

Hidden Polynomial Equations and Substitution (C)

Just as there are hidden quadratic equations, there are also hidden polynomial equations. These are best solved by substitution. Often the substitution will contribute a side condition. For example, to find all real numbers x so that

$$e^{3x} - 4e^{2x} + 5 = 0$$

we introduce the substitution $u = e^x$ which introduces the side condition $u > 0$ to go along with the cubic equation

$$u^3 - 4u^2 + 5 = 0.$$

Since $u^3 - 4u^2 + 5 = (u+1)(u^2 - 5u+5)$, the cubic equation in u can be solved completely. However, there is no x corresponding to $u = -1$ since $e^x = -1$ has no solution for x a real number.