## Polynomials and Rational Functions (C)

A polynomial in the variable $z$ is an expression of the form

$$
a_{0}+a_{1} z+a_{2} z^{2}+\cdots+a_{M} z^{M}
$$

where $\mathrm{a}_{0}, \mathrm{a}_{1}, \cdots, \mathrm{a}_{N}$ are complex numbers. The numbers $\mathrm{a}_{0}, \mathrm{a}_{1}, \cdots, \mathrm{a}_{N}$ are called the coefficents of the polynomial. $a_{0}$ is called the constant, and $A_{N}$ is called the leading coeffient, provided $\mathrm{A}_{N} \neq 0$. The convention is not to write any term in the polynomial whose coeficient is unless all the coefficients are zero, in which case we write, and call this polynomial the zero polynomial. If $p(z)$ is a polynomial other than the zero polynomial, the highest power of the variable is called the degree of the polynomial. non-zero onstants are regarded as polynomials of degree. The degree of the zero polnomial is taken to be $-\infty$, so that degrees add when polynomials are multiplied. A polynomial whose lead coefficient is 1 is called a monic polynomial. A polynomial whose coeffients are all real numbers is called a real polynomial.

A rational function in the variable $z$ is the ratio of two polynomials in $z$. The values of $z$ which make the denominator of the rational function equal to are called the poles or singularities of the rational function. The values which make the rational function equal to are called its roots or zeros. A rational function whose numerator and denominator are polnomials of degree 1 or less are called fractional linear transformations. Of course, every polynomial is a rational function.

