



## **EFFECT OF SERVOVALVE/ACTUATOR DYNAMICS ON DISPLACEMENT CONTROLLED TESTING**

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### **SUMMARY**

This paper presents a study of the use of servo-hydraulic actuators in the implementation of dynamic testing techniques in displacement control. Mathematical models for a displacement-controlled test system are presented and used to investigate the influences of the servo-system on the overall system. The influence investigated includes response amplitude reduction, system stability and controller gain setting, system response delay, and nonlinearities in servo-systems. Linear system analysis and computer simulation were conducted to explain and predict the system response. While the emphasis throughout is on the development of an understanding of the influences, a simple first-order phase-lead network was used to compensate for the amplitude reduction and response delay for tests with small hydraulic demands. The results indicated that the influences can be predicted and compensation schemes can be developed with the presented models.

### **INTRODUCTION**

The performance of many seismic mitigation devices (i.e., passive and semi-active dampers) is highly dependent on the rate of loading. High-speed/real-time dynamic loading is necessary for testing such devices [1-4]. In addition, nonlinear structural behaviors are, to a certain extent, sensitive to strain rates [5]. Shake tables are often used to simulate the dynamic effects of earthquakes on structural models. However, the size of a structure that can be tested is often limited by the size and payload capacity of the existing tables. Testing methods such as the real-time pseudodynamic testing (RPsD) method have been studied to facilitate the investigation of the dynamic effects of earthquakes on structures and structural subassemblages. In an RPsD test, hydraulic actuators are used to apply calculated displacements to the test structure, and the displacement commands are updated continuously by solving the governing differential equation of the test structure using measured restoring forces. Meanwhile, high-speed cyclic

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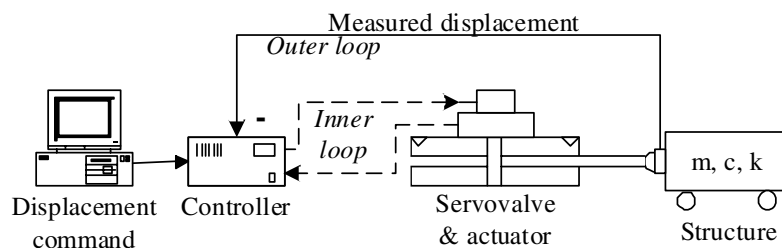
tests have been reported in the literature to investigate the strain-rate effects on concrete beam-column joints [6].

In these testing systems, hydraulic actuators are in displacement control (i.e., the command signal to the servovalve is based on the difference between the command displacement and the measured displacement). Servo-systems (i.e., actuators, servovalves and their controllers) can have significant influence on a displacement-controlled testing system. For example, the actuator response delay can significantly aggravate the accuracy of RPsD, and even cause instability [8]; the servovalve controller gains required can be affected by the size of the structural mass [7, 9]; and the maximum achievable loading rate is limited by the servovalve size [6]. Although individual studies have provided some solutions to these problems, little has been done to systematically investigate the effect of servo-systems on dynamic tests in displacement control.

This paper first presents mathematical models for a displacement-controlled test system along with experimental validation. Computer simulation and linear system analysis are used to investigate the potential influences of the servo-system on the behavior of the test system. The influences investigated include response amplitude reduction, system stability and controller gain setting, system response delay, and nonlinearities in servo-systems. The emphasis throughout is on the development of an understanding of these influences rather than detailed compensation techniques.

## TESTING SYSTEMS IN DISPLACEMENT CONTROL

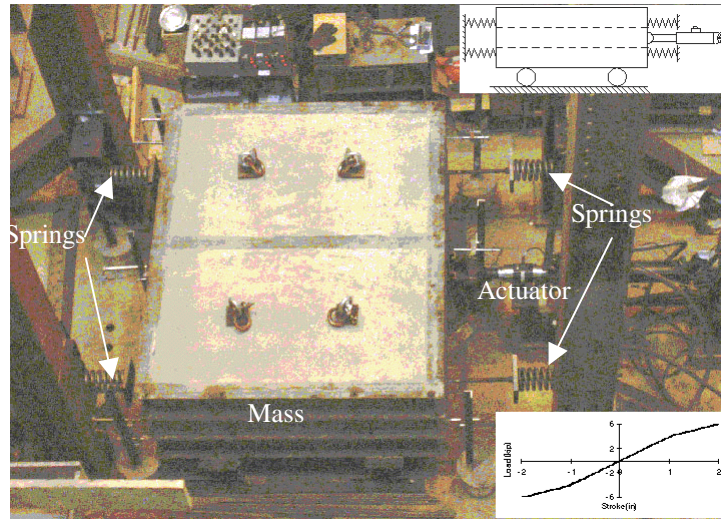
Figure 1 schematically shows a test system using servo-hydraulic actuation: a servovalve controller compares a command signal (target displacement) to a feedback signal (measured displacement) and sends a current proportional to the difference (DC error) to a servovalve to drive the valve spool. The spool regulates the hydraulic flow entering the actuator, causing differential fluid pressure across the actuator piston. The pressure difference between the two chambers multiplied by the actuator piston area produces the force applied to the test structure. The structure/piston displacement measured by an LVDT mounted in the actuator house is fed back to the controller to close the outer control loop. For the three-stage servovalve used in this study, an inner control loop exists to control the servovalve spool position.



**Figure 1 Schematics of a displacement-controlled test system**

The servo-system in this study included a 156 kN (35 kip) MTS 244.23 actuator with a 341 lpm (90gpm) MTS 256.09 servovalve controlled by an MTS 407 analog controller. To facilitate the comparison of experiment and simulation the test structure simply consisted of a 7080 kg (15.6 kip) concrete mass atop four caster wheels with two springs (175.2 kN/m (1 kip/in.)) on each side of the structure in the direction of motion as shown in Figure 2. The initial natural frequency of the system was measured through free vibration tests as 1.58 Hz. The springs were arranged such that the structural stiffness would decrease from 700.8 kN/m (4kips/in.) to 350.4 kN/m (2 kips/in.) when the structural displacement exceeded 2.5 cm (1 in.) as shown in the lower right inset of Figure 2. An automobile

suspension strut was used as a damper, and the measured viscous damping and friction force of the system were 0.8% of critical damping and 95.7 N (21.5 lbs), respectively. An equivalent viscous damping of 1.1% was used to simplify the analysis.

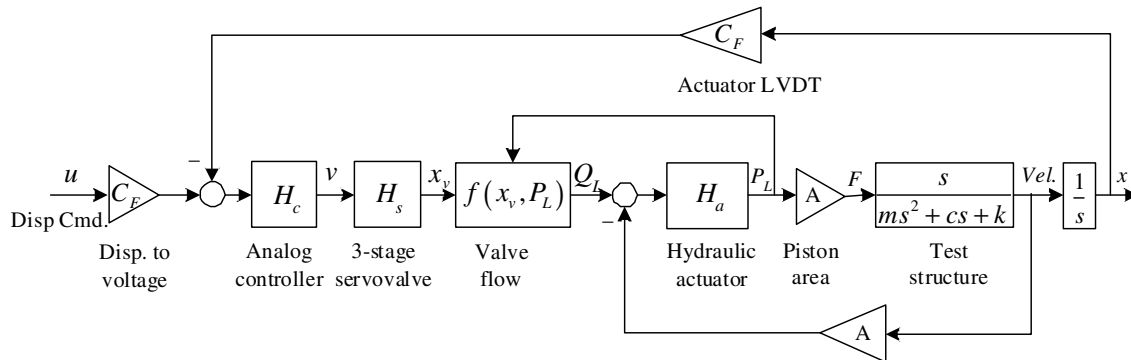


**Figure 2** Test structure with the servo-hydraulic actuator

## TEST SYSTEM MODELS

### Models for computer simulation

The test system is represented using block diagrams in Figure 3, which shows the relations between system components (blocks with inputs and outputs labeled). To simplify the presentation, the component models are shown in transfer functions (representations of differential equations in the frequency domain). The mathematical models of the system have been derived in Zhao et al. [10] and Zhao [11] based on the formulations by Merritt [12]. This section briefly reviews these models.



**Figure 3** Block diagram model of the test system

The main function of the analog controller is Proportional-Integral-Derivative (PID) control, which applies appropriate gains to the DC error to determine the spool drive current to be sent to the servovalve. The integral control usually reduces the steady-state tracking error. The transient response of the structure/system was of interest; hence the integral gain was set to zero. The dynamics of the PID control with zero I gain were represented by

$$H_c = G_p + G_d s, \quad (1)$$

where  $G_p$  and  $G_d$  are the proportional and derivative gain of controller, respectively and  $s$  is the complex variable, which represents the frequency of displacement commands.

The servovalve dynamics relate the valve spool positions ( $x_v$ ) to valve commands ( $v$ ). Based on the physical operation of the servovalve, the dynamics of the three-stage servovalve were modeled by a second-order transfer function,

$$H_s = \frac{K_{vp}}{\tau A_v s^2 + A_v s + K_3 K_{vp}} \frac{1}{x_{vmax}}, \quad (2)$$

where  $\tau$  is the equivalent time constant of the pilot-stage valve,  $K_{vp}$  is the pilot-stage valve flow gain,  $A_v$  is the main-stage spool area,  $K_3$  is the sensitivity factor of the internal LVDT, and  $x_{vmax}$  is the maximum spool stroke. Refer to Zhao [11] for a detailed description and identification of the parameters.

The servovalve spool positions determine the hydraulic flow to the actuator, and the nonlinear servovalve flow characteristics were represented by

$$Q_L = f(x_v, P_L) = K_v x_v \sqrt{1 - \frac{x_v}{|x_v|} \frac{P_L}{P_s}}, \quad (3)$$

where  $x_v$  is the spool opening of the servovalve (-1 to 1),  $K_v$  is the no-load flow gain of the servovalve, which is a function of spool opening,  $P_L$  is the load pressure ( $P_L A$  is approximately the force applied to the structure, and  $A$  is the actuator piston area), and  $P_s$  is the supply pressure. The servovalve flow relation includes two types of nonlinearity: the **load pressure influence** expressed by the square root term and the **nonlinear no-load flow gain** ( $K_v$ ) [11].

The actuator dynamics are controlled by the law of conservation of mass: the hydraulic flow into the actuator needs to counteract the fluid compressibility, system leakage, and actuator chamber volume change. Hence,

$$H_a = \frac{1}{K_a s + C_l}, \quad (4)$$

where  $K_a$  is the compressibility coefficient of the hydraulic fluid inside both actuator chambers and  $C_l$  is the total leakage coefficient of the servovalve/actuator combination.

With these models and the carefully identified parameters [11], computer simulations were conducted with SIMULINK<sup>®</sup> 3.0, a dynamic system simulation toolbox for MATLAB<sup>®</sup> version 6.0. In addition, linear system analysis was conducted to provide closed form derivations and solutions.

### Models for linear system analysis

The dynamic system modeled by high-order nonlinear differential equations can be approximated within a useful, though limited, range as a simple linear system. The controller D gain was usually set very small (e.g., 0.2 ms) in this study because too large of a D gain would amplify high-frequency signal

(e.g., electric noise). Hence, the controller dynamics were simplified as a pure gain for the frequency range of interest (0-10 Hz),

$$H_c = G_p \quad (5)$$

For low frequencies (i.e., 0-10 Hz in this study), the servovalve dynamics shown in Eq. (2) can be represented by a linear relation,

$$H_s = K_s . \quad (6)$$

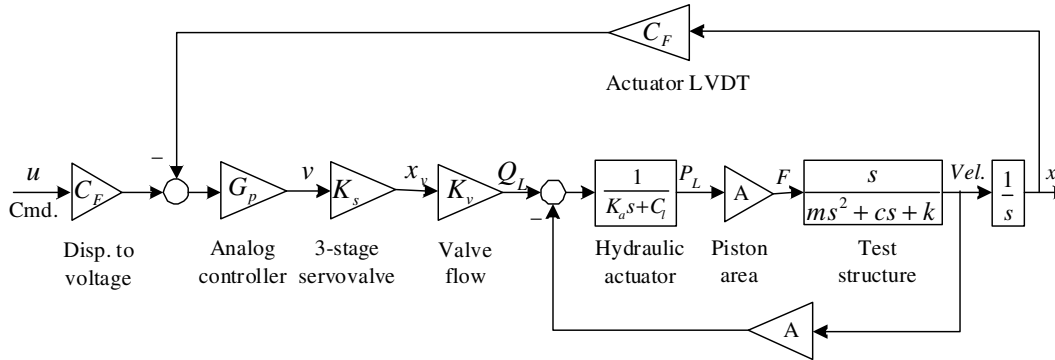
Note that close-formed analysis with this simplified relation needs to be verified using computer simulations with the higher-order model because the servovalve dynamics omitted here can be important.

When the load pressure is small such that the square root term is close to unity, the nonlinear flow model shown in Eq. (3) can be simplified to the linearized approximation about the null position of the servovalve spool; hence,

$$Q_L = K_v x_v . \quad (7)$$

In addition, when the servovalve spool operates near its neutral position, the initial slope of the flow vs. spool opening curve can be used for  $K_v$ .

The simplified models shown in Eqs. (5)–(7) can greatly simplify the system analysis near the system neutral position, where the system operates most of the time. Figure 4 shows the system model with the simplified component dynamics. Using control engineering techniques such as frequency response, this model can provide insight to the system behavior and the effect of the servovalve system on the test system. The information obtained through computer simulation and linear analysis was validated in the laboratory using the SDOF structural model.



**Figure 4 Simplified block diagram model of the test system**

## OVERALL SYSTEM DYNAMICS

For the test system in displacement control as shown in Figure 4, a transfer function ( $G_{xu}$ ) from the command position ( $u$ ) to the displacement response ( $x$ ) in the frequency domain was derived as

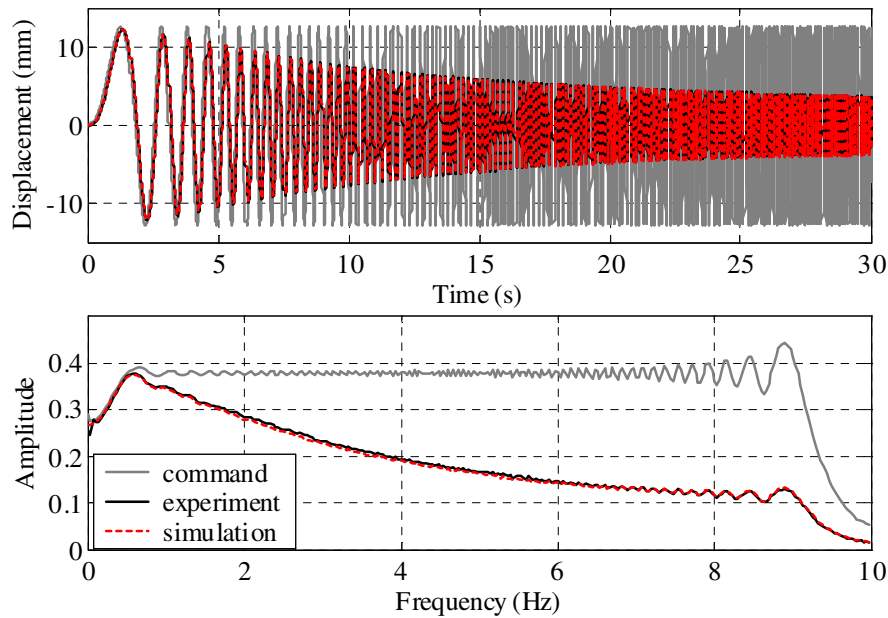
$$G_{xu}(s) = \frac{AK_v C_F K_s G_p}{(C_l + K_a s)(k + cs + ms^2) + A^2 s + AK_v C_F K_s G_p}. \quad (8)$$

The first term of the denominator,  $(C_l + K_a s)(k + cs + ms^2)$ , is much smaller than the rest especially when the excitation frequency is around the natural frequency of the test structure. Thus the system can be approximated using a first-order transfer function with a cut-off frequency ( $\omega_c$ ) formulated as

$$G_{xu}(s) = \frac{1}{s/\omega_c + 1} \text{ and } \omega_c = \frac{K_v C_F K_s G_p}{A}. \quad (9)$$

The cut-off frequency was calculated as 1.96 Hz for the system in this study.

Figure 5 shows both the simulation and experimental results of the test system under a 12.7 mm sinesweep excitation (0-10 Hz). The close match between the simulation and experimental results indicates that the above models can accurately represent the physical system under certain conditions.



**Figure 5 Responses of the system subjected to a 12.7-cm sinesweep input (0-10 Hz)**

## EFFECTS OF SERVO-SYSTEM ON TEST SYSTEM

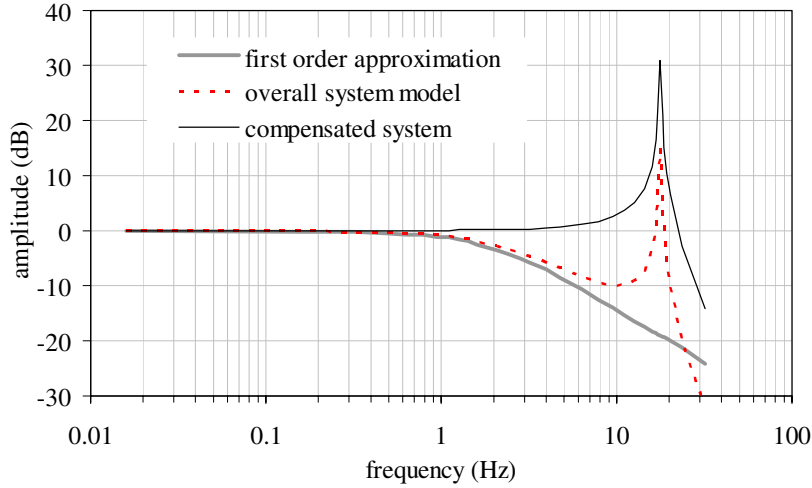
### Amplitude reduction

The amplitude of the measured and simulated displacement reduces as the excitation frequency increases as shown in Figure 5. The reduction begins at zero frequency and reaches approximately 30% at 2 Hz. Other researchers have also observed the amplitude reduction of displacement-controlled systems, similar to the results shown in Figure 5 [6]. In tests where the structural mass to which the actuator is attached is small compared to the actuator loading capacity, the system dynamics is first-order dominant for low frequencies (e.g., less than 10 Hz in this study). Hence, the amplitude reduction may be estimated using

$$AR(\omega) = 1 - \frac{1}{\sqrt{1 + (\omega/\omega_c)^2}} \quad (10)$$

where  $\omega$  is the frequency of the displacement command [7].

Computer simulation shows that the servovalve capacity cannot be fully utilized if the test does not incorporate any compensation. The amplitude reduction may be compensated by modifying the command signal using a first-order phase-lead network [7]. Figure 6 shows the frequency response of the test system (Figure 4), the simplified system with the first-order approximation, and the compensated system. It can be seen that the response roll-off is removed with the amplitude compensation. It should be noted the compensation cannot be applied to tests where the displacement command is close to the actuator capacity (e.g., tests with 0-10 Hz sinesweep command with full actuator stroke), when the amplified command signal might exceed the limit of the voltage signal the controller can handle.



**Figure 6 Frequency response of the system with amplitude compensation**

Meanwhile, the amplitude reduction is affected by a slightly damped vibration mode due to the first term in the denominator of the overall system transfer function (Eq. (8)) that was neglected in the amplitude reduction estimation (Eq. (10)). This vibration mode is related to the so-called oil-column resonance with a frequency of

$$\omega_n = \sqrt{A^2/mK_a} \quad (11)$$

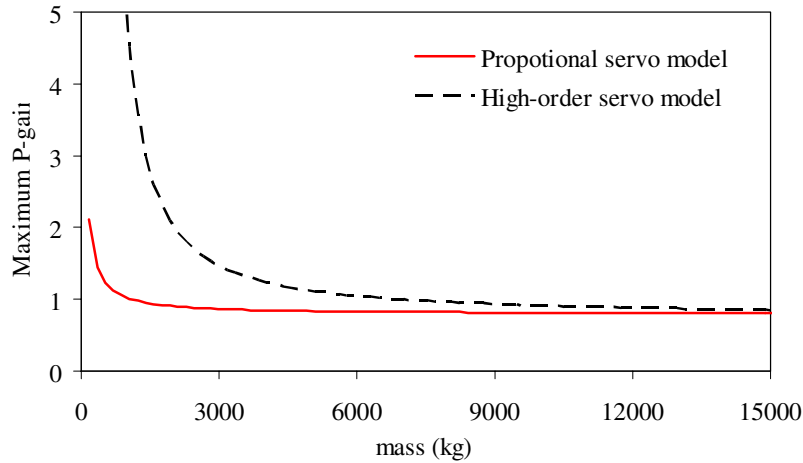
where  $K_a$  is the compressibility coefficient of the hydraulic fluid inside both actuator chambers. The amplitude reduction to be compensated is typically smaller than that estimated using Eq. (10). As a result, the time constant required by the phase-lead network needs to be finalized through both linear system analysis and computer simulation when the frequency range of the command signal to be compensated is known.

### Stability and controller gain setting

Equation (9) indicates that the system performance may be improved by tuning up the controller P-gain ( $G_p$ ) such that the cut-off frequency shifts far away from the frequency of interest. However, there exists a limit on  $G_p$  for a given testing system because larger P-gain can cause instability. Similar

observations of limited maximum achievable P-gain have been made in the literature [9, 13]. For example, Thewalt et al. found that the maximum P-gain setting was affected by the mass to which the actuator was attached, while the test system with a small P-gain had poor responses. The problem was solved by hanging part of the structural mass, such that the mass to which the actuator is attached was reduced while preserving the correct gravity loading [9].

Instead of determining the controller gains through trial and error, system analysis can be used to provide guidelines for controller gain setting. Following Routh's stability criterion [14], a relation between  $G_p$  and the system parameters can be formulated. Theoretically,  $G_p$  can be set large if the physically installed mass is very small (e.g. where only actuator piston mass is present). However as indicated in Figure 7, the maximum P-gain decays exponentially with an increase of the physical mass (the mass attached to the actuator). In practice, the physical mass, even when the substructuring technique is used, may be large enough to limit the P-gain setting such that the system performance is significantly affected.



**Figure 7 Relation between the maximum P-gain and the physical mass in terms of stability criterion**

### Time delay of the system

The first-order approximation of the overall system dynamics shown in Eq. (9) indicates that the influence on the position-tracking ability of actuators due to overall system dynamics has an equivalent aspect, response delay, in addition to amplitude reduction. For a first-order system with a cut-off frequency of  $\omega_c$  (in rad/s), the system response delay can be estimated by

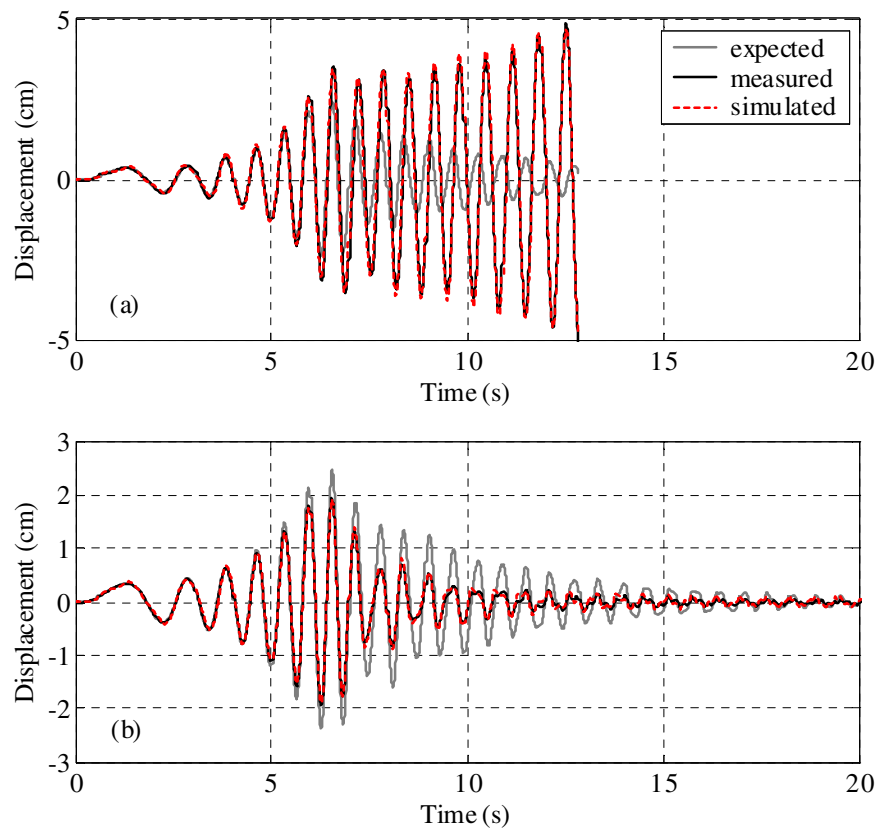
$$T_d = \frac{1}{\omega_c} = \frac{A}{K_v C_F K_s G_p}. \quad (12)$$

This value was calculated as 81 ms for the system in this study. Because the compensation of the amplitude reduction using a first-order phase-lead network also adjusts the phase of the command signal, the system response delay has been partially compensated when the amplitude reduction compensation is incorporated. The reason that the response delay cannot be fully compensated using the amplitude reduction compensation is the impact the physical mass has on the amplitude reduction, that is, the oil column resonance affects the system amplitude response, while it has little influence on the system phase response.



The remaining part of the response delay would merely cause a phase delay in the structural response with respect to the displacement command in the tests where displacement commands are predetermined as in Dhakal and Pan [6]. However, the remaining response delay would affect an RPsD system because the online determination of the displacement command requires measured structural responses [8, 15-16]. Horiuchi et al. noted that the total system energy would increase if delayed responses were used to calculate restoring forces. The concept was proven by computer simulations with an electronic delay element rather than experimental studies such that the influence of the high-order system dynamics (i.e., the oil-column resonance) was not reflected in their study. The studies by Nakashima et al. [1] showed that compensation for a portion of the total delay was made to obtain a good amplitude match to command signals. The effect of the remaining part of the delay was not observed because the calculated displacement instead of the measured displacement was used to compute the restoring force somewhat similar to the case of predetermined displacements.

The influence of the response delay on displacement-controlled testing systems was investigated experimentally by Zhao et al. using an elastic structure [7]. Because the forces directly measured by the actuator load cell include inertial forces and damping forces in addition to restoring forces, the measured displacement multiplied by a constant structural stiffness was used to determine the restoring force. Although the study was limited to an elastic structure, the measured response was used in solving the governing differential equation of the structure; hence, the influence of the system response delay was clearly demonstrated as shown in Figure 8.



**Figure 8. System response with (a) undercompensation of delay and (b) overcompensation of delay.**

A similar phase-lead network to the one used for the amplitude compensation was used to implement delay compensation, and the test results are shown in Figure 8 along with simulation results. The response of the undercompensated system (i.e., the compensated delay was less than the response delay) indicated a reduced system damping, which corroborated with Horiuchi's hypothesis that using delayed response can introduce negative damping to the test system. The negative damping in the test eventually caused the system response to become unstable. On the other hand, the test results for the overcompensated case (i.e., the compensated delay was more than the response delay) indicated that overcompensation for the response delay would increase the system damping and result in incorrect structural responses.

### **Nonlinearity in servo-systems**

Significant nonlinearities such as the nonlinear servovalve flow property and load pressure influence can affect the system behavior when the test involves large flow demands [7, 11]. As the required spool opening increases, the flow gain  $K_v$  reduces, leading to reduced performance of the servo-system. Thus the response delay determined using the linearized system models and the initial flow gain may not be adequate throughout the operating range of the servo-system. The load pressure also affects the servo-system performance by limiting the hydraulic flow into the actuator.

These nonlinearities were observed in some tests that required large spool opening. For example, in the test shown in Figure 8 (a), the maximum spool opening reached 28%. Both nonlinear flow properties of the servovalve and load pressure influence were needed in the computer simulation to match the measured results. These results indicate that the nonlinearities of servo-systems can have significant impact on the implementation of displacement-controlled testing systems, and more advanced compensation schemes are necessary for these cases.

## **CONCLUSIONS**

For displacement-controlled testing, overall system dynamics cause the amplitude reduction and response delay. The system dynamics includes that of the test structure as well as the servo-system though the contribution from the servo-system is usually dominant. The effect of the test structure becomes important when the oil-column resonant frequency is close to the frequency of interest. With properly identified parameters, the models presented in this paper can be used to estimate the amplitude reduction and response delay.

The compensation for amplitude reduction is necessary for high frequency cyclic testing with predetermined displacement commands to fully utilize the capacity of the test equipment. Furthermore, the response delay must be compensated if the measured response is used in determining the command displacement as in RPsD testing for the next step, otherwise, the system damping can be greatly reduced, thus causing stability problems. Meanwhile, overcompensation of the response delay would provide additional energy dissipation in the system, and cause incorrect test results.

The amplitude reduction/response delay cannot be eliminated by increasing the controller P gain because the mass to which the actuator is attached affects the system dynamics, and too large of a P gain may cause instability. The system response delay cannot be fully negated by the amplitude reduction compensation because of a vibration mode in the system (so-called oil column resonance). Simple first-order phase-lead networks can be used to compensate the system if hydraulic demands are small during the test; otherwise, more advanced compensation schemes are necessary to account for nonlinearities in servo-systems.

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