Rationalizing Expressions (C)

The formula for summing a geometric progression can be used to rewrite expressions containing the difference of two roots of the same order.

For example, we have for $a \geq 0$ and $b \geq 0$

$$a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$$

so, for example,

$$\begin{aligned}
\frac{\sqrt{x+h} - \sqrt{x}}{h} &= \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\
&= \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \frac{1}{\sqrt{x+h} + \sqrt{x}}.
\end{aligned}$$

Another example is

$$a - b = (\sqrt[3]{a} - \sqrt[3]{b}) \times ((\sqrt[3]{a})^2 + \sqrt[3]{a}\sqrt[3]{b} + (\sqrt[3]{b})^2)$$

so

$$\begin{aligned}
\frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} &= \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \times \frac{(\sqrt[3]{x+h})^2 + \sqrt[3]{x+h}\sqrt[3]{x} + (\sqrt[3]{x})^2}{(\sqrt[3]{x+h})^2 + \sqrt[3]{x+h}\sqrt[3]{x} + (\sqrt[3]{x})^2} \\
&= \frac{x + h - x}{h((\sqrt[3]{x+h})^2 + \sqrt[3]{x+h}\sqrt[3]{x} + (\sqrt[3]{x})^2)} \\
&= \frac{1}{(\sqrt[3]{x+h})^2 + \sqrt[3]{x+h}\sqrt[3]{x} + (\sqrt[3]{x})^2}
\end{aligned}$$