Sign Changes (C)

From calculus we have the following useful result.

Theorem 2728

If $p(z)$ is a polynomial with real coefficients, $a < b$ and the sign of $p(a)$ is different from the sign of $p(b)$ then p has a zero between a and b.

We also have some bits of common sense:

- If all of the exponents in the polynomial $p(z)$ are even or all of the exponents are odd and $p(a) = 0$ then $p(-a) = 0$.
- If all of the coefficents of the polynomial have the same sign, then the polynomial has no positive roots.
- If all coefficient of the even powered terms of $p(z)$ are positive and all the odd powered terms have coefficients which are negative, or vice-versa, then the polynomial has no negative roots.

Exercises

1.

Find all the zeros of each of the following polynomials:

(a)

$x^3-6x^2+11x-6$;

(b)

$x^4-7x^3+17x^2-17x + 6$;

(c)

$10x^3-37x^2+37x-6$;

(d)

$5x^4 -7x^3 - 3x^2 + 7x-2$;

(e)

$54x^3-21x^2-7x+2$;

(f)

$x^4+4x^3 + 4x^2 - 4x - 5$;

2.

Suppose that $p(z) = z^4 + 3z^3 + 2z^2 + 3z + 1$. Show that $p(i) = 0$ and then find all the zeros of p.