# Synoptic Meteorology II: The Quasi-Geostrophic Omega Equation

**Readings:** Sections 2.3 and 2.5 of *Midlatitude Synoptic Meteorology*.

## Why are we interested in vertical motion?

Before we derive and discuss the quasi-geostrophic omega equation, it is prudent to ask: why are we interested in vertical motion? Firstly, recall our discussion of the quasi-geostrophic vorticity equation. The $\partial\omega/\partial p$ term contained within that equation is responsible for the amplification and/or deamplification of middle tropospheric troughs and ridges. As a result, in the quasi-geostrophic system, if we want to know something about how the amplitude of the midlatitude trough/ridge pattern is evolving, we need to know how the vertical velocity varies with respect to pressure.

Secondly, recall that mixing ratio is conserved for dry-adiabatic ascent. Thus, ascent over a deep enough vertical layer or prolonged period brings about condensation. This is substantially aided if the layer in which the ascent occurs is relatively moist prior to the ascent beginning. Naturally, the overlap of ascent and moisture implies cloud and precipitation formation. Thus, we are interested in vertical motion because, even though synoptic-scale vertical motion is typically small in magnitude, it plays a role in the formation and evolution of clouds and precipitation.

## Obtaining the Quasi-Geostrophic Omega Equation

Recall that the quasi-geostrophic vorticity equation is given by the following:

$$\nabla^2\left(\frac{\partial\Phi}{\partial t}\right) = f_0\left(-\vec{\mathbf{v}}_g\cdot\nabla\left(\zeta_g + f\right)\right) + f_0^2\frac{\partial\omega}{\partial p} - f_0K\zeta_g \qquad (1)$$

Likewise, recall that the quasi-geostrophic thermodynamic equation is given by the following:

$$-\frac{\partial}{\partial p}\left(\frac{\partial\Phi}{\partial t}\right) = -\vec{\mathbf{v}}_g\cdot\nabla\left(h\theta\right) + \sigma\omega + \frac{R}{pc_p}\frac{dQ}{dt} \qquad (2)$$

The quasi-geostrophic vorticity (1) and thermodynamic (2) equations are two equations containing two unknowns – vertical motion $\omega$ and geopotential height $\Phi$. Other variables such as geostrophic relative vorticity $\zeta_g$ and potential temperature $\theta$ that appear in (1) and (2) can be diagnosed using the geopotential height field and thus are not independent unknown variables.

To obtain the quasi-geostrophic omega equation, we wish to combine (1) and (2) to eliminate $\Phi$, leaving a single equation for $\omega$ that describes the vertical motion on a given isobaric surface.

To do so, we need to obtain $\partial/\partial p$ of (1). Doing so, and commuting the derivatives on the left-hand side of the resultant equation, we obtain:

$$\nabla^2\left(\frac{\partial}{\partial p}\frac{\partial \Phi}{\partial t}\right) = f_0\frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right)\right) + f_0^2\frac{\partial^2 \omega}{\partial p^2} - f_0\frac{\partial}{\partial p}\left(K\zeta_g\right) \tag{3}$$

Likewise, we need to obtain $\nabla^2$ of (2). Doing so, and slightly re-writing the first term on the right-hand side of the resultant equation, we obtain:

$$-\nabla^2\left(\frac{\partial}{\partial p}\frac{\partial \Phi}{\partial t}\right) = h\nabla^2\left(-\vec{\mathbf{v}}_g \cdot \nabla\theta\right) + \sigma\nabla^2\omega + \frac{R}{pc_p}\nabla^2\left(\frac{dQ}{dt}\right) \tag{4}$$

If we add (3) and (4) and bring the terms involving $\omega$ to the left-hand side of the resultant equation, we obtain the *quasi-geostrophic omega equation*:

$$\sigma\nabla^2\omega + f_0^2\frac{\partial^2 \omega}{\partial p^2} = -f_0\frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right)\right) - h\nabla^2\left(-\vec{\mathbf{v}}_g \cdot \nabla\theta\right) + f_0\frac{\partial}{\partial p}\left(K\zeta_g\right) - \frac{R}{pc_p}\nabla^2\left(\frac{dQ}{dt}\right) \tag{5}$$

(5) is a partial differential equation describing the vertical motion $\omega$ on an isobaric surface. There are four forcing terms on the right-hand side of (5). From left to right, these represent differential geostrophic-vorticity advection, the Laplacian of potential-temperature advection, differential friction, and the Laplacian of diabatic heating.

As with the quasi-geostrophic height tendency equation, this equation is typically applied in the middle troposphere and not at the surface. When combined with (1), however, (5) can be used to examine the evolution of synoptic-scale weather features – including surface pressure systems. We will tackle this in a future lecture.

The left-hand side of (5) expresses $\omega$ in terms of the Laplacian ($\nabla^2$) as well as a second derivative with respect to pressure. Because the second partial derivative of a local maximum is negative and that of a local minimum is positive, the left-hand side of (5) can be approximated as:

$$\sigma\nabla^2\omega + f_0^2\frac{\partial^2 \omega}{\partial p^2} \propto -\omega \tag{6}$$

As before, the $\propto$ symbol means "is proportional to," such that the left-hand side of (6) is proportional to $-\omega$. Therefore, where the right-hand side of (5) is positive, $\omega$ is negative. Because of the sign convention on $\omega$, this implies local *ascent*. Likewise, where the right-hand side of (5) is negative, $\omega$ is positive, implying local *descent*.

## Basic Interpretation of the Quasi-Geostrophic Omega Equation

*Differential Geostrophic-Vorticity Advection*

The contribution to vertical motion exclusively due to differential geostrophic-vorticity advection can be expressed by:

$$\omega \propto f_0 \frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right)\right) \tag{7}$$

To interpret (7), we consider the cases where (a) cyclonic geostrophic-vorticity advection increases upward (i.e., positive advection becoming more positive upward) and (b) anticyclonic geostrophic-vorticity advection increases upward (i.e., negative advection becoming more negative upward).

In case (a), the numerator on the right-hand side of (7) is positive. The denominator, the change in pressure, is negative – as it always is, because pressure decreases upward. Therefore, $\omega$ is negative, which implies middle tropospheric *ascent*. In case (b), the numerator on the right-hand side of (7) is negative. The denominator here is also negative. Therefore, $\omega$ is positive, which implies middle tropospheric *descent*.

Note the $f_0$ preceding the partial derivative in (7). This suggests that the magnitude of this forcing term increases with latitude. However, we typically neglect this relationship for simplicity.

*Laplacian of Potential-Temperature Advection*

The contribution to vertical motion exclusively due to the Laplacian of potential-temperature advection can be expressed by:

$$\omega \propto h\nabla^2\left(-\vec{\mathbf{v}}_g \cdot \nabla\theta\right) \tag{8}$$

The right-hand side of (8) includes a Laplacian operator, which is difficult to readily interpret. To simplify, we make use of a generalized form of the relationship posed by (6), such that:

$$\omega \propto -h\left(-\vec{\mathbf{v}}_g \cdot \nabla\theta\right) \tag{9}$$

The definition of $h$, which is positive-definite, is provided in the quasi-geostrophic height tendency equation lecture notes. We typically neglect its influence for simplicity and instead focus on the geostrophic potential-temperature advection term in the parentheses. Given the leading negative, warm (positive) geostrophic potential-temperature advection on a given isobaric surface results in $\omega < 0$ (or ascent) and cold (negative) geostrophic potential-temperature advection on a given isobaric surface results in $\omega > 0$ (or descent).

One interesting digression before proceeding: a hallmark of a baroclinic atmosphere is the presence of horizontal temperature (or potential-temperature) gradients. The *baroclinicity* can be viewed as a measure of the strength of those gradients, or how rapidly the temperature changes over a given distance. Because of the proportionality in (9), we can state that the magnitude of $\omega$ is proportional to the magnitude of the baroclinicity in the synoptic-scale environment.

*Differential Friction*

The contribution to vertical motion exclusively due to differential friction can be expressed by:

$$\omega \propto -f_0 \frac{\partial}{\partial p}\left(K\zeta_g\right) \tag{10}$$

In (10), $K$ represents the effects of friction and is positive-definite. $K$ is non-zero only within the boundary layer (i.e., close to the surface), where the frictional effects of the land-surface can be meaningfully communicated to the troposphere.

The right-hand side of (10) contains a partial derivative with respect to pressure. However, because $K$ is non-zero only in the boundary layer, $K\zeta_g$ is zero in the middle to upper troposphere. Thus, the sign of the right-hand side of (10) depends entirely on the sign of the geostrophic relative-vorticity $\zeta_g$ in the lower troposphere.

For the case of lower-tropospheric cyclonic geostrophic relative-vorticity ($\zeta_g > 0$), the numerator on the right-hand side of (10) is negative. The denominator, the change of pressure, is negative. Per the leading negative on the right-hand side of (10), this implies $\omega < 0$, or *ascent*. This is what is known as *Ekman pumping*.

For lower-tropospheric anticyclonic geostrophic relative vorticity ($\zeta_g < 0$), the numerator on the right-hand side of (10) is positive. The denominator, the change of pressure, is negative. Per the leading negative on the right-hand side of (10), this implies $\omega > 0$, or *descent*. This is what is known as *Ekman suction*.

As with the differential geostrophic-vorticity advection term, there is an $f_0$ preceding the partial derivative in (7). Thus, the magnitude of this forcing term increases with latitude, but we typically also neglect this relationship for simplicity.

*Laplacian of Diabatic Heating*

The contribution to vertical motion exclusively due to the Laplacian of diabatic heating can be expressed by:

$$\omega \propto \frac{R}{pc_p}\nabla^2\left(\frac{dQ}{dt}\right) \tag{11}$$

$dQ/dt$ is the diabatic heating rate. Diabatic warming refers to the situation where $dQ/dt > 0$, while diabatic cooling refers to the situation where $dQ/dt < 0$. This term is non-zero only in the presence of diabatic heating, such as from radiation and latent heat release. On the synoptic-scale, where motions are primarily adiabatic in nature and the atmosphere is unsaturated, this term can often be neglected.

As with the Laplacian of potential temperature advection, the right-hand side of (11) includes a Laplacian operator that is difficult to readily interpret. To simplify, we assume that the Laplacian of a quantity is proportional to the negative of that quantity, such that:

$$\omega \propto -\frac{R}{pc_p}\frac{dQ}{dt} \tag{12}$$

Thus, the presence of diabatic warming leads to $\omega < 0$, implying *ascent*. The presence of diabatic cooling leads to $\omega > 0$, implying *descent*.

Note that (12) indicates that $\omega$ is inversely proportional to pressure, such that the forcing magnitude is greater at higher altitudes (i.e., lower pressures). As with the similar relationships for other terms in the quasi-geostrophic height tendency and omega equations, we often neglect this relationship for simplicity.

**The Physical Reasoning Behind the Quasi-Geostrophic Omega Equation's Forcing Terms**

In the preceding section, we focused on a basic explanation of the signs of the forcings that result in ascent and descent. However, this explanation is meaningless unless we know *why*, physically, these forcings result in ascent and descent.

Let us start with the differential geostrophic-vorticity advection term. We start with equation (19) from our "Quasi-Geostrophic Vorticity Equation" lecture:

$$\frac{D_g\varsigma_g}{Dt} = f_0\frac{\partial\omega}{\partial p} - \beta v_g$$

This equation describes changes in geostrophic relative-vorticity following the geostrophic flow. Neglect the β right-hand side term and use the continuity equation to rewrite the remaining right-hand side term to obtain:

$$\frac{D_g \varsigma_g}{Dt} \approx -f_0 \left( \nabla_h \bullet \vec{\mathbf{v}} \right) \tag{13}$$

Let us consider an air parcel approaching the base of a trough (i.e., it is moving through the trough) from the west, in a region of anticyclonic geostrophic-vorticity advection. The air parcel must increase its geostrophic relative vorticity as it approaches the base of the trough, such that the left-hand side of (13) must be positive. From (13), this means that there must be convergence within the region of anticyclonic geostrophic-vorticity advection.

Now, let us consider an air parcel moving away from the base of a trough toward the east (i.e., also moving through the trough), in a region of cyclonic geostrophic-vorticity advection. The air parcel must decrease its geostrophic relative vorticity as it departs the base of the trough, such that the left-hand side of (13) must be negative. From (13), this means that there must be divergence within the region of cyclonic geostrophic-vorticity advection.

However, the quasi-geostrophic omega equation is concerned with how the geostrophic-vorticity advection changes (in sign and/or magnitude) as a function of pressure. Consider two cases:

- Low-level cyclonic vorticity advection and upper-level anticyclonic vorticity advection, or anticyclonic geostrophic-vorticity advection increasing with height. The above discussion implies low-level divergence and upper-level convergence, such that vertically integrating divergence results in mid-level descent.
- Low-level anticyclonic vorticity advection and upper-level cyclonic vorticity advection, or cyclonic geostrophic-vorticity advection increasing with height. The above discussion implies low-level convergence and upper-level divergence, such that vertically integrating divergence results in mid-level ascent.

Both interpretations are consistent with our basic insight derived above. Note, however, an implicit middle-tropospheric evaluation of the omega equation (i.e., close to the level of non-divergence). Indeed, the omega equation is typically evaluated in the middle troposphere using low- and upper-level forcings (e.g., $\omega$ at 500 hPa using forcings from 700 hPa and 300 hPa).

Next, consider the potential-temperature advection term. Air parcels conserve (maintain their value of) potential temperature following the motion for (dry) adiabatic motions; i.e., no diabatic heating. For warm potential-temperature advection on an isobaric surface, air must flow from warm to cold air. With potential temperature increasing upward, the higher-valued isentropes must ascend over their lower-valued counterparts associated with the cold air mass. Thus, air parcels constrained to a higher-valued isentropic surface must ascend over the cold air mass that they approach under the condition of warm potential-temperature advection. Conversely, air parcels constrained to a lower-valued isentropic surface must descend as they approach a warmer air mass under the condition of cold potential-temperature advection.

Next, consider the differential friction term. As discussed last semester, friction causes air parcels to flow inward (i.e., converge) toward surface cyclones ($\zeta_g > 0$). Given a lower boundary condition of zero divergence, upward integration from the surface results in net friction-induced convergence with surface cyclones. From the continuity equation, this leads to lower-tropospheric ascent. This is consistent with our basic interpretation above. Conversely, friction causes air parcels to flow out (i.e., diverge) from surface anticyclones ($\zeta_g < 0$). Given a lower boundary condition of zero divergence, upward integration from the surface results in net friction-induced divergence with surface anticyclones. From the continuity equation, this leads to lower-tropospheric descent, again consistent with our basic interpretation above.

Finally, consider the diabatic heating term. In this case, diabatic heating is primarily a diagnostic proxy for *existing* vertical motions, consistent with the quasi-geostrophic omega equation being a diagnostic equation (as discussed later in this lecture). Since we evaluate mid-tropospheric vertical motions with the quasi-geostrophic omega equation, diabatic warming is generally associated with latent heat release due to condensation, freezing, or deposition, each of which generally occur with ascent. Conversely, diabatic cooling is generally associated with latent cooling due to evaporation, melting, or sublimation, each of which generally occur with and/or force descent.

## The Quasi-Geostrophic Omega Equation and Geostrophic Balance

*Preliminary Considerations*

For convenience, let us restate the quasi-geostrophic omega equation:

$$\sigma\nabla^2\omega + f_0^2\frac{\partial^2\omega}{\partial p^2} = -f_0\frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g\cdot\nabla\left(\zeta_g + f\right)\right) - h\nabla^2\left(-\vec{\mathbf{v}}_g\cdot\nabla\theta\right) + f_0\frac{\partial}{\partial p}\left(K\zeta_g\right) - \frac{R}{pc_p}\nabla^2\left(\frac{dQ}{dt}\right) \quad (14)$$

If diabatic heating is absent or can be estimated somehow, the right-hand side of (14) includes only geostrophic quantities. Obviously, the geostrophic wind $\boldsymbol{v}_g$ and the geostrophic relative vorticity $\zeta_g$ are geostrophic quantities. However, with Poisson's relationship, the hydrostatic equation, and the definition of geostrophic relative vorticity in terms of the geopotential, the potential temperature $\theta$ can also be viewed as a geostrophic quantity.

As a result, synoptic-scale vertical motions – weak as they may be – are **forced** entirely by the geostrophic flow. Let us consider this thought in the context of the continuity equation, however:

$$\nabla \cdot \vec{\mathbf{v}}_{ag} + \frac{\partial \omega}{\partial p} = 0 \tag{15}$$

The continuity equation states that the vertical motion is intricately tied to the divergent component of the ageostrophic wind. Because the geostrophic flow exclusively forces vertical motions, (15) implies that it also forces the divergent component of the ageostrophic wind. As the ageostrophic wind inherently implies a departure from geostrophic balance, *this means that the geostrophic flow is responsible for bringing departures from geostrophic balance!*

The ageostrophic circulation – that comprised of the ageostrophic wind and vertical motion – acts to restore the geostrophic (and hydrostatic) balance that the geostrophic wind itself destroyed. In the quasi-geostrophic system, this can be understood with the quasi-geostrophic vorticity equation in the form given by (13). In the following, we demonstrate these concepts in the context of each of the forcing terms to the quasi-geostrophic omega equation.

*Application to the Quasi-Geostrophic Omega Equation*

Differential Geostrophic-Vorticity Advection

Consider the case of cyclonic geostrophic-vorticity advection increasing upward, associated with ascent per earlier discussion. The continuity equation, given by (15), relates the partial derivative of the vertical motion $\omega$ with respect to pressure to the divergence of the ageostrophic flow.

In this discussion, assume that the ascent is maximized in the middle troposphere, where we have applied the quasi-geostrophic omega equation, and decays to zero at the rigid boundaries of the surface and tropopause. Thus, $\partial\omega/\partial p$ in the lower troposphere is positive and $\partial\omega/\partial p$ in the upper troposphere is negative. From (15), we know that the former is associated with *convergence* and the latter with *divergence*.

Next, consider the quasi-geostrophic vorticity equation. In the lower troposphere, where $\partial\omega/\partial p$ is positive, $\zeta_g$ must increase following the flow. This implies that middle-tropospheric ascent forces increasing geostrophic relative vorticity in the lower troposphere! In the upper troposphere, where $\partial\omega/\partial p$ is negative, $\zeta_g$ must decrease following the flow. This implies that middle-tropospheric ascent forces decreasing geostrophic relative vorticity in the upper troposphere! Thus, *ascent counteracts the initial situation of cyclonic geostrophic-vorticity advection increasing upward that forced the vertical motion in the first place!*

Similar arguments can be made to understand what happens for anticyclonic geostrophic-vorticity advection increasing upward. The geostrophic flow forces synoptic-scale descent; the ageostrophic circulation associated with such descent, however, counteracts the initial situation of anticyclonic geostrophic-vorticity advection increasing with height.

Potential-Temperature Advection

Consider the case of warm potential-temperature advection and thus ascent on the isobaric level on which the quasi-geostrophic omega equation is applied. Recall that because pressure is constant on an isobaric surface, warm potential temperature advection implies warm temperature advection.

In the absence of diabatic processes, dry-adiabatic ascent results in cooling (of temperature, though not potential temperature, which is conserved). This can be viewed in the context of ascent along a dry adiabat on a skew-T. As a result, *ascent counteracts the initial situation of warm (potential) temperature advection that forced the vertical motion in the first place!* Similar arguments can be made to describe the response to cold potential-temperature advection.

Differential Friction

Let us consider the case where there is cyclonic geostrophic relative-vorticity and thus ascent near the surface. As above, this results in $\partial\omega/\partial p > 0$ in the lower troposphere and $\partial\omega/\partial p < 0$ in the upper troposphere. This leads to $\zeta_g$ increasing following the flow in the lower troposphere and decreasing following the flow in the upper troposphere. Thus, *ascent counteracts the initial situation of friction dissipating cyclonic geostrophic relative vorticity in the lower troposphere that forced the vertical motion in the first place!* Similar arguments can be made for the case where friction acts on anticyclonic geostrophic relative-vorticity within the boundary layer.

Diabatic Heating

Finally, consider the case of diabatic warming ($dQ/dt > 0$). The physical interpretation is identical to that for warm potential-temperature advection above. Ascent-driven adiabatic cooling resulting from diabatic warming counteracts the warming. Similar arguments can be made in the inverse for diabatic cooling.

## The Quasi-Geostrophic Omega Equation Applied to the Four-Quadrant Jet Model

One can invoke the quasi-geostrophic omega equation to interpret vertical motions associated with the four-quadrant jet model. Considering only the differential geostrophic absolute-vorticity advection term, the quasi-geostrophic omega equation can be approximated by:

$$\omega \propto f_0 \frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g \cdot \nabla\left(\zeta_g + f\right)\right) \tag{16}$$

Making this approximation implies that the other terms of the quasi-geostrophic omega equation are negligible (i.e., no potential-temperature advection, friction, or diabatic heating).

An upper-tropospheric trough, with a maximum of cyclonic geostrophic absolute vorticity in its base, is typically found poleward of a westerly jet streak. An upper-tropospheric ridge, with a maximum of anticyclonic geostrophic absolute vorticity in its apex, is typically found poleward of a westerly jet streak. This configuration results in upper-tropospheric cyclonic vorticity advection in the left-exit and right-entrance regions of the jet streak and upper-tropospheric anticyclonic vorticity advection in the right-exit and left-entrance regions of the jet streak. If vorticity advection is small near the surface, this indicates that there should be middle-tropospheric ascent in the right-entrance and left-exit regions of the jet streak and middle-tropospheric descent in the left-entrance and right-exit regions of the jet streak.

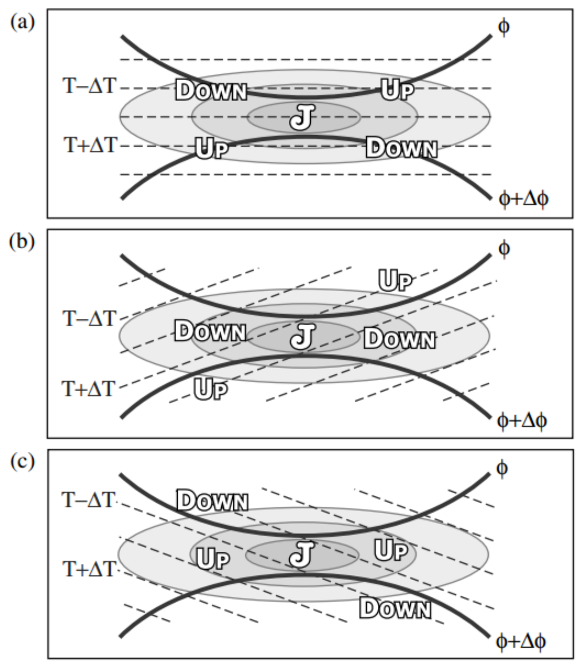


**Figure 1**. Schematic illustrations of an upper-tropospheric jet associated with (a) no potential-temperature advection, (b) cold potential-temperature advection, and (c) warm potential-temperature advection. In each panel, isentropes are given by the dashed lines and isohypses are given by thick solid lines. Labels of "Up" and "Down" indicate middle-tropospheric ascent and descent, respectively. From Lang and Martin (2012, *Quart. J. Roy. Meteor. Soc.*), their Fig. 3.

If we consider only the potential-temperature advection forcing term, the quasi-geostrophic omega equation can be approximated by:

$$\omega \propto -h\left(-\vec{\mathbf{v}}_g \cdot \nabla \theta\right) \tag{17}$$

Making this approximation implies that the other terms of the quasi-geostrophic omega equation are negligible (i.e., no differential geostrophic relative vorticity advection, friction, or diabatic heating).

Consider two cases: (1) isentropes (or, equivalently on an isobaric surface, isotherms) are oriented such that there is cold advection through the jet streak and (2) isentropes are oriented such that there is warm advection through the jet streak. These are depicted in Figs. 1(b,c), respectively. For reference, the case with no potential temperature advection through the jet streak is depicted below in Fig. 1(a).

As stated before, cold potential-temperature advection on an isobaric surface implies sinking motion across that isobaric surface. Thus, the first case is associated with quasi-geostrophic forcing for descent. This forcing is maximized in the jet core, where the wind component perpendicular to the isentropes or isotherms is maximized (and, thus, cold advection is maximized). Conversely, warm potential-temperature advection on an isobaric surface implies rising motion across that isobaric surface. Thus, our second case is associated with quasi-geostrophic forcing for ascent. This forcing is again maximized in the jet core, where the wind component perpendicular to the isentropes or isotherms is maximized (and, thus, warm advection is maximized).

Thus, there are two contributors to vertical motions we must consider: (1) parcel accelerations contributing to ageostrophic flow and vertical motion and (2) potential temperature advection by the geostrophic wind contributing to ageostrophic flow and vertical motion. In our first case, the combined effects of these two contributors result in descent aligned with the jet core; in the second case, the combined effects of these two contributors result in ascent aligned with the jet core. The interpretation of the four-quadrant jet model is otherwise similar to that which we have considered before, as can be inferred from a comparison of Figs. 1(b,c) to Fig. 1(a).

## Evaluating the Quasi-Geostrophic Omega Equation

The quasi-geostrophic omega equation contains no partial derivatives with respect to time. As a result, this equation ***cannot*** be used to make a forecast! Instead, it may only be used to *diagnose* vertical motions (on the synoptic-scale under the constraints of the quasi-geostrophic system) at a given time.

Like the quasi-geostrophic height tendency equation, the vertical motion $\omega$ on the left-hand side of (5) depends upon the second derivatives of $\omega$ with respect to $x$ and $y$ (as manifest through the Laplacian operator) and $p$. In other words, the local value of the vertical motion depends upon its value at adjacent locations in both the horizontal and vertical. Thus, to solve this system requires an iterative approach. Also as before, it is difficult to accurately compute the frictional and diabatic heating forcing terms that make up part of the right-hand side of (5).

However, the differential geostrophic-vorticity advection and Laplacian of potential-temperature advection terms may be computed or estimated readily from any available source of atmospheric data, such as a numerical model analysis or forecast. This, along with the general proportionality from (6), enables us to *diagnose* synoptic-scale, mid-tropospheric vertical motions. Computational approaches are preferable over estimation approaches as the two primary forcing terms can – and often do! – oppose one another in sign, making their relative magnitudes important.