Level Payment Problem (C)

One important application of formula for the sum of a geometric progression is the **Level Payment Problem**. You borrow A dollars for N periods at a rate of r per period. What should your payment P per period be?

Observe that at the end of the first period, when you make your payment you will owe

$$A(1+r) - P$$

since your new balance was $A+rA$ and you paid off P of it. At the end of the second period you will owe

$$(A(1+r) - P)(1+r) - P = A(1+r)^2 - P((1+r) + 1).$$

At the end of the third period you will owe

$$(A(1+r)^2 - P((1+r) + 1))(1+r) - P = A(1+r)^3 - P((1+r)^2 + (1+r) + 1).$$

Continuing in this fashion, we see that at the end of the N^{th} period you owe

$$A(1+r)^N - P((1+r)^{N-1} + \cdots + (1+r) + 1) = A(1+r)^N - P\frac{(1+r)^N - 1}{r}$$

if $r \neq 0$ and

$$A - NP$$

if $r = 0$. If you are to owe zero after the N^{th} payment then

$$P = A\frac{r(1+r)^N}{(r+1)^N - 1} = \frac{Ar}{1-(1+r)^{-N}}$$

if $r \neq 0$ and

$$P = \frac{A}{N}$$

if $r = 0$.