NONLINEAR ANALYSIS OF RC STRUCTURES WITH STRAIN PENETRATION EFFECTS

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ABSTRACT

The strain of the longitudinal reinforcement of flexural members penetrates into the adjacent concrete member, causing a slip of the reinforcement and an end rotation for the flexural member at the intersection. Analysis ignoring the strain penetration effects will underestimate the deflections and member elongation, and overestimate the member stiffness and local response parameters that are used to quantify structural damage (e.g., strains and section curvature). Focusing on the strain penetration of reinforcing bars anchored in footings and bridge joints, this paper describes a hysteretic model for the reinforcing bar stress vs. slip response that can be integrated at the end of fiber-based flexural concrete members. It is shown that the proposed modeling technique is capable of capturing the strain penetration effects by simulating the measured global and local responses of a concrete column, a flexural wall, and a bridge tee-joint system.

Introduction

For reinforced concrete structures subjected to moderate to large earthquakes, capturing the structural response and associated damage require accurate modeling of localized inelastic deformations occurring at the member end regions as identified by shaded areas in Figure 1. These member end deformations consist of two components: 1) the flexural deformation that causes inelastic strains in the longitudinal bars and concrete, and 2) the member end rotation, as indicated by arrows in Figure 1, due to reinforcement slip at the connection interface. The reinforcement slip in well-designed structures results from the accumulative steel strain difference between the bar and concrete within the connecting member.

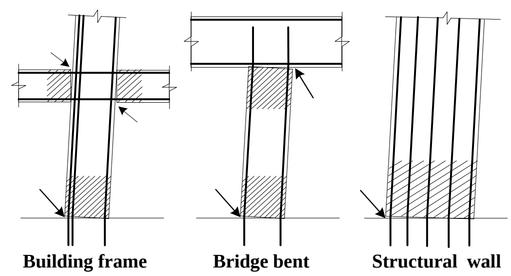

Figure 1. Typical inelastic regions in well-designed concrete structures

Experimental studies have generally reported that this end rotation contributes up to 35

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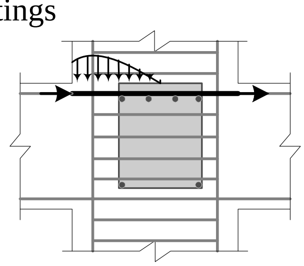
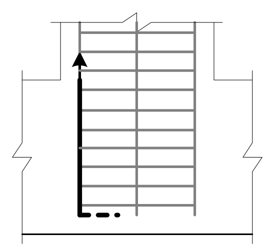
percent to the lateral deformation of flexural members (Calderone et al., 2000, Kowalsky et al., 1999, and Saatcioglu et al., 1992). The strain penetration and the associated end rotation also greatly influence the localized strains and curvature in the critical regions, and stiffness of the flexural member. Historically, strain penetration effects have been neglected in frame analyses of concrete structures. Due to compensation of errors that occur to the resultant force magnitudes and moment arms at the section level, the lateral force resistance of a flexural member may not be significantly affected by this approach. However, it has been shown that neglecting strain penetration effects will substantially overestimate member stiffness, steel strains, and section curvature, all of which are important variables required in modern performance-based engineering (Zhao and Sritharan, 2005, Sritharan et al. 2000). Since the objective of the finite element analyses is to produce satisfactory global and local responses, an accurate representation of the strain penetration effects is critical when developing finite element models of concrete structures.

The efforts to date to consider the end rotation caused by the slip of beam bars anchored in **building joints** range from simple rotational spring models to interface elements formed by nonlinear uniaxial springs to special beam-column elements (Zhao and Sritharan, 2005). On the other hand, studies on the strain penetration effects of longitudinal bars into footings and bridge joints are very scarce. The existing models developed for building joints were not further pursued in this study due to following reasons: 1) the bond-slip behavior of the bars anchored in joints may be greatly different form those in footings as shown in Table 1; 2) the existing models have not proven appropriate for capturing localized parameters needed in performance-based engineering; and 3) the models may not be reliably extended to capture the bond-slip rotation of a general flexural member (that has an arbitrary cross-section and is subjected to multi-directional loading).

Table 1. Summary of different bond-slip behaviors of bars anchored in building joints and footings





Beam bars in building joints	*Column bars in footings and bridge joints*
• Anchorage length limited by column width	• Adequate anchorage length
• Confinement steel parallel to the bars	• Confinement steel perpendicular to the bars
• Tension at one end and compression the other	• Tension/compression at one end
• End bearing not available	• End bearing helps to transfer compression
• Entire bar likely experiences slip	• A portion of the bar experiences slip

To capture the strain penetration effects and the corresponding the member end rotation at the footing and bridge joint interfaces, the fiber-based analysis of concrete members was used in conjunction with a zero-length section element. As shown in Figure 2, the zero-length section element is located at the intersection between a flexural member and a footing or a cap beam of a bridge bent. Because of not imposing any limits to the shape of the zero-length section element and the loading direction, this approach can be used to model the end rotation that occurs to any flexural member subjected to multi-directional loading. In addition, detailed local response information, such as concrete/steel strain and section curvature, can be easily obtained from the analysis. In this paper, the analysis technique is briefly reviewed followed by a description of a bar stress vs. slip relationship that is critical to suggested analysis technique. Using a cantilever column, a flexural wall, and a bridge t-joint system, it is shown that fiber-based analyses incorporating zero-length section elements with the proposed constitutive models can accurately capture both the global and local responses of concrete structures.

Modeling Strain Penetration in Fiber-Based Analysis

Fiber-based Analysis

The lateral load response of concrete flexural members was modeled using the nonlinear fiber-based beam-column element available in OpenSees (2005). In the fiber-based analysis method, the flexural member is represented by unidirectional steel and concrete fibers. Because the steel and concrete fiber responses are specified in the direction of the member length, the fiber analysis concept is suitable for modeling any flexural members. Although flexure-shear interaction is not typically integrated in the element formulation and the built-in plane section assumption may not be appropriate for modeling response of some flexural members and response of shear-dominant members, fiber analysis remains the most economic and accurate means to capture seismic behavior of concrete structures (Spacone et al. 1996a, 1996b).

The fiber analysis typically follows the direct stiffness method, in which solving the equilibrium equation of the overall system yields the nodal displacements. After the element displacements are extracted from the nodal displacements, the element forces are determined and the member stiffness is upgraded, based on which the global stiffness matrix is assembled for the next time step. The stiffness and forces of the fiber-based elements are obtained by numerically integrating the section stiffness and forces corresponding to a section deformation (i.e., axial strain $\bar{\varepsilon}$ and curvature ϕ).

The section deformation is calculated through interpolating the element end deformations (i.e., displacement and rotation) at the integration points. From the section deformation, the strain in each fiber (ε) is obtained using the plane sections remain plane assumption. For example, the fiber strain is calculated using, $\varepsilon = \bar{\varepsilon} + \phi y$, where y is the distance of the fiber from the centroid of the section. The fiber stress and stiffness are updated according to the material models, followed by upgrading of the section force resultant and the corresponding stiffness. The neutral axis position of the section at an integration point is determined through an iterative procedure, which balances the force resultants at the section level as well as at the member level.

A zero-length section element contains one section (that corresponds to one integration point), which defines the stress resultant–deformation response of the whole element. A zero-length section element is usually used to calculate the moment-curvature response of a flexural member. The zero-length section element available in OpenSees is assumed to have a unit length such that the element deformations (i.e., elongation and rotation) are equal to the section deformations (i.e., axial strain and curvature). Therefore, a zero-length section element inserted between the flexural member and its footing can be used to model the member end rotation.

Unlike the material models required by regular fiber-based beam-column elements, a stress vs. displacement relationship is required for each steel fiber to upgrade the forces and stiffness of the zero-length section element. This relationship essentially represents the bond-slip behavior of fully anchored steel reinforcements.

Bar Stress vs. Slip Model

To capture the strain penetration effects under multi-directional load using the zero-length section element available in OpenSees, a model to describe the bar stress vs. loaded end slip response has been recently introduced using pull out test data and measured response of well-designed columns(Zhao and Sritharan 2005). In this model, the envelope of the bar stress vs. the slip response at the end of the flexural member is characterized using the function included in Figure 3 with a hysteresis rule presented in Figure 4. The slip when the bar stress reaches the yield (s_y) and ultimate strengths (s_u) are obtained from:

$$s_y = 0.1\left(\frac{d_b}{4}\frac{f_y}{\sqrt{f_c'}}(2a+1)\right)^{1/a} + 0.0134 \tag{1}$$

$$s_u = 35 s_y \tag{2}$$

where d_b = bar diameter (in.); f_y = yield strength of bar (ksi); f'_c= concrete strength of footing (ksi); and α = the parameter used in the local bond-slip relation and was taken as 0.4 in this study in accordance with CEB-FIP Model Code 90 (FIB Task Group 5.2 2000).

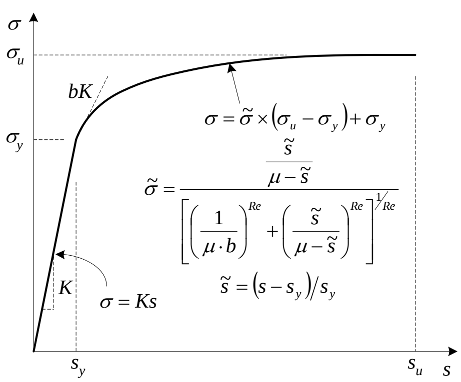


Figure 3. Bar stress vs. loaded end slip for fully anchored reinforcing bars into footings.

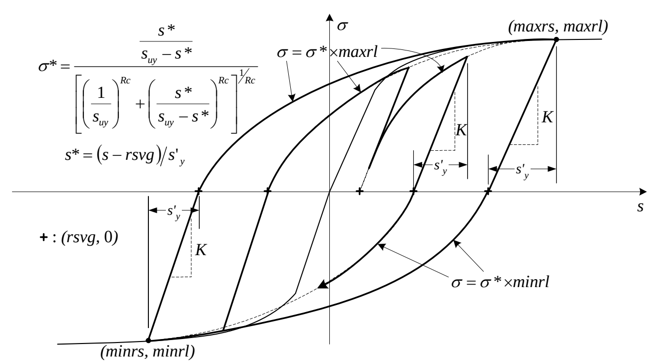


Figure 4. Hysteric rule for the bar stress vs. loaded end slip of fully anchored bars into footings.

Summary of Sample Analyses

To demonstrate the applicability of the zero-length section element with the proposed material models and the corresponding improvements to the analysis results, cyclic responses of three structural components were simulated using OpenSees (Ver. 1.5) and the results were compared with the experimental data. For all examples, the existing Concrete01 and Steel02 elements in OpenSees were used, respectively, to model the concrete and steel fibers of the beam-column elements.

Cantilever Column

The column investigated here was that tested by Smith (1996), which served as the reference column for an investigation on strategic relocation of plastic hinges in bridge columns. The column had a circular section with a 24-inch diameter as shown in the insert of Figure 5 and contained 22 # 7, Grade 60 longitudinal bars and # 3 spiral reinforcement at a spacing of 2.25 inches. The clear height of the column was 144 in. above the column footing. Under constant axial load of 400 kips, the yield displacement of the column was reported to be 1.6 in. and the corresponding lateral load resistance was 58 kips. The failure of the column occurred due to fracture of the longitudinal reinforcing bars at the column base, after attaining lateral displacement of 12.7 in. with lateral resistance of 80 kips.

Figure 5(a) compares the measured column top lateral displacement versus lateral force resistance with the analysis results, which were obtained with and without the zero-length element to capture the strain penetration effects and by modeling the column using five fiber-based beam-column elements. The analysis with the zero-length section element (with model parameters of s_y = 0.022 in., f_y = 66 ksi, b = 0.5, and R_c = 1.0) more closely captured the measured response. In the pull-direction of loading, this analysis accurately predicted the lateral force resistance at the yield and maximum lateral displacements. In the push-direction, the analysis appears to have somewhat overestimated the maximum force resistance due to the measured load resistance in this direction being slightly smaller than the pull direction. On the other hand, the analysis that ignored the strain penetration effects overestimated the ultimate lateral load resistance and greatly underestimated the column lateral deflection for a given lateral load.

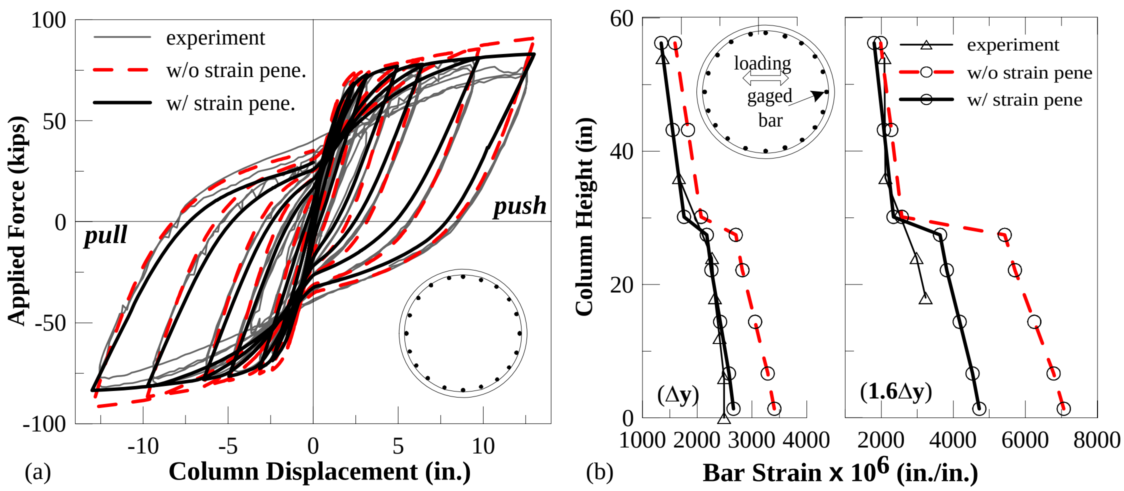


Figure 5. Comparison of experimental and analytical results for a cantilever column
(a) force vs. displacement; (b) strain distribution of an extreme bar

The column end rotation due to strain penetration reduces stress in the column longitudinal bars, which is evident in Figure 5(b). At the column yield displacement, the analysis that included the strain penetration effects correctly captured the strain distribution along a longitudinal extreme bar. The corresponding analysis without the strain penetration effects overestimated the bar strains in the plastic hinge region by about 30 percent. The strain gages in the hinge regions gradually failed when the column was subjected to inelastic displacements. Using the available data obtained at a column lateral displacement of 2.5 in., Figure 5(b) compares the measured stain data with the calculated strain profiles. Again the analysis with the zero-length section element produced strains that closely matched with the measured strains along the bar. The analysis that ignored the strain penetration effects overestimated the bar strains by as much as 50%. The measured strains at the two locations are smaller than the predicted values by the analysis that included the strain penetration effects. This discrepancy may be due to the effects of tension stiffening, which were ignored in the OpenSees analysis.

Flexural Wall

Presented below is the fiber-based analysis of RW2 tested by Thomsen and Wallace (1995). RW2 was the second of two rectangular walls with the cross-section details as shown in Figure 6. The height of RW2 was 144 in., and the wall was tested under cyclic lateral displacements and constant axial load of 85 kips. The RW2 was modeled in OpenSees using nonlinear beam-column elements and existing material models for concrete and steel reinforcement fibers. The zero-length bond slip element described above (with the model parameters of s_y = 0.022 in., f_y = 66 ksi, b = 0.5, and R_c = 1.0) was used to capture the strain penetration effects; the shear deformation of the wall was, however, not modeled.

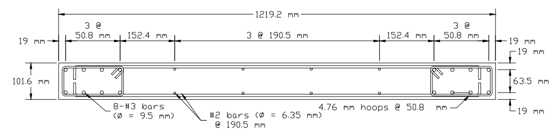


Figure 6. Cross section of RW2 tested by Thomsen and Wallace.

A comparison of measured force-displacement response against the simulated response for RW2 under cyclic lateral loading is shown in Figure 7. The analysis result shows good agreement with the measured response. Small discrepancies exist, which was believed to be due to the fact that shear-flexure interaction was not considered in the modeling. The inadequacies of the material models in the current version of OpenSees might have also contributed to the discrepancies shown in the comparison.

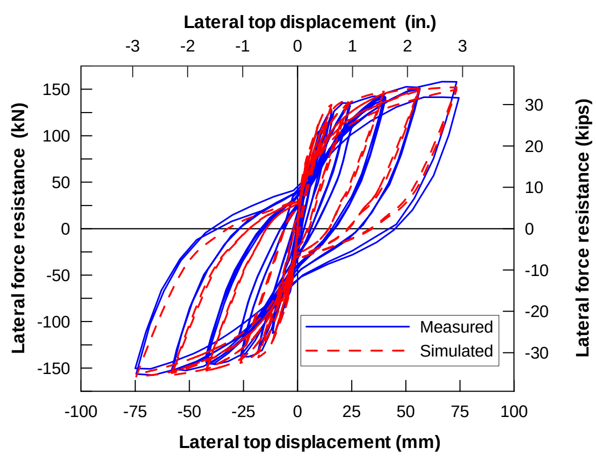


Figure 7. Measured vs. simulated response of RW2.

Bridge Tee-Joint System

A bridge tee-joint system (specimen IC1) tested in an inverted position by Sritharan et al. (1996) was studied to verify the feasibility of the proposed model for analyzing a structural system. This specimen with a conventional reinforced concrete cap beam, as schematically shown in Figure 8, evaluated a new design method suitable for bridge cap beam-to-column joints. Under constant axial load of 90 kips, the column was subjected to cyclic lateral loading at a height of 72 in. above the column-to-cap beam interface. The yield lateral displacement for the tee-joint system was reported to be 0.7 in. with the corresponding lateral resistance of 56 kips. The test joint experienced strength deterioration at lateral displacement of 4 in. due to formation of large joint cracks and subsequent joint damage.

The simulation model included six fiber-based beam-column elements for the cap beam and four beam-column elements for the column. An additional fiber-based beam-column element with the elastic column section properties modeled the joint. The zero-length section element (with the model parameters of s_y = 0.02 in., f_y = 65 ksi, b = 0.5, and R_c = 0.7) was located between this elastic element and the adjoining column element. Because of the straight end anchorage of the column longitudinal bar, a reduced R_c value was used as recommended in Zhao and Sritharan (2005).

Figure 8(a) compares the measured force-displacement hysteresis response of the test unit with the analytical results obtained with and without the strain penetration effects. The analysis, which included the strain penetration effects, produced the force-displacement response that closely matched with the measured response in both loading directions. The joint shear failure experienced by the test unit towards the end of testing was not included in the analytical model, and hence the analysis slightly overestimated the force resistance at the maximum displacement. On the other hand, the analysis that did not include the strain penetration effects overestimated both the lateral load resistance and the unloading and reloading stiffness.

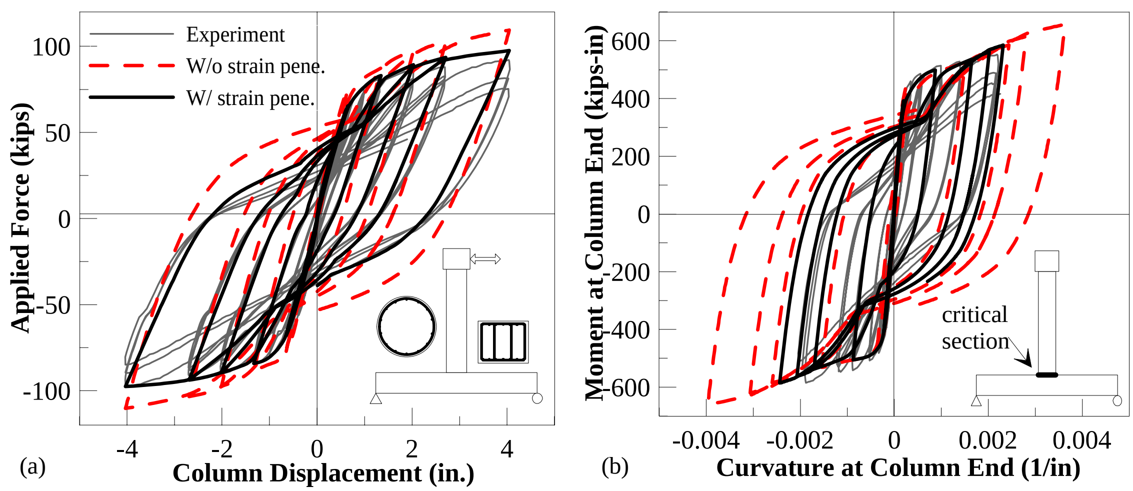


Figure 8. Comparison of experimental and analytical results for T-joint specimen IC1
(a) force vs. displacement; (b) moment vs. column end curvature

The advantages of incorporating the strain penetration effects in the analysis is more pronounced in Figure 8(b), in which the column moment vs. curvature histories at the beam-to-column intersection are compared. The analysis that ignored the strain penetration effects overestimated the column end curvature by approximately 90% towards the end of the test, indicating that the bar slip due to strain penetration significantly affects the local response measures that are indicative of damage to the plastic hinge region. A significant improvement to the moment-curvature response prediction was obtained when the analysis included the strain penetration effects. However, the predicted moment-curvature hysteretic loops are somewhat broader along the reloading path prior to intersecting the curvature axis. This discrepancy is expected to be diminished when the values of the model parameters, especially s_u, b, and R_c, are refined.

Conclusion Remarks

Well-designed flexural concrete members experience rotations at the fixed end(s) due to the slip of longitudinal reinforcements at the member interfaces. The slip results from strain penetrating along fully anchored longitudinal bars into the adjoining concrete members. Focusing on column and wall longitudinal bars anchored in footings and bridge joints, an efficient method is presented in this paper to model the bond slip rotation using a zero-length section element in nonlinear fiber-based analysis of concrete structures. A constitutive model that expresses the bar stress vs. loaded-end slip response was described for the steel fibers of the zero-length section element.

Advantages of the proposed method to improve fiber-based analysis of concrete structures were demonstrated by simulating cyclic response of a concrete cantilever column, a flexural wall, and a bridge tee-joint system. Simulated responses were compared with the observed responses at both global and local levels. The analyses that utilized the proposed method to model the strain penetration effects satisfactorily captured the deflections, force vs. displacement hysteresis responses, strains in the longitudinal reinforcing bar and section curvature of the test units. When the strain penetration effects were ignored, the force resistance at a given lateral displacement was overestimated, along with portraying larger hysteresis loops. Most importantly, the local response parameters such as the steel strain and section curvature, which indicate the extent of structural damage, were grossly overestimated, indicating that the strain penetration effects should not be ignored in the analysis of concrete members. The zero-length section element incorporating the proposed constitutive model for the steel fibers can be used in nonlinear fiber-based analysis to accurately capture the strain penetration effects and thus the global and local responses of concrete flexural members. The presented method is versatile because it can be used for modeling concrete flexural members without limiting cross-sectional shapes or direction of the lateral load.

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