When the function  $P(\cdot)$  is monotonically decreasing there is a unique dominant strategy Nash equilibrium quantity. An advantage to this specification is that the individual players incentives to contribute to the public good are independent of the levels of public good provision chosen by the other players. The dominant strategy in Kesar (1996) was for players to contribute 7 out of their endowment of 20 tokens to the public good. She observed an average over-contribution rate of 25% relative to the dominant strategy solution across all observations in an experiment where the groups remained fixed. Over-contribution rates were initially around 50% and declined non-monotonically to about 20%, a typical pattern of decay. This contribution level and pattern of time decay are similar to the earlier corner equilibria experiments mentioned above.

Palfrey and Prisbrey (1997) conducted an experiment where the subjects' return on the private good,  $P(\cdot)$ , was a random variable that changed each period, to investigate players contributions to the public good as a function of their cost. The marginal value of the public good was held constant for all periods and was identical for all players. They found a strong negative relationship between contributions and the cost of contributions, as well as concluding that time decay and experience also play significant roles in reducing subjects' over-contributions.

The alternative design method that yields an interior solution is to incorporate declining marginal benefit to the public good,  $\frac{\partial G(e_i,q_i)}{\partial q_i} < 0$ , such as Isaac, McCue

and Plott (1985), Sefton and Steinberg (1996) and Laury, Walker and Williams (1999). Issac, McCue and Plotts' experiment was characterized by declining value of the marginal contribution to the public good in an environment with asymmetric agents. Group size was 10 individuals, 5 of which received a high marginal payoff and the remaining 5 received a low marginal payoff to the public good. The return to the private good was constant. They found that there was significant overcontribution, and that this over-contribution was greater for individuals in the high payoff condition. Their results were typical in that there was a significant decay in the over-contribution as the number of periods increased.

An experiment that simultaneously incorporated both interior Nash and Pareto levels as well as asymmetric endowments and asymmetric shares of the benefits from the public good would more clearly isolate the sources of over-contribution, relative to the Nash, if they did indeed appear in the data. Over-contribution could be the result of "rule-of-thumb" allocation decisions, so interior Nash equilibrium experiments with varying endowments, such as Laury, Walker and Williams (1999), allow for testing such a hypothesis. Theoretically, the Nash equilibrium is independent of endowments given an endowment level sufficient to obtain the Nash equilibrium. Incorporating benefit share asymmetry, as well as endowment asymmetry allows for matched-pairs hypothesis tests to isolate the factors generating public good provision levels. This design most closely mirrors reality in an environment when individuals have different income levels and realize different benefit shares of a public good that is subject to declining marginal benefits, and as such should provide greater insight into global public goods problems.

## 3.2 Theory

The experimental design is VCM with declining value of the marginal contribution to the public good, and constant value of the marginal contribution to the private good. Each individual is a member of a group composed of N-1 other subjects. The payoff function for each individual is designed so that the value of the marginal contribution to the private good is one for all units<sup>24</sup>.

$$\pi_i = (e_i - q_i) + b\left(aQ - \frac{Q^2}{2}\right)\alpha_i \tag{72}$$

This functional form implies declining marginal per capita return  $MPCR \equiv -\frac{\left(\frac{\partial C}{\partial q_i}\right)}{\left(\frac{\partial P}{\partial q_i}\right)} = b\alpha_i \, (a-Q)$ . Indeed the term MPCR is a bit of a misnomer in the presence of asymmetric shares. A more appropriate term is marginal return for player i.

### 3.2.1 Nash Equilibrium

The individual optimum is the choice of  $q_i$  that maximizes (72). Manipulating the first order condition yields the reaction function for player i,  $q_i^r$ .

$$q_{i}^{r}(a, b, \alpha_{i}, Q_{-i}) = (a - Q_{-i}) - \frac{1}{b\alpha_{i}}$$
(73)

Where  $Q_{-i} = \sum_{j \neq i}^{N} q_j$  is the sum of the other players contributions to the public good. The symmetric Nash equilibrium is the simultaneous solution to (73), where

<sup>&</sup>lt;sup>24</sup>This is a simplified version of the functional forms in chapter 1 of this dissertation, regarding provision of a pure public good. The benefit functions are identical, but here the marginal cost of providing the public good are constant and symmetric. Chapter 1 investigates public good provision when marginal costs are linear and increasing at asymmetric rates, implying  $MPCR = \frac{b\alpha_*(a-Q)}{c_*q_*}$ .

 $\alpha_i = \frac{1}{N} \text{ for } i = [1, ..., N].$ 

$$q_i^* = \frac{a}{N} - \frac{1}{b}$$

$$Q^* = a - \frac{N}{b}$$
(74)

Asymmetric benefit shares result in the same total Nash equilibrium quantity, but the individual level is a function of player i's benefit share of the public good,  $\alpha_i$ . The asymmetric Nash equilibrium level of provision for player i is

$$q_i^* = a\alpha_i - \frac{1}{b} \tag{75}$$

### 3.2.2 Pareto Optimal Provision

To player i the value of the marginal contribution the public good is:  $b(a-Q)\alpha_i$ , but the social value is b(a-Q). The Pareto optimal outcome is such that the social marginal value of the public good is set equal to the value of the private good, which has been normalized to unity for all players, on all units. The Pareto optimal outcome is the choice of Q that maximizes the sum of all players payoffs, denoted  $\Pi \equiv \sum_{i=1}^{N} \pi_i$ , while the Nash equilibrium is the sum of the  $q_i$  that maximize the individual players payoff functions.

$$\Pi = \sum_{i=1}^{N} (e_i - q_i) + \sum_{i=1}^{N} b \left( aQ - \frac{Q^2}{2} \right) \alpha_i$$

$$\Pi = E - Q + b \left( aQ - \frac{Q^2}{2} \right)$$
(76)

Where  $E \equiv \sum_{i=1}^{N} e_i$ , is the sum of the players endowments. Maximization of (76) yields the Pareto optimal level of public good provision, denoted  $Q^o$ .

$$Q^o = a - \frac{1}{b} \tag{77}$$

$$99$$

The Pareto optimal quantity is strictly greater than the Nash quantity for all group size, N, greater than one. Since the marginal cost (opportunity cost) of providing the public good is equal to one for all units, and across all players, any allocation of the Pareto level is efficient<sup>25</sup>. An equitable solution would be to provide the public good in proportion to benefit shares.

## 3.3 Experimental Design

The treatment variables are the individuals endowments,  $e_i$ , and their share of the public good,  $\alpha_i$ . The design is a two by two factorial design in which both the endowment and the benefit shares are either symmetric or asymmetric<sup>26</sup>. For the asymmetric treatments endowments and benefit shares are either high or low. Theoretically the distribution of the endowments is irrelevant, due to the constant unit cost of public good provision. The sufficient condition for an interior Pareto quantity is that the total level of endowment be strictly greater than the Pareto optimal level,  $E \equiv \sum_{i=1}^{N} e_i > Q^o$ . If each player contributed their entire endowment to the public good the marginal benefit of the last unit could be set equal to zero. Denote this quantity  $Q_{\text{max}}$ . Equation (76) determines the value where b(a-Q)=0, as  $a=E=Q_{\text{max}}$ . Since the optimal quantity of public good provision is independent of the levels of the endowment, it is desirable to have the same total level of the endowment across all treatments to be able to compare the effects of varying the distribution of endowments and shares on provision levels. Within this theoretical framework the total endowment is potentially a treatment variable

<sup>&</sup>lt;sup>25</sup>For strictly increasing marginal costs of public good provision there is a unique Pareto level where the marginal costs of the last unit of the public good are equated.

<sup>&</sup>lt;sup>26</sup>A factorial design tests all four possible combinations in the asymmetric endowment, asymmetric share case; (high, high) and (low, low), as well as (high, low) and (low, high).

in future experiments that could be scaled to investigate the hypothesis that players are allocating certain "rule-of-thumb" proportions of their endowments to the public good.

For an interior Nash equilibrium the parameters of the experiment need to be such that for the low benefit share the value of the marginal contribution on the first unit of public good provision is greater than the marginal value of a contribution to the private good. This is equivalent to the restriction:  $ab(\alpha_{low}) > 1$ . The parameters for the experiment are

- a = 120,  $b = \frac{1}{12}$  and N = 6.
- For symmetric endowments  $e_i = 20$  for i = [1, ..., 6]. For asymmetric endowments  $e_i = 15$  for i = [1, ..., 3],  $e_j = 25$  for j = [4, ..., 6], so that  $E = Q_{\text{max}} = 120$  across all treatments.
- For symmetric shares  $\alpha_i = \frac{1}{N} = \frac{1}{6}$  for i = [1, ..., 6]. For asymmetric shares<sup>27</sup>  $\alpha_i = \frac{1}{9}$  for i = [1, ..., 3],  $\alpha_j = \frac{2}{9}$  for j = [4, ..., 6].
- The number of periods was randomly chosen from a uniform distribution on the interval [7, .., 14].
- Subjects had information on their endowment, the distribution and the total
  endowment of the group, their share and the distribution of shares. Subjects
  had a record of their allocation of their endowment and their payoff for that
  and all previous periods and the total level that the group contributed to the

<sup>&</sup>lt;sup>27</sup>Due to budget constraints the pilot experiment only tested (high, high), (low, low) in the asymmetric endowment, asymmetric share treatment. A complete factorial design would test the (low, high), (high, low) treatment as well.

public good. However, subjects did *not* have information on the distribution of contributions by the other members of their group, nor did they have information on which of the other people in the room were in their group.

Average earnings for the first 12 subjects were between \$13.30 and \$25.25 for approximately two hours. There were 20 data points for each subject making the pilot data set around 240 observations.

For these parameter values the payoff function for the symmetric endowment, symmetric share case simplifies to

$$\pi_i = (20 - q_i) + \left(10Q - \frac{Q^2}{24}\right) \frac{1}{6} \tag{78}$$

The parameters imply that the Nash equilibrium is:  $Q^* = a - \frac{N}{b} = 48$ , and that  $q_i^* = \frac{a}{N} - \frac{1}{b} = 8$  for the symmetric endowment, symmetric share treatment. The Pareto optimum is  $Q^o = a - \frac{1}{b} = 108$ . As noted, any division of the Pareto level is optimal given that marginal cost of public good provision is constant for all players on all units. If the Pareto optimal level is provided equally then  $q_i^o = \left(a - \frac{1}{b}\right)\frac{1}{N} = 18$ .

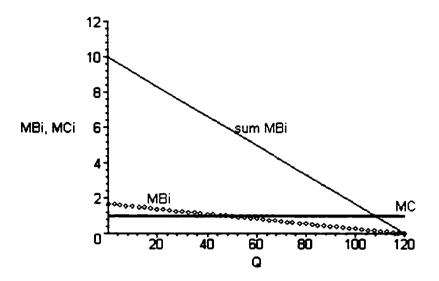


Figure 3.1: Nash and Pareto levels for a = 120,  $b = \frac{1}{12}$  and p = 1.

The asymmetric endowment case has exactly the same Nash and Pareto levels for the group and individual players. However, for the asymmetric shares the individual Nash quantity depends on the whether the individual has a high or a low share. From equation (75) the Nash quantity for the individual is:  $q_i^* = a\alpha_i - \frac{1}{b}$ . For the low share this is  $q_{low}^* = \frac{120}{9} - \frac{1}{\left(\frac{1}{12}\right)} = \frac{4}{3}$ , and for the for the high share  $q_{high}^* = \frac{240}{9} - \frac{1}{\left(\frac{1}{12}\right)} = \frac{44}{3}$ , eleven times greater. The total from the three low share players is 4 units and the total from the high share players is 44 units for a total of 48, the same total as identical shares. The difference is that the high and low share groups are coordinating on different amounts. For example, if a player expects the other members of the group to contribute such that the right hand side of the reaction function  $q_i^r = (a - Q_{-i}) - \frac{1}{b\alpha_i}$  is less than one, than the optimal strategy is to completely free-ride and contribute zero to the public good. The critical values as a function of the three different benefit share values are  $Q_{-i} = 120 - \frac{1}{\left(\frac{1}{12}\right)\alpha_i}$ . For

the low share players this implies a critical value of  $Q_{-i} = 12$ . For the high payoff share a similar strategy exists, but the crucial value of  $Q_{-i} = 66$ . Figure 2 shows the incentives of individual players given their benefit share type and conjecture regarding  $Q_{-i}$ .

Since the design is such that even in the high endowment case the other two high share players can contribute a maximum of 50 units of the public good, the minimum self-interested contribution from a high share player is 16. Given this, a low share players optimal strategy is to completely free-ride. If the high share players optimally coordinate their actions in a repeated game setting then the high share players would contribute 22 units of the public good. This amount is attainable by a coalition of the three high share players in the high share, high endowment treatment, but not in the high share, symmetric endowment treatment. For the symmetric payoff share treatments all players will have an incentive to coordinate on the Nash quantity of 48.

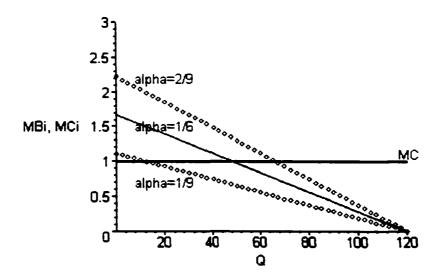
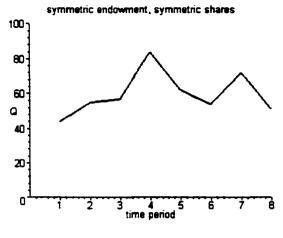


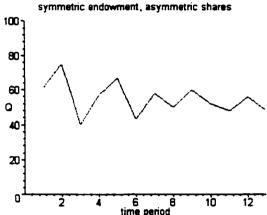
Figure 3.2: Nash equilibria for low, symmetric and high benefit share groups.

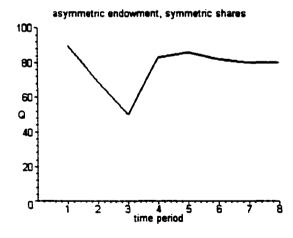
### 3.4 Pilot Results

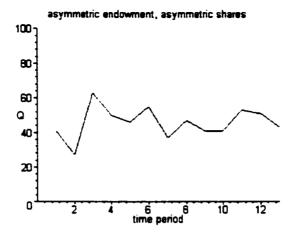
A pilot experiment was conducted using twelve subjects, randomly assigned to two separate groups. The experiment was run on the two groups simultaneously, so that the subjects did not know the identity of the other group members. Each run of the experiment lasted from seven to fourteen periods, randomly drawn from a uniform distribution. The session consisted of two runs of the experiment, and was completed in under two hours. The results for the overall level of public good provision are given below, where the first row is the time period and the treatments are listed as (endowment, share). Symmetric endowment is 20 for i = [1, ..., 6], asymmetric endowment is 15 for i = [1, ..., 3] and 25 for j = [4, ..., 6]. Symmetric shares are  $\frac{1}{6}$  for i = [1, ..., 6], asymmetric shares are  $\frac{1}{9}$  for i = [1, ..., 3] and  $\frac{2}{9}$  for j = [4, ..., 6], so that the high endowment subjects also received a high share. The actual levels of public good provision are relative to the Nash prediction of 48 and the Pareto optimum of 108.

Table 3.1: Total Public Goods Provision													
Treatment	1	2	3	4	5	6	7	8	9	10	11	12	13
sym,sym	44	55	57	84	62	54	72	51					
sym,asy	61	75	40	57	67	43	58	50	60	52	48	56	48
asy,sym	90	69	50	83	86	82	80	80					
asy,asy	41	27	63	50	46	55	37	47	41	41	53	51	43









While this is only a pilot, some results are striking and warrant further investigation. First, there does not appear to be systematic over-contribution relative to the Nash equilibrium in three of the treatments, contrary to previous work. Only the asymmetric endowment, symmetric share case appears significantly higher, nearly approaching the Pareto optimal level. Furthermore, this treatment appears to exhibit a slight decline in provision over time, but still nowhere close to the levels of decay found in previous experiments. Both of the asymmetric endowment treatments converge to the Nash equilibrium and exhibit a type of oscillatory behavior that would be predicted by implying negative autocorrelation. This effect may be due to the group members gradually coordinating on their desired level, possibly the Nash equilibrium. Along the same lines, all treatments appear to have smaller variance as the number of time periods increases.

There are different interpretations of the sources of these patterns, some of which are hinted at in the responses to the exit questions, given in the Appendix. Many of the individual players contributions in the asymmetric benefit share treatments were remarkably similar to the Nash prediction. This was true for both the

low and high benefit share groups. Since three of the four treatments (the asymmetric endowment, symmetric share being the exception) roughly converged to the Nash prediction of 48, there was a distinct lack of over-provision that has characterized previous studies. In fact, it is hard to discern a systematic pattern of decay across any of the four treatments, another result that contradicts the stylized facts. One interpretation for these results is that the concept of fairness plays a significant role in public goods provision.

Another interesting point is that the level of provision appears higher for the symmetric share treatment. This raises the possibility that subjects tend to increase provision when there is an equitable, or "fair" distribution of the benefits of the public good. Further runs of the experiment are necessary to provide an adequate number of observations for hypothesis testing to discern the sources of this variation.

### 3.4.1 Hypothesis Tests

The first point of inquiry is if the overall level of provision by the entire group is different from the Nash level of 48. There are a total of 20 observations of public good provision by the entire group. The value of the test statistic is t = 0.9852, therefore we can not reject the null hypothesis that the overall level of public good provision is different from the Nash equilibrium. A greater number of observations would increase the power of the test providing stronger evidence on this point.

Taking the time periods across treatments as matched pairs, we can test if the two samples are drawn from the same distribution. The matched pairs test statistic is:

$$t_m = \frac{n^{.5} (\bar{x}_D)}{s_D} \tag{79}$$

108

Table 3.2: Test Statistics					
Treatment	$t_m$				
(sym,sym-sym,asy)	1.26				
(sym.sym-asy,sym)	-2.85**				
(asy,sym-asy,asy)	4.64***				
(sym,asy-asy,asy)	1.88*				

Even with such low powered hypothesis tests all of the treatments that contained asymmetric endowments are significantly different than the symmetric endowment contributions. Clearly, the symmetric benefit shares led to a greater level of provision when endowments were asymmetric, suggesting a strong role for "fairness" in public goods provision. Interestingly, there appears to be a higher level of public good provision by both low and high endowment subjects relative to symmetric endowments. This indicates that there might be a lower tendency to free-ride when individuals have different wealth levels. While it is difficult to draw any clear conclusions from such a small amount of pilot data, the values of the test statistics indicate future runs of the experiment are certainly warranted, and may make a substantial contribution to the public goods provision literature.

A possible extension of this framework is to incorporate increasing marginal cost of public good provision. Increasing marginal cost implies that there is a unique allocation of the overall level of public good provision that is efficient. In the constant symmetric marginal cost framework any allocation is efficient. An increasing marginal cost design would also reveal the efficiency of individual contributions across treatments.

# 4 Appendix

## Chapter 1

Proof of Proposition 1: The Pareto optimal level of abatement is strictly greater than the Nash equilibrium.

- (i)  $\sum_{j=1}^{N} \frac{1}{c_j} > \sum_{j=1}^{N} \theta_j$  for all  $0 < \alpha_i < 1$ ,  $\sum_{j=1}^{N} \alpha_j = 1$ .
- (ii) If either the benefit shares, the MAC curve slopes, or both are symmetric then:  $\sum_{j=1}^{N} \frac{1}{c_j} = N \sum_{j=1}^{N} \theta_j$ .

Proof of (i)  $\sum_{j=1}^{N} \theta_{j} \equiv \sum_{j=1}^{N} \frac{\alpha_{j}}{c_{j}}$  is an arithmetic series that, for  $0 < \alpha_{j} < 1$ . contains a positive fraction,  $\alpha_{j}$ , in the numerator and  $c_{i}$  in the denominator.  $\sum_{j=1}^{N} \frac{1}{c_{j}}$  is a series that contains the same elements in each denominator and 1 in the numerator. Therefore each  $\theta_{j}$  of the sum  $\sum_{j=1}^{N} \theta_{j}$  is strictly less than the corresponding element,  $\frac{1}{c_{j}}$ , of the sum  $\sum_{j=1}^{N} \frac{1}{c_{j}}$ , and therefore  $\sum_{j=1}^{N} \frac{1}{c_{j}} > \sum_{j=1}^{N} \theta_{j}$ . This completes the proof of (i).

Proof of (ii) By definition the benefit shares  $\sum_{j=1}^{N} \alpha_j = 1$ . When the MAC are symmetric then  $c_i = c$  for i = [1, ..., N] and the sum  $\sum_{j=1}^{N} \frac{1}{c_j} = \frac{N}{c}$ . The sum  $\sum_{j=1}^{N} \theta_j \equiv \sum_{j=1}^{N} \frac{\alpha_j}{c} = \frac{1}{c} \sum_{j=1}^{N} \alpha_j = \frac{1}{c}$ . Therefore  $\sum_{j=1}^{N} \frac{1}{c_j} = N \sum_{j=1}^{N} \theta_j$ .

When only the benefit shares are symmetric then  $\alpha_i = \frac{1}{N}$  for i = [1, ..., N] the sum  $\sum_{j=1}^{N} \theta_j$  reduces to  $\frac{1}{N} \sum_{j=1}^{N} \frac{1}{c_j}$ . Therefore  $\sum_{j=1}^{N} \frac{1}{c_j} = N \sum_{j=1}^{N} \theta_j$ . This completes the proof of Proposition 1.

Proof of Proposition 2: For the case of symmetric marginal abatement cost curve slopes, the global net benefit of the Pareto optimal solution over the Nash equilibrium is increasing in the variance of the benefit shares.

Proof of Proposition 2 relies on Lemma 1.

Lemma 1: The sum  $\sum_{j=1}^{N} (\alpha_j)^2$  is minimized for benefit share symmetry,  $\alpha_j = \frac{1}{N}$  for j = [1, ..., N], and the variance of the benefit shares is strictly increasing in  $\sum_{j=1}^{N} (\alpha_j)^2$ .

Proof of Lemma 1: showing the sum  $\sum_{j=1}^{N} (\alpha_j)^2$  is minimized for symmetry follows from the same procedure as the proof of lemma 2. The Lagrangian is  $\min_{\alpha_1,\dots,\alpha_N} \left\{ L = \sum_{j=1}^{N} (\alpha_j)^2 + \lambda \left(1 - \sum_{j=1}^{N} \alpha_j\right) \right\}$ . The variance of  $\alpha$  is defined as  $var(\alpha) \equiv \frac{\sum_{j=1}^{N} (\alpha_j - \bar{\alpha})^2}{N-1}$ , where  $\bar{\alpha} \equiv \frac{\sum_{j=1}^{N} \alpha_i}{N} = \frac{1}{N}$ , by definition of the benefit shares.  $var(\alpha) = \frac{\sum_{j=1}^{N} (\alpha_j - \bar{\alpha})^2}{N-1} = \frac{\sum_{j=1}^{N} (\alpha_j)^2 - \left[\frac{1}{N}\right]}{N-1}$ , and therefore the variance of the benefit shares is strictly increasing in  $\sum_{j=1}^{N} (\alpha_j)^2$ .

Lemma 2: For all mean preserving distributions of the slopes of the MAC curves.  $c_i$ , the sum  $\sum_{j=1}^{N} \frac{1}{c_j}$  is minimized for identical  $c_i$ .

Proof of Lemma 2: requires showing that the sum  $\sum_{j=1}^{N} \frac{1}{\bar{c}+d_j}$ , where  $\bar{c}$  is the arithmetic mean of the MAC slopes and  $d_j$  is the deviation of country j's MAC slope from the mean, is minimized for  $d_j = 0$  for j = [1, ..., N]. A mean preserving distribution of the  $c_i$  implies the constraint  $\sum_{j=1}^{N} d_j = 0$ . The Lagrangian for this problem is:  $\min_{d_1,...,d_N} \left\{ L = \sum_{j=1}^{N} \frac{1}{\bar{c}+d_j} + \lambda \left(0 - \sum_{j=1}^{N} d_j\right) \right\}$ . The N first order conditions are of the form  $\frac{-1}{(\bar{c}+d_j)^2} = \lambda$ . The second order condition for a minimum is satisfied,  $\frac{\partial^2 L}{(\partial d_j)^2} = \frac{2}{(\bar{c}+d_j)^3} > 0$ , for any deviation smaller than the mean, or equivalently for all  $c_j > 0$ . Any two first order conditions then imply  $\frac{1}{\bar{c}+d_i} = \frac{1}{\bar{c}+d_j}$  implying that the sum is minimized for equal deviations,  $d_1 = d_2 = \ldots = d_N$ . The mean preserving distribution constraint  $\sum_{j=1}^{N} d_j = 0$ , implies  $d_1 = d_2 = \ldots = d_N = 0$ , and thus the sum  $\sum_{j=1}^{N} \frac{1}{\bar{c}+d_j}$  is minimized for identical  $c_j$ . This completes the proof of Lemma 2.

Lemma 3: For all mean preserving distributions of the slopes of the MAC curves,  $c_j$ , the sum  $\sum_{j=1}^{N} \frac{(\alpha_j)^2}{c_j}$  is minimized for complete symmetry;  $c_j = \bar{c}$  for j = [1, ..., N] and  $\alpha_j = \frac{1}{N}$  for j = [1, ..., N].

Proof of Lemma 3: requires showing that the sum  $\sum_{j=1}^{N} \frac{(\alpha_i)^2}{c_i}$  is minimized subject to the constraints  $\sum_{j=1}^{N} \alpha_j = 1$  and  $\sum_{j=1}^{N} c_j = k$ , where k is an arbitrary constant. The Lagrangian for this problem is:

 $\min_{d_1,\ldots,d_N} \left\{ L = \sum_{j=1}^N \frac{(\alpha_j)^2}{c_j} + \lambda \left( 1 - \sum_{j=1}^N \alpha_j \right) + \mu \left( k - \sum_{j=1}^N c_j \right) \right\}$ . The N first order conditions are of the form  $\frac{2\alpha_i}{c_j} + \lambda + \mu = 0$ . The second order condition for a minimum is satisfied since the determinant of the Hessian is  $\frac{4\left[ (\alpha_i)^3 - (\alpha_i)^2 \right]}{(c_j)^4} < 0$  for all  $\alpha_j \in (0,1)$ . Any two first order conditions then imply  $\frac{\alpha_i}{c_j} = \frac{\alpha_k}{c_k}$  implying that the sum is minimized for identical  $\theta's$ . Complete symmetry is one, but not the only, situation in which the  $\theta's$  are identical. This completes the proof of Lemma 3.

Proof of Proposition 3: For all possible coalitions with at least one non-signatory, global abatement in the Stackelberg equilibrium is strictly less than that of the Nash equilibrium which is strictly less than Pareto optimal.

(i) The Stackelberg level of signatory abatement, for any given coalition of signatories, is strictly less than the Nash level of abatement. Define  $S_c^s \equiv \sum_{j=1}^M \frac{1}{c_j}$ ,  $S_\alpha^s \equiv \sum_{j=1}^M \alpha_j^s$  and  $S_\theta^n \equiv \sum_{j=M+1}^N \theta_j^n$  to conserve on space. For the signatories the Stackelberg is  $Q^s = \frac{abS_c^s S_\alpha^s}{\left[1+bS_\theta^n\right]^2 + bS_c^s S_\alpha^s}$  while the Nash is  $Q^s = \frac{abS_c^s S_\alpha^s}{1+b\left(S_\theta^n + S_c^s S_\alpha^s\right)}$ . The numerators are identical while the Stackelberg has a strictly larger denominator for at least one non-signatory.

However, for the non-signatories the opposite is true, the Stackelberg level of abatement  $Q^n = \frac{abS_{\theta}^n \left[1+bS_{\theta}^n\right]}{\left[1+bS_{\theta}^n\right]^2 + bS_{\varepsilon}^2 S_{\alpha}^2}$  is strictly greater than the Nash  $Q^{n*} = \frac{abS_{\theta}^n}{\left[1+b\left(S_{\theta}^n+S_{\varepsilon}^nS_{\alpha}^n\right)\right]}$ . Dividing by the Stackelberg quantity by the  $\left[1+bS_{\theta}^n\right]$  term shows that the numerators are identical while the Nash has a strictly greater denominator.

(ii) The global level of abatement is strictly greater in the Nash equilibrium for at least one non-signatory. The difference between the Nash minus the Stackelberg level of abatement is  $\frac{ab^2 S_c^* S_c^* S_d^*}{\left[1+b(S_\theta^n+S_c^*S_c^*)\right]\left\{\left[1+bS_\theta^n\right]^2+bS_c^*S_c^*\right\}} > 0.$  Furthermore, the Pareto optimal level of abatement,  $Q^o = \frac{abS_c^*}{1+bS_c^*}$ , in equation (13), is strictly greater than the Nash  $\frac{S_c^*-S_\theta^n-S_c^*S_c^*}{\left[1+bS_c^*\right]\left[1+b(S_\theta^n+S_c^*S_c^*)\right]} > 0$  for at least one non-signatory. This completes the proof of Proposition 3.

Proof of Proposition 4 is trivial.

Proof of Proposition 5: If no country has  $\alpha_i > \frac{1}{2}$  then the Nash equilibrium concept implies that all sub-coalitions will desire full participation, and hence the Pareto level of abatement.

In the Nash coalition formation becoming a signatory implies an increase in abatement if  $q_i^{s,M+\{i\}}-q_i^{n*,M}>0$ 

$$q_i^{s,M+\{i\}} - q_i^{ns,M} = \frac{ab \left\{ \begin{array}{c} \left[ S_{\alpha}^s + \alpha_i \right] \left[ 1 + b \left( S_{\theta}^n + S_c^s S_{\alpha}^s \right) \right] \\ -\alpha_i \left[ 1 + b \left( S_{\theta}^n - \theta_i \right) + b \left( S_c^s + \frac{1}{c_i} \right) \left( S_{\alpha}^s + \alpha_i \right) \right] \end{array} \right\}}{\Delta_{(iv)}}$$
(80)

Where 
$$\Delta_{(iv)} \equiv c_i \left[ 1 + b \left( S_{\theta}^n + S_c^s S_{\alpha}^s \right) \right] \left[ 1 + b \left( S_{\theta}^n - \theta_i \right) + b \left( S_c^s + \frac{1}{c_i} \right) \left( S_{\alpha}^s + \alpha_i \right) \right] > 0.$$

The denominator is strictly positive so inclusion in the coalition implies an increase in the level of abatement for country i when the numerator is positive, or when

$$\begin{split} \left[S_{\alpha}^{s} + \alpha_{i}\right]\left[1 + b\left(S_{\theta}^{n} + S_{c}^{s}S_{\alpha}^{s}\right)\right] - \alpha_{i}\left[1 + b\left(S_{\theta}^{n} + S_{c}^{s}S_{\alpha}^{s} + \left(\frac{1}{c_{i}}S_{\alpha}^{s} + \alpha_{i}S_{c}^{s}\right)\right)\right] > 0 \\ S_{\alpha}^{s}\left[b\left(S_{\theta}^{n} + S_{c}^{s}S_{\alpha}^{s}\right)\right] - \alpha_{i}b\left[\frac{1}{c_{i}}S_{\alpha}^{s} + \alpha_{i}S_{c}^{s}\right] > 0 \end{split}$$

Collecting terms

$$S_{\alpha}^{s} + bS_{c}^{s} \left[ \left( S_{\alpha}^{s} \right)^{2} - \left( \alpha_{i} \right)^{2} \right] + bS_{\alpha}^{s} \left[ S_{\theta}^{n} - \theta_{i} \right] > 0$$

For the Stackelberg equilibrium  $q_i^{s,M+i} - q_i^{n,M} > 0$  if the numerator is positive, or

$$\begin{split} \left[ S_{\alpha}^{s} + \alpha_{i} \right] \left[ (1 + b S_{\theta}^{n})^{2} + b S_{c}^{s} S_{\alpha}^{s} \right] \\ - \alpha_{i} \left( 1 + b S_{\theta}^{n} \right) \left[ \begin{array}{c} (1 + b S_{\theta}^{n})^{2} + b^{2} \left( \theta_{i} \right)^{2} - 2b \theta_{i} \left( 1 + b S_{\theta}^{n} \right) \\ + b \left( S_{c}^{s} S_{\alpha}^{s} + \frac{1}{c_{i}} S_{\alpha}^{s} + \alpha_{i} S_{c}^{s} + \theta_{i} \right) \end{array} \right] > 0 \end{split}$$

$$\begin{split} &b\left[S_{\theta}^{n}\left(2S_{\alpha}^{s}-\alpha_{i}\right)+S_{c}^{s}\left(S_{\alpha}^{s}\right)^{2}-\theta_{i}S_{\alpha}^{s}-\left(\alpha_{i}\right)^{2}S_{c}^{s}+\alpha_{i}\theta_{i}\right]\\ &+b^{2}\left[S_{\theta}^{n}\alpha_{i}\left(-S_{c}^{s}S_{\alpha}^{s}-\frac{1}{c_{i}}S_{\alpha}^{s}-\alpha_{i}S_{c}^{s}+3\theta_{i}\right)+\left(S_{\theta}^{n}\right)^{2}\left(S_{\alpha}^{s}-2\alpha_{i}\right)-\alpha_{i}(\theta_{i})^{2}\right]\\ &+b^{3}\left[\alpha_{i}\left(2\theta_{i}\left(S_{\theta}^{n}\right)^{2}-\left(S_{\theta}^{n}\right)^{3}-\left(\theta_{i}\right)^{2}\right)\right]+S_{\alpha}^{s}>0 \end{split}$$

## Chapter 2

Interior Autarchy EE Restriction

- (i)  $q_i > 0, \forall i, \forall s \in (0,1)$
- (a) The first condition is that the non-negativity constraints on the Kuhn-Tucker conditions are not binding. The home autarchy Nash-equilibrium quantities for an interior solution are

$$q_c^{NE} = \frac{\beta(1 + (1 - s)N)\theta_c - \delta(1 - s)N\theta_d}{\Delta}$$

$$q_d^{NE} = \frac{\beta(1 + sN)\theta_d - \delta sN\theta_c}{\Delta}$$
(81)

Where  $\Delta \equiv \beta^2 (1+sN)(1+(1-s)N) - \delta^2 (s(1-s)N^2)$  and  $\theta_c \equiv \alpha_c - c_c$ ,  $\theta_d \equiv \alpha_d - c_d$ .  $\Delta$  is strictly positive for all values of the state variable  $s \in [0,1]$  since  $\beta > \delta$ . In the limit,  $\beta = \delta$ , as the two types of the goods are perfect substitutes,  $\Delta = 2$ .

For the clean quantity the Kuhn-Tucker conditions will be slack for all parameter values such that

$$\beta(1 + (1 - s)N)\theta_c - \delta(1 - s)N\theta_d > 0$$

$$\frac{\beta}{\delta} > \frac{\theta_d}{\theta_c} \left[ \frac{1}{\left[ \frac{1}{(1 - s)N} \right] + 1} \right]$$
(82)

The term in square brackets above is a positive fraction for all  $s \in [0, 1)$ . For imperfect substitutes the own effect,  $\beta$ , is assumed to dominate the cross effect,  $\delta$ , so that  $\frac{\beta}{\delta}$  is greater than one. Since all of the parameters are positive,  $q_c^{NE}$  is strictly positive for all  $\theta_c > \theta_d$ , and for some values of  $\theta_d > \theta_c$ , as s gets sufficiently close to 1.

(b) The home autarchy Nash-equilibrium dirty quantities will be slack for all quantities such that:

$$\beta(1+sN)\theta_d - \delta sN\theta_c > 0$$

or

$$\frac{\beta}{\delta} > \frac{\theta_c}{\theta_d} \left[ \frac{1}{\left(\frac{1}{\epsilon N}\right) + 1} \right] \tag{83}$$

The term in square brackets above is a positive fraction for all  $s \in (0, 1]$ . This means that  $q_d^{NE}$  is strictly positive for all  $\theta_c < \theta_d$ , and for some values of  $\theta_c > \theta_d$ , as s gets sufficiently close to 0.

(c) Combining the results in (a) and (b) we have a single parameter restriction for strictly positive quantities of both  $q_c^{NE}$  and  $q_d^{NE}$ .

$$\frac{\beta}{\delta} > \frac{\theta_d}{\theta_c} \left[ \frac{1}{\left[\frac{1}{(1-s)N}\right] + 1} \right], \frac{\theta_c}{\theta_d} \left[ \frac{1}{\left(\frac{1}{sN}\right) + 1} \right]$$

The closer the  $\theta's$  are, meaning that the relative differences in demand minus marginal cost are not too large, and the more imperfect substitutes the goods are

(greater the difference between  $\beta$  and  $\delta$ ), the more likely the Nash equilibrium quantities will be positive for all values of  $s \in [0,1]$ . For the parameters used in the paper,  $\alpha_c = 200$ ,  $\alpha_d = 190$ ,  $c_c = 10$ ,  $c_d = 8$ , N = 7,  $\alpha_c^* = 150$ ,  $\alpha_d^* = 145$ ,  $c_c^* = 9$ ,  $c_d^* = 6$ ,  $N^* = 7$ ,  $\beta = 1$  and  $\delta = 0.9$ , the above condition for the home market is:  $1.11 > 0.958 \left[ \frac{1}{\left[\frac{1}{(1-s)N}\right]+1}\right]$ ,  $1.04 \left[\frac{1}{\left(\frac{1}{sN}\right)+1}\right]$ , which is satisfied for all  $s \in [0,1]$ , and for all N. This suggests that the conditions for an interior EE are not sensitive to the number of firms.

(ii) 
$$\Pi_D > 0$$
 at  $s = 0$ ,  $\Pi_D < 0$  at  $s = 1$ 

(iia) The denominator of the profit differential equation (55) is strictly positive, so its sign is determined by the numerator. Evaluating (55) at s = 0, yields the condition that must be satisfied for the profit differential to be positive at s = 0:

$$\theta_c^2 \beta^3 \left[ 1 + N \right]^2 - \theta_d^2 \left[ \beta^3 - \beta \delta^2 N^2 \right] - 2\theta_c \theta_d \beta^2 \delta \left[ N(N+1) \right] > 0$$

Dividing by  $\beta$ , and collecting terms this condition reduces to:

$$\beta^{2}(\theta_{c}^{2} - \theta_{d}^{2}) + 2\beta^{2}\theta_{c}^{2}N - 2\beta\delta\theta_{c}\theta_{d}N + N^{2}[\beta^{2}\theta_{c}^{2} + \delta^{2}\theta_{d}^{2} - 2\theta_{c}\theta_{d}\beta\delta] > 0$$

$$\beta^{2}(\theta_{c}^{2} - \theta_{d}^{2}) + 2\beta\theta_{c}^{2}N[\beta\theta_{c} - \delta\theta_{d}] + N^{2}[(\beta\theta_{c} - \delta\theta_{d})^{2}] > 0$$
(84)

The second part of the condition is satisfied for all  $\beta\theta_d > \delta\theta_c$ , or  $\frac{\beta}{\delta} > \frac{\theta_c}{\theta_d}$ .

(iib) Evaluating (55) at s = 1, yields the condition that must be satisfied for the profit differential to be negative at s = 1:

$$\theta_c^2 \left[ \beta^3 - \beta \delta N^2 \right] - \theta_d^2 \left[ \beta^3 (1+N)^2 \right] + 2\theta_c \theta_d \beta^2 \delta \left[ N(N+1) \right] < 0$$

After some manipulation we get:

$$\beta^2(\theta_d^2 - \theta_c^2) + 2\beta\theta_d^2N[\beta\theta_d - \delta\theta_c] + N^2[(\beta\theta_d - \delta\theta_c)^2] > 0$$
 (85)

The second part of the condition is satisfied for all  $\beta\theta_d > \delta\theta_c$ , or  $\frac{\beta}{\delta} > \frac{\theta_c}{\theta_d}$ . Taken together, conditions (i) and (ii) are satisfied when

$$\frac{\beta}{\delta} > \frac{\theta_d}{\theta_c}, \frac{\theta_c}{\theta_d} \tag{86}$$

The intuition behind this result is that the lower the relative substitutability of the two goods, meaning the greater that  $\frac{\beta}{\delta}$  is above 1, the larger the relative differences in the  $\theta$ 's, which are the demand curve intercepts minus the marginal cost, that can sustain a unique stable interior EE.

### Trade Parameter Restrictions

The trade parameter restrictions for the Nash-equilibrium quantities to be non-negative comes directly from the inverse of the coefficient matrix of the reaction functions. For example, the Kuhn-Tucker restriction for  $q_c^d$  will be slack when the first line of the inverse of the coefficient matrix in (61) times the vector on the right hand side of (61) is strictly greater than zero. Since the common denominator,  $\Delta$ , is greater than zero, we have

$$\begin{split} & \left\{ [s^*N^* + 1]\beta^2[Y + 1] - \delta^2 s^*N^*Y \right\} [\alpha_c - c_c] \\ & - \left\{ s^*N^* \left\{ \beta^2[Y + 1] - \delta^2 Y \right\} \right\} \left[ \alpha_c - c_c^* - t_c \right] \\ & - \left\{ \beta \delta (1 - s)N \right\} [\alpha_d - c_d] \\ & - \left\{ \beta \delta (1 - s^*)N^* \right\} [\alpha_d - c_d^* - t_d] \\ & > 0 \end{split}$$

where  $Y = (1 - s)N + (1 - s^{\bullet})N^{\bullet}$ , which is the global number of dirty firms. The above condition reduces to:

$$\beta^2[Y+1]\left[\alpha_c-c_c\right]$$

$$> \beta \delta \left\{ (1-s)N \left[ \alpha_d - c_d \right] + (1-s^*)N^* \left[ \alpha_d - c_d^* - t_d \right] \right\}$$

$$+ \left\{ s^*N^* \left[ \beta^2 [Y+1] - \delta^2 Y \right] \right\} \left[ c_c - c_c^* - t_c \right]$$

All terms in brackets are positive for the parameters chosen in the paper. In general the left hand side is greater due both to the fact that the [Y+1] term is greater than  $(1-s)N+(1-s^*)N^*$ , and the fact that the own effect on price is greater than the cross effect,  $\beta > \delta$ . If the clean type faces a higher marginal cost of production in the home country than the foreign country,  $c_c > c_c^*$ , there will be a greater likelihood that the Kuhn-Tucker condition will be binding for  $q_c^d$ , although this third term is generally smaller in magnitude than the other two terms. The greater the price advantage for clean is,  $\alpha_c > \alpha_d$ , and the less substitutable the goods are.  $\beta > \delta$ , the less likely that the Kuhn-Tucker condition will be binding for  $q_c^d$ .

Similarly, the Kuhn-Tucker restriction for  $q_c^{e^*}$  will be slack when the first line of the inverse of the coefficient matrix in (61) times the vector on the right hand side of (61) is strictly greater than zero. As above that condition reduces to

$$\beta^{2}[Y+1] \left[\alpha_{c} - c_{c}^{\bullet} - t_{c}\right] + \left\{sN\left[\beta^{2}[Y+1] - \delta^{2}Y\right]\right\} \left[c_{c} - c_{c}^{\bullet} - t_{c}\right]$$
>  $\beta\delta\left\{\left[(1-s)N\right]\left[\alpha_{d} - c_{d}\right] + (1-s^{\bullet})N^{\bullet}\left[\alpha_{d} - c_{d}^{\bullet} - t_{d}\right]\right\}$ 

The greater the price advantage for clean,  $\alpha_c > \alpha_d$ , and the less substitutable the goods are,  $\beta > \delta$ , the less likely that the Kuhn-Tucker condition will be binding for  $q_c^d$ . However, if the clean type faces a higher marginal cost of production in the home country than the foreign country,  $c_c > c_c^*$ , as in the numerical example, there is an increased likelihood that the Kuhn-Tucker condition will not be binding for  $q_c^{e*}$ .

The Kuhn-Tucker restriction for  $q_d^d$  will be slack when the first line of the inverse of the coefficient matrix in (61) times the vector on the right hand side of (61) is strictly greater than zero. The condition reduces to

The greater the price advantage for clean is,  $\alpha_c > \alpha_d$ , the more likely that the Kuhn-Tucker condition will be binding for  $q_d^d$ . Also, when the dirty type faces a higher marginal cost of production in the home country than the foreign country,  $c_d > c_d^*$ , there is a greater likelihood that the Kuhn-Tucker condition will be binding for  $q_d^d$ . Taken together with the parameter restriction for the clean type, a necessary condition for an interior solution is that the intercepts for the demand curves and the unit cost differentials not be too different as in the autarchy case. Furthermore, we can say that there is a greater likelihood that there will be an interior solution when the substitutability of the goods decreases.

The Kuhn-Tucker restriction for  $q_d^{ee}$  will be slack when the fourth line of the same equation is strictly greater than zero.

$$\beta^{2}[X+1] \left[\alpha_{d} - c_{d}^{*} - t_{d}\right] + \left\{ (1-s)N \left[\beta^{2}[X+1] - \delta^{2}X\right] \right\} \left[c_{d} - c_{d}^{*} - t_{d}\right]$$
>  $\beta\delta \left[sN \left(\alpha_{c} - c_{c}\right) + s^{*}N^{*} \left(\alpha_{c} - c_{c}^{*} - t_{c}\right)\right]$ 

Again, this condition is more easily satisfied as the foreign country has a greater cost advantage.

Reaction Function Coefficient Matrix Inverse

$$A^{-1}\Delta = \begin{cases} a_{11}^{-1} = & [s^*N^* + 1]\beta^2[Y+1] - \delta^2s^*N^*Y \\ a_{12}^{-1} = & -s^*n^*\left\{\beta^2[Y+1] - \delta^2Y\right\} \\ a_{13}^{-1} = & -\beta\delta(1-s)N \\ a_{14}^{-1} = & -\beta\delta(1-s^*)N^* \\ a_{21}^{-1} = & sN\left\{-\beta^2[Y+1] + \delta^2Y\right\} \\ a_{22}^{-1} = & [sN+1]\beta^2[Y+1] - \delta^2sNY \\ a_{23}^{-1} = & -\beta\delta(1-s)N \\ a_{24}^{-1} = & -\beta\delta(1-s^*)N^* \\ a_{31}^{-1} = & -\beta\delta sN \\ a_{31}^{-1} = & -\beta\delta s^*N^* \\ a_{31}^{-1} = & (1-s^*)N^* + 1]\beta^2[X+1] - \delta^2(1-s^*)N^*X \\ a_{34}^{-1} = & (1-s^*)N^*\right]\left\{-\beta^2[X+1] + \delta^2X\right\} \\ a_{41}^{-1} = & -\beta\delta sN \\ a_{42}^{-1} = & -\beta\delta s^*N^* \\ a_{43}^{-1} = & [(1-s)N]\left\{-\beta^2[X+1] + \delta^2X\right\} \\ a_{44}^{-1} = & [(1-s)N]\left\{-\beta^2[X+1] + \delta^2X\right\} \\ a_{44}^{-1} = & [(1-s)N+1]\beta^2[X+1] - \delta^2(1-s)NX \end{cases}$$

Where  $\Delta = \beta^3[sN + s^*N^* + 1][(1-s)N + (1-s^*)N^* + 1] - \beta\delta^2[sN + s^*N^*][(1-s)N + (1-s^*)N^*], X = sN + s^*N^*$  is the global number of clean firms, and  $Y = (1-s)N + (1-s^*)N^*$  is the global number of dirty firms.

## Chapter 3

### Instructions

This experiment is a study of individual and group decision making. The instructions are straightforward. If you follow them carefully and make good investment decisions you may earn a considerable amount of money. If you have any questions raise your hand and one of the monitors will assist you. Do not communicate with any other individual in the room other than the monitors. You will be paid in cash at the end of the experiment.

#### **Overview**

You have been randomly assigned to a group of six people. The are two sixperson groups in this room. Each of you will be given a specific number of tokens
at the beginning of each period. Each period you will choose how to invest your
tokens between two accounts. You must invest all your tokens each period. An
experiment will consist of a random number of periods, between 7 and 14. The
number of periods has been written on a card held by one of the monitors. No one
other than the monitors knows this number.

Each period you will be choosing between two investment opportunities: the Individual Fund and the Group Fund. Every token you invest in the Individual Fund will earn a return of \$0.01. What you earn from the Group Fund will depend on the total number of tokens that you and the other five members of your group invest in the Group Fund.

Each member of the group will receive a predetermined share of the Group Fund. If you are assigned an equal share, you, and each member of your group, will receive  $\frac{1}{6}$  of the earnings from the Group Fund.

If you are assigned to a group with unequal shares, then three members of your group will receive  $\frac{1}{9}$  of the earnings from the Group Fund and three members of your group will receive  $\frac{2}{9}$  of the earnings from the Group Fund.

For each experiment you, and each member of your group, will receive either an equal or unequal number of tokens to invest each period. If you are assigned an equal number of tokens then you, and the other members of your group, will receive 20 tokens to invest each period. If you are assigned to a group with unequal number of tokens then three group members will receive 15 tokens each period and three group members will receive 25 tokens each period. In every period the total number of tokens that the entire group has to invest will be the same, 120.

If you are in a group with both unequal shares of the Group Fund and an unequal number of tokens then the three group members that receive a  $\frac{2}{9}$  share will receive 25 tokens each period. The three group members that receive a  $\frac{1}{9}$  share of the earnings from the Group Fund will receive 15 tokens each period.

You will remain in the same group for all periods of the experiment. You will participate in at least two experiments this evening.

#### **Investment Opportunities**

Each period you will be choosing between two investment opportunities:

### 1) The Individual Fund

Every token you invest in the Individual Fund will earn a return of 1 cent.

Example: Suppose you invested 10 tokens in the Individual Fund. Then you would earn \$0.10 from this fund.

Example: Suppose you invested 18 tokens in the Individual Fund. Then you would earn \$0.18 from this fund.

Example: Suppose you invested 0 tokens in the Individual Fund. Then you would earn \$0.00 from this fund.

### 2) The Group Fund

The return from the group fund is a little more difficult to determine.

Your earnings from the Group Fund only depend on the total amount invested by your group and your share of the group fund, *not* on the amount that you invested in the Group Fund.

There is a payoff table attached that shows your earnings from the Group Fund, depending on the total amount invested by the group and your share of the Group Fund. This is best explained by a number of examples:

### **Examples**

Equal shares of the Group Fund:

Example: You begin each period with 20 tokens. Suppose that your share of the group exchange is  $\frac{1}{6}$ , and you decide to invest no tokens in the Group Fund, but that the other five members of your group invested a total of 100 tokens. Then, as you can see from the middle column of the payoff table, your earnings from the Group Fund would be \$0.97, and your earnings from the Individual Fund would be \$0.20. Your total earnings for the period would be \$1.17. The other members of your group would also receive \$0.97 from the Group Fund.

Example: You begin each period with 15 tokens. Suppose that your share of the group exchange is  $\frac{1}{6}$ , and you decide to invest 12 tokens in the Group Fund,

and the other five members of your group invested a total of 60 tokens. This makes a total of 72 tokens in the Group Fund. Then your earnings from the Group Fund would be \$0.84, and your earnings from the Individual Fund would be \$0.03. Your total earnings for the period would be \$0.87. The other members of your group would also receive \$0.84 from the Group Fund.

Example: You begin each period with 25 tokens. Suppose that your share of the group exchange is  $\frac{1}{6}$ , and you decide to invest 20 tokens in the Group Fund, but that the other five members of your group invested no tokens in the Group Fund. Then your earnings from the Group Fund would be \$0.31, and \$0.05. Your total earnings for the period would be \$0.36. The other members of your group would also receive \$0.31 from the Group Fund.

Unequal shares of the Group Fund:

Example: You begin each period with 20 tokens. Suppose that your share of the group exchange is  $\frac{1}{9}$ , and you decide to invest no tokens in the Group Fund, but that the other five members of your group invested a total of 74 tokens. Then your earnings from the Group Fund would be \$0.59, and your earnings from the Individual Fund would be \$0.20. Your total earnings for the period would be \$0.79. The other two members of your group that have a  $\frac{1}{9}$  share would also receive \$0.59 from the Group Fund. The other three members of your group that have a  $\frac{2}{9}$  share of the Group Fund would receive \$1.19.

Example: You begin each period with 15 tokens. Suppose that your share of the group exchange is  $\frac{2}{9}$ , and you decide to invest 14 tokens in the Group Fund, and that the other five members of your group invested a total of 48 tokens. This

makes a total of 64 tokens. Then your earnings from the Group Fund would be \$1.04. and your earnings from the Individual Fund would be \$0.01. Your total earnings for the period would be \$1.05. The other two members of your group that receive a  $\frac{2}{9}$  share would also receive \$1.04 from the Group Fund. The other three members of your group that have a  $\frac{1}{9}$  share of the Group Fund. would receive \$0.52.

#### **Procedure**

Each period you will decide how many tokens to invest in the Individual Fund and how many to invest in the Group Fund. You will write these amounts on the Investment Decision Form, which will then be collected by the monitors. Do not discuss your investment decision with anyone in the room. Only the monitors will learn of your decisions. The monitors will total the amount that your group invested in the Group Fund and write this amount on your Investment Decision Form.

The monitors will return the Investment Decision Form to you with your earnings from the Individual Fund and the Group Fund. Your earnings for the period are your earnings from the Individual Fund and Group Fund added together.

Please carefully record your earnings form the Individual Fund and Group Fund on the Record Sheet after every period. The monitors will also keep track of this information. It is important that both you and the monitors make this calculation and agree. Each experiment will last between 7 and 14 periods, depending on the number written on the card at the front of the room. At the end of the first experiment you will be randomly assigned to a new group with the possibility of

a different number of tokens and a different share of the Group Fund. You will be paid in cash at the end of the experiment.

### Exit Questions and Responses

Subjects were recruited from undergraduate Economics courses at the University of California, Santa Cruz.

- 1) Why/how did you decide to allocate your funds between the Individual Fund and the Group Fund?
- I gambled at first and hoped the group would put in a lot of money. I also experimented to see the kinds of effects different decisions would have. I found a decision making process that seemed pretty stable and I kept using that method until the end of the game, or until I gambled again.
- In the first game I saw that the ROI of the group fund was better than for the individual fund when the group fund has less than 56 tokens, about the same for 56-80 tokens, and worse for more than 80 tokens.
- Depends on my tokens and share. With a higher share and tokens I will put more tokens in the group fund.
- I saw the average the group was contributing and I added more if people added more (and visa versa). First I tried to contribute most of my tokens, but it didn't work.
- First period I looked at the group fund and guessed it would probably be the same next time. Second period I only put money in the individual fund, I think that's the best I could do.
- The individual fund was just playing it safe. I thought that the group would put more tokens towards the end of the game, so I thought I should do that too.

The group fund was riskier but worth more per token.

- 2) Any thoughts/comments?
- Interesting, I assume it's a study relating to the tragedy of the commons.
- If we could do it on computer.
- It seems like the conditions in the second (run) forced me to choose the individual fund every time.
- I think I would put more tokens in the group fund, again, assuming the whole group would do the same to get more money.
  - 3) If you had known the last period would you have done anything differently?
- Yes, switch my group and individual. Just because I got a lower than average result.
  - -Yes, maybe put all my tokens in one fund.
- No, because I could not guess what anyone else was going to do. Anything too extreme would have been too risky.

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