<u>Lateral Boundary Conditions</u>

Learning Objectives

Following this lecture, students will be able to:

- Understand the meteorological phenomena and simulation configurations for which different lateral boundary condition formulations are most appropriate.
- Conceptualize how lateral boundary conditions can influence a model forecast.
- Design a model simulation with appropriately placed and specified lateral boundaries.

Introduction

For any non-global numerical simulation, the simulation domain is finite. Consequently, some means of handling the outermost extent of the simulation domain – its *lateral (or side) boundaries* – must be employed. There are five general classes of lateral boundary conditions:

- **Specified**: Time-varying values of all dependent variables are prescribed on each lateral, or side, of the simulation domain. Specified lateral boundary conditions are widely used in real-data model simulations.
- **Periodic**: Information exiting the simulation domain on one end returns into the simulation domain on the opposite end. These lateral boundary conditions are often used in idealized model simulations but are never used in real-data model simulations.
- **Rigid**: One or more sides are represented by impervious walls through which nothing can pass. Unless mitigated by artificial damping, phenomena that impinge on a rigid boundary are typically reflected away from the boundary back into the model domain. These lateral boundary conditions are used only in idealized simulations on laterals where all simulated phenomena travel tangent to the rigid boundary (known as *channel model simulations*).
- **Open**: Most appropriate in idealized simulations of meteorological phenomena isolated in the simulation domain's interior, with zero prescribed flow tangent to the lateral boundary. These lateral boundary conditions allow for outward-propagating waves, especially gravity waves, to exit the simulation domain rather than be reflected into the domain.
- **Symmetric**: Appropriate only for idealized simulations of meteorological phenomena that are quasi-symmetric along and do not have flow across their major axis, such as an intense tropical cyclone. These lateral boundary conditions specify that there is no flow normal to the boundary, with all other fields mirrored across the boundary.

Specified lateral boundary conditions may be obtained from one of two sources. In the first, data at the analysis and all forecast times from a *previously run* larger-area model simulation is used to specify the lateral boundary conditions. This is known as a *one-way* nest, where no interaction is permitted between the model simulation that provides the lateral boundary conditions and the simulation you wish to run over a limited area. Single-domain limited-area model simulations and the outermost domain of multiple-domain limited-area model simulations are examples of one-way nests.

In the second, lateral boundary condition data are provided from a model simulation *run concurrently* with your simulation. This is known as a *two-way* nest, or one in which the inner nested domain and the outer parent domain can interact during a simulation. Two-way interactive nesting is typically used with the inner domain(s) of a multiple-domain limited-area model simulation when sufficient computational resources exist.

Fig. 1 provides an illustration of how specified lateral boundary conditions are implemented with the WRF-ARW model; it is generally representative of lateral boundary formulations in grid-based models. Values of the model's dependent variables are specified by the lateral boundary conditions at each time step on the outermost grid points of the simulation domain (depicted in yellow in Fig. 1). Since the data used to prescribe these values typically have coarser spatial and temporal resolution than does the limited-area model simulation, these data are interpolated to the grid spacing (in all three dimensions) and time step of the limited-area model simulation.

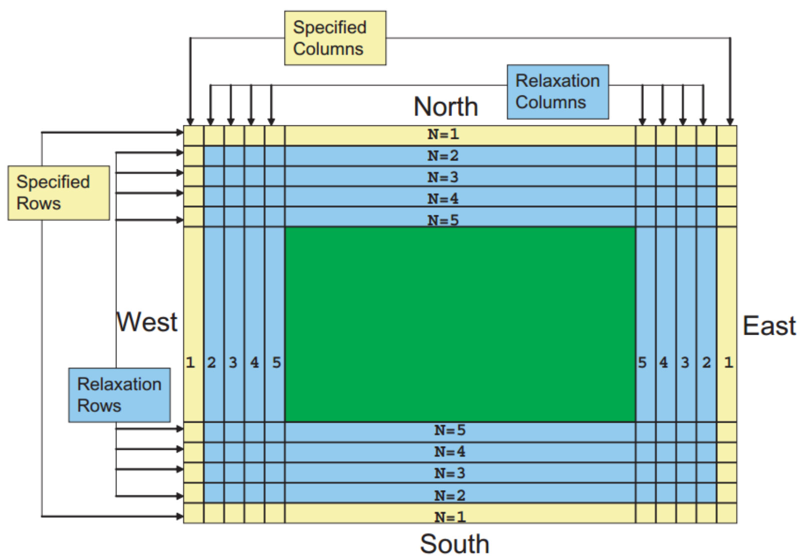

Figure 1. Illustration of specified lateral boundary conditions (yellow), including the model grid points (blue) over which the prescribed values provided by the lateral boundary conditions are relaxed to their values within the interior of the limited-area simulation domain (green), as used within real-data numerical simulations conducted using the WRF-ARW model. Figure reproduced from Skamarock et al. (2019), their Fig. 6.1.

Since the prescribed values of model variables on the lateral boundaries will differ from those on the interior of the limited-area model simulation, a *relaxation zone* is used near the specified lateral boundaries to blend the specified lateral boundary values with those on the domain's interior and mitigate sharp gradients (short wavelength features) that may exist along the lateral boundaries due to the specified lateral boundary data differing from the interior of the simulation domain. The relaxation zone's width is often a user-specifiable parameter.

There are multiple ways by which a relaxation zone may be specified. One that we consider here, as manifest in the WRF-ARW model, was formulated by Davies and Turner (1977, *Quart. J. Roy. Meteor. Soc.*) and is described in Skamarock et al. (2008). For any prognostic model variable *f*, its time tendency within the relaxation zone is given by:

$$\frac{\partial f}{\partial t} = F_1\left(f_{LBC} - f\right) - F_2\Delta^2\left(f_{LBC} - f\right)$$

In the above, *f* is predicted by the model, f_{LBC} is the value of *f* specified on the lateral boundary, and Δ^2 represents a five-point horizontal smoother applied on model coordinate surfaces. F_1 and F_2 are weighting coefficients given by:

$$F_1 = \frac{1}{10\Delta t}\frac{n_{spec} + n_{relax} - m}{n_{relax} - 1}$$

$$F_2 = \frac{1}{50\Delta t}\frac{n_{spec} + n_{relax} - m}{n_{relax} - 1}$$

In the above, n_{spec} is the number of rows or columns of grid points over which specified lateral boundary conditions are applied, n_{relax} is the number of the row or column of grid points at which the relaxation zone ends, and *m* is the number of rows or columns in from the lateral boundary. Note that *m* has a minimum value of $n_{spec} + 1$ and a maximum value of $n_{spec} + n_{relax} - 1$. It is also possible for these weighting coefficients to be multiplied by an exponential function to make smoother the transition from the lateral boundaries to the interior for large relaxation zones. The weighting coefficients thus relax, or nudge, the values of model dependent variables specified on the lateral boundaries to their values on the interior of the model domain.

Not all model dependent variables have their values specified from another data set on the lateral boundaries. For instance, in WRF-ARW, vertical velocity, all microphysical species except water vapor, and all non-conserved scalars for one-way nests have their values specified by other means. For vertical velocity, a zero-*gradient* boundary condition over the specified zone is applied. For the other variables, this zero-*gradient* boundary condition is applied when exiting the domain and a zero-*valued* boundary condition is applied when entering the domain.

Desired Characteristics of and Sources of Error with Lateral Boundary Conditions

There are several desirable characteristics for any specified lateral boundary. In particular, meteorological phenomena moving into or out of simulation domains that use specified lateral boundary conditions should do so without significant distortion. For those phenomena moving *into* a nested simulation domain, both interpolation from the coarser lateral boundary conditions and the probable existence of short-wavelength variability between the lateral boundaries and the simulation domain should not negatively impact simulation quality. Likewise, phenomena should be able to move *out of* a nested simulation domain without being reflected into or trapped in the domain.

As denoted within the course text, there are six primary potential sources of error associated with specified lateral boundary conditions:

- **Coarse resolution of the specified lateral boundary conditions**. Data used for specified lateral boundary conditions are typically at a coarser resolution in both time and space than the time step and horizontal grid spacing of your simulation domain. Smaller-scale variability thus cannot be accurately captured by the lateral boundary conditions, which may impact forecast quality at and near the lateral boundaries.

- **Group and phase speed errors**. Features that are coarsely resolved by the specified lateral boundary conditions may be better resolved in your finer-resolution simulation domain. Consider a domain with Δx = 15 km and lateral boundary conditions with Δx = 45 km. A feature of wavelength 200 km will be crudely resolved by the lateral boundary conditions (less than $5\Delta x$) but will be well-resolved within the simulation domain (greater than $13\Delta x$). As we will demonstrate in a later lecture, finite-difference methods result in greater errors in a wave-like feature's propagation speed and energy transports for smaller-wavelength features than they do for longer-wavelength ones.

- **Errors in the specified lateral boundary conditions**. Lateral boundary conditions come from numerical model analyses or forecasts, both of which are imperfect approximations to the real atmosphere. Consequently, specified lateral boundary conditions are to some extent erroneous, in turn impacting simulation quality.

- **For one-way nests, a lack of feedback between the simulation domain and the specified lateral boundary conditions**. In these simulations, the domain's interior cannot influence or change the values of the specified lateral boundary conditions. The resulting short-wavelength variability between the lateral boundaries and the domain's interior can influence forecast accuracy.

- **Dynamical imbalance between specified lateral boundary conditions and the solution on the interior of the simulation domain**. Discrepancies between the specified lateral boundary conditions and domain's interior can result in imbalanced kinematic and mass fields near the lateral boundaries. To attempt to restore balance, the model will generate non-physical gravity waves. The extent to which these waves influence the model solution depends on how close the primary feature of interest is to the lateral boundaries and how large of a difference is seen between the specified lateral boundary conditions and the model solution on the interior of the simulation domain.

- **Inconsistencies between physical parameterizations used to generate the specified lateral boundary conditions and within the simulation domain**. The model from which the lateral boundary conditions are obtained is generally not identical to that which you are using for your forecast. One way in which this is manifest is by the choice of physical parameterizations. For example, turbulent vertical mixing in the planetary boundary layer differs in how it is handled between atmospheric boundary layer parameterizations, in turn impacting near-surface thermodynamic and kinematic fields.

Practical Recommendations for Lateral Boundary Conditions

The error sources noted above motivate several practical recommendations for simulations in which specified lateral boundary conditions are used to mitigate their deleterious influences on model forecasts. These recommendations include:

- **Use a buffer zone between the primary area or phenomenon of interest in the simulation domain and the lateral boundaries**. To first order (i.e., excluding any influence of gravity waves), the time it takes for lateral boundary information to reach a given grid point on the domain's interior is controlled by how many grid points are used by the model's chosen finite-differencing scheme and how many model time steps it will take for this finite-differencing scheme to impact this location (Ancell et al. 2018, *Bull. Amer. Meteor. Soc.*). Thus, if the lateral boundary conditions are desired to have as small of an impact as possible on the forecast in an area of interest, the area of interest should be placed as far away from the lateral boundaries as is feasible. In practice, however, the influence of lateral boundary conditions on the model solution is somewhat unavoidable.

- **Minimize interpolation error with the specified lateral boundary conditions**. The specified lateral boundary conditions should be updated frequently and be provided at a grid spacing as close to that of the simulation domain as is possible so that small-scale variability (in time and space) is not altogether missed by the lateral boundary conditions.

- **Maintain as much consistency as possible between the model configuration used to provide lateral boundary conditions and to integrate the model**. Inconsistencies between the physical parameterizations used by your simulation and those used by the model that generated your lateral boundary conditions can impact forecast quality. Thus, it is desirable to use similar (if not identical) physical parameterizations for your forecasts as were used by the model that generated the lateral boundary conditions.

- **Use well-tested and effective lateral boundary condition formulations**. To large extent, lateral boundary condition formulations available within modern numerical weather prediction models are well-tested and effective. Of course, “well-tested and effective” does not mean error-free, but rather associated with reasonably small errors.

- **Avoid strong forcing of either meteorological or geophysical origin near the lateral boundaries**. Sharp gradients in a model’s dependent variables can originate with static geophysical features (e.g., sloped terrain and ocean currents) and transient meteorological phenomena (e.g., cyclones). Placing the lateral boundaries near such features would add to the magnitude of the sharp gradients in the dependent variables that results from the lateral boundary specifications themselves, potentially impacting the forecast.

- **Where possible, use two-way nests rather than one-way nests**. One-way nests are unavoidable for limited-area model domains on the outermost simulation domain. Two-way nests for inner simulation domains allow for their lateral boundary conditions to be updated based on what occurs within the inner domain. Further, the lateral boundary conditions are typically directly updated more frequently – once every time step on the outer simulation domain rather than once every history output time – using two-way nests than if they are specified using a one-way nest.

- **Evaluate the extent to which the chosen lateral boundary configuration influences the model solution**. The methods introduced below highlight ways in which lateral boundary condition influences on numerical model forecasts can be quantified. In as much as is possible, one or more of these or other related methods should be used to inform lateral boundary condition formulation and placement.

The Influence of Lateral Boundary Conditions upon Limited-Area Model Simulations

In the following, we describe *possible* influences of specified lateral boundary conditions on limited-area model simulations. Note that the specifics of these influences are *not* necessarily generalizable to a wide range of model applications. As a result, we focus on the *methods* that can be used to examine the influence of specified lateral boundary conditions on limited-area model simulations rather than the findings obtained from such methods. Furthermore, note that variants of these methods exist. The intention here is not to describe every possible method by which the influence of specified lateral boundary conditions can be identified but rather to describe a subset of these methods and their applications.

One way to examine the influence of specified lateral boundary conditions is to conduct a series of simulations in which the lateral boundaries are placed at increasingly large distances from an area or feature of interest. Output from these simulations is then verified over the area covered by the smallest simulation domain. An example of this is provided by the Treadon and Petersen (1993) study cited in the course text. Therein, they found that forecast error – defined in their study by the domain-averaged root-mean squared error of 500 hPa geopotential height – increased as the distance between the verification area and the lateral boundaries decreased. Their study also depicts how the model solution near the lateral boundaries is constrained to that provided by specified boundary conditions (Fig. 2). Since these specified lateral boundary data are typically coarser than your simulation's horizontal grid spacing, features will also appear with less short-wavelength variability near to the lateral boundaries.

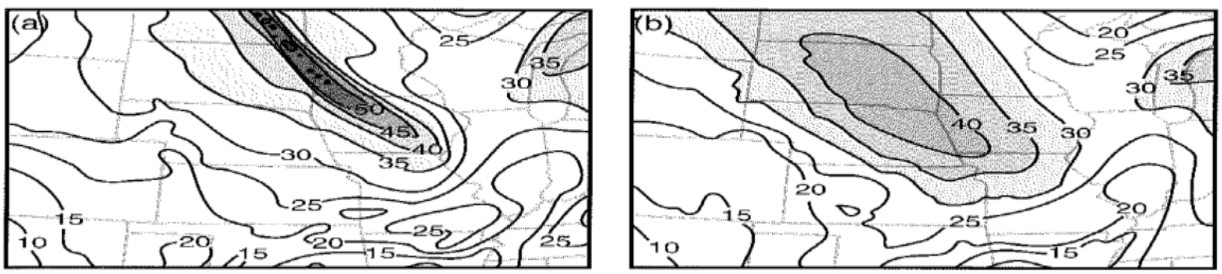

Figure 2. 250 hPa isotachs (m s^{-1}) from two 12-h Eta model simulations with Δx = 40 km initialized at 1200 UTC 3 August 1992. In (a), the lateral boundaries are placed approximately 2,000 km away from the edges of what is plotted. In (b), the lateral boundaries are placed at the outermost periphery of what is plotted. Figure reproduced from Warner (2011), their Fig. 3.44.

One can also examine the influence of specified lateral boundary conditions by conducting a simulation in which the lateral boundary conditions are provided at identical grid spacing to that used by your simulation, which uses the same model configuration as that from which the lateral boundary conditions are drawn. Differences between these two simulations would thus quantify the influence of the lateral boundary condition formulation on the forecast. A variant of this approach is the so-called "Big Brother-Little Brother" experiment, as briefly described in both Sections 3.5 and 10.4 of the course text.

The lateral boundary conditions' influence on the model forecast grows over time. There are multiple ways by which this influence can be quantified. These methods do not directly quantify the forecast error due to the specified lateral boundary conditions, but rather help assess the influence of the specified lateral boundary conditions on the model forecast. One means of doing so is though using adjoint methods, as illustrated in Fig. 3.

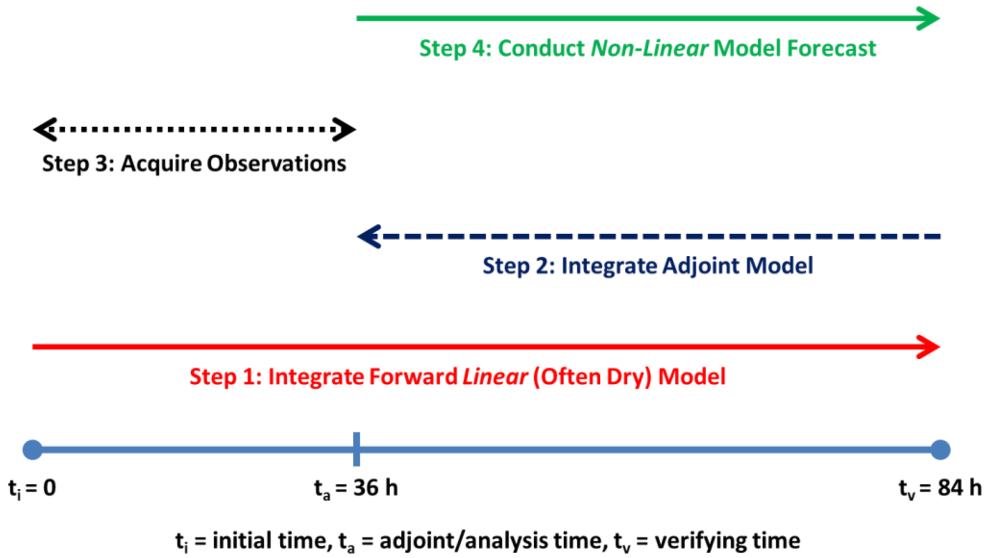

Figure 3. Schematic representation of adjoint methods. At an initial time, the linear version of a numerical model is integrated forward in time (red arrow). A feature of interest at a later forecast time is then identified in the linear model's output. Next, the adjoint (or inverse linear) version of the model is integrated backward in time from the later forecast time to an earlier time, either the simulation's start time or some other time of interest (blue-dashed arrow). The resulting analysis provides data from which forecast sensitivity can be determined. Adjoint methods are often used with observation targeting and data assimilation, wherein the sensitivity metric can provide a way of identifying what observations (when, where, and what; black-dashed arrow) are needed to improve the non-linear model's forecast (green arrow). Applied to lateral boundary condition sensitivity studies, adjoint methods can be used to identify to where along the lateral boundaries the forecast is most sensitive.

The influence of specified lateral boundary conditions on the model forecast can also be illuminated by conducting two model simulations in which only the lateral boundary conditions are varied. Such studies allow for the illumination of how lateral boundary condition information spreads inward from the lateral boundaries with time. An example is provided by the Vukicevic and Errico (1990) study cited in the course text, in which two limited-area model simulations encompassing Europe and northern Africa were conducted. The simulations were identical expect for the specified lateral boundary conditions. Integrating the model forward for 6 h, differences in the 500 hPa geopotential height of in excess of 5 m are evident across much of the domain, even well-removed from the lateral boundaries (Fig. 4). As the advective velocity is far too small to account for such differences on the interior of the simulation domain, this example highlights how numerical noise – the "seeding chaos" effect related to the finite-differencing approximations described by Ancell et al. (2018, *Bull. Amer. Meteor. Soc.*) – and/or gravity waves generated along the lateral boundaries can spread lateral boundary information over a large area in a short amount of time.

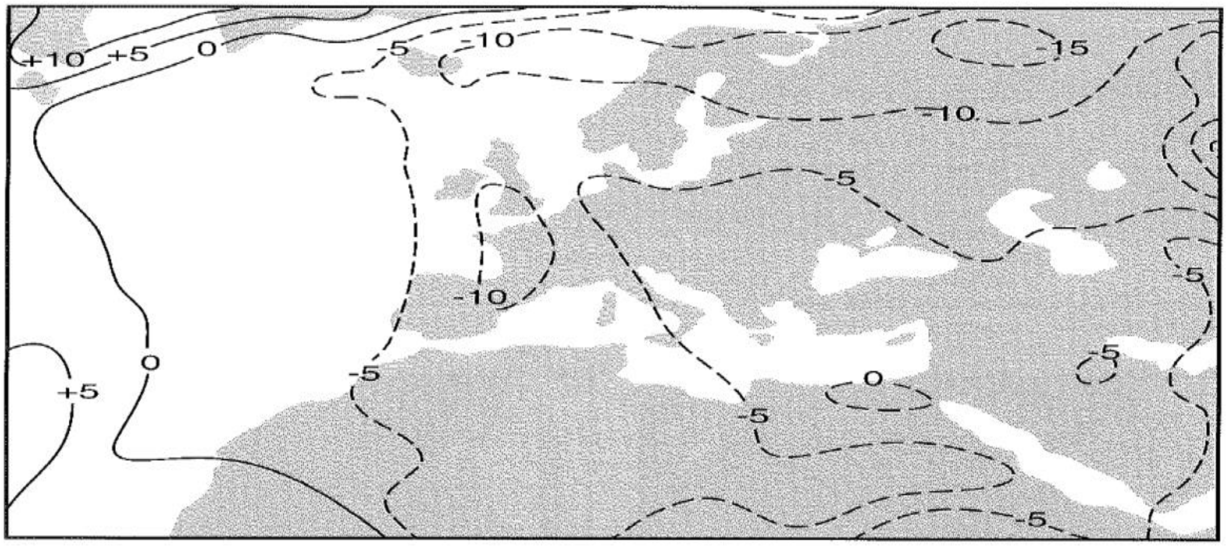

Figure 4. Difference in 500 hPa geopotential height (m) between two model simulations conducted using a limited-area model, one using unperturbed and one using perturbed lateral boundary conditions, at t = 6 h. Figure reproduced from Warner (2011), their Fig. 3.46.