Quadratic Formula (C)

A quadratic equation is an equation of the form

$$Az^2 + Bz + C = 0 \tag{1}$$

where A ≠ 0, *B*, *C* and *z* are complex numbers. We regard *z* as unknown, and *A*, *B*, and *C* as known. The ojective is to determine *z*. Completing the square gives us

Theorem 2622 (Quadratic Formula)

If A ≠ 0, *B* and *C* are complex numbers and *z* satisfies $Az^2 + Bz + C = 0$ then

$$z = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \ or \ z = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

where $(\sqrt{B^2 - 4AC})^2 = B^2 - 4AC..$

Note that we have to be careful about the meaning of $\sqrt{B^2 - 4AC}$ when $B^2 - 4AC$ is not a non-negative real number. If *B* is a real number and AC≤ 0, then the **discriminant** $B^2 - 4AC$ is a non-negative real number, and so is its square root. If either *B* or *C* is zero, it is more efficient to solve quadratic equations by factoring.

Exercises

1.

Solve each equation for the indicated variable:

(a)
$z^2 + 3z + 2 = 0$, solve for *z*.

(b)
$x^2 + 7x + 2 = 0$, solve for *x*.

(c)
$x^2 + 7x - 2 = 0$, solve for *x*.

(d)
$x^2 + 4x + 5 = 0$, solve for *x*.

(e)
$2x^2 - 9x + 11 = 0$, solve for x.
(f)
$(2+i)x^2 + 20x + (2-i) = 0$, solve for x.
(g)
$(2+3i)x^2 + 2ix + (2-3i) = 0$, solve for x.
(h)
$2z^2 + iz + 11 = 0$, solve for z.
(i)
$2x^2 + 4xy + y^2 + 2y + 3x - 12 = 0$, solve for y.
(j)
$16x^2 + 4xy + y^2 + 2y + 3x - 12 = 0$, solve for y.
(k)
$5x^2 + 4xy + 4y^2 + 2y + 3x - 12 = 0$, solve for y.

2.
For which real numbers x are there real numbers y so that $2x^2 + 4xy + y^2 + 2y + 3x - 12 = 0$?

3.
Derive the quadratic formula by completing the square.