1. Suppose that $(\Omega, \mathcal{F}, \Pr)$ is a probability space. Prove that $\Pr(\emptyset) = 0$.

2. Suppose that $(\Omega, \mathcal{F}, \Pr)$ is a probability space. Prove that Pr is finitely additive.

3. Suppose that $(\Omega, \mathcal{F}, \Pr)$ is a probability space, and $A \in \mathcal{F}$. Prove that $\Pr(A^c) = 1 - \Pr(A)$.

4. Suppose that $(\Omega, \mathcal{F})$ is a measurable space, and $Q : \mathcal{F} \to [0, 1]$ satisfies

   - $Q(\Omega) = 1$;
   - Q is finitely additive;
   - For each sequence of events $\{A_k\}_{k=1}^{\infty}$ with $A_{n+1} \subset A_n$ for each positive integer n and

     $$\cap_{n=1}^{\infty} A_k = \emptyset$$

     we know $Q(A_n) \to 0$ as $n \to \infty$.

   Prove that Q is a probability measure.

5. Prove that all distribution functions are non-decreasing, are right-continuous with left-hand limits, converge to 1 at infinity and 0 at negative infinity.

6. Suppose that $\Omega = [0, 1]$, $\mathcal{F}$ is the set of all Lebesgue measurable subsets of Ω and $\Pr(F)$ is the integral of $f(x) = x$ with respect to Lebesgue measure over F if F does not contain $1/3$, and is the integral of $f(x) = x$ with respect to Lebesgue measure over F plus $1/2$ if F contains $1/3$. Let $X : \Omega \to (-\infty, \infty)$ defined by $X(\omega) = \omega^2$.

   Verify that X is a real valued random variable, find and graph its distribution function, and compute its mean, variance and characteristic function.