Factoring (C)

In most cases, however, we do not have even a disguised quadratic equation, and our only recourse is to factor the polynomial expression. We have the following useful result:

Theorem 2685

If $p(z)$ is a polynomial then $p(a) = 0$ if and only if there is a polynomial $q(z)$ so that $p(z) = (z-a)q(z)$.

This is easy to prove. If we can write $p(z) = (z-a)q(z)$ then $p(a) = (a-a)q(a) = 0$. On the other hand, suppose that

$$p(z) = A_N z^N + A_{N-1} Z^{N-1} + \cdots + A_1 z + A_0$$

and $p(a) = 0$. Then

$$p(z) = p(z) - p(a) = A_N(z^N - a^N) + A_{N-1}(z^{N-1} - a^{N-1}) + \cdots + A_1(z - a)$$

and for every positive integer M we have

$$z^M - a^M = (z-a)(z^{M-1} + z^{M-2}a + z^{n-3}a^2 + \cdots + za^{M-2} + a^{M-1})$$

so we can see that z-a divides $p(z)$. (Another use of geometric progressions!) **QED**

Once one has found one zero, say $z=a$, one finds (by long division), the polynomial q so that $p(z) = (z-a)q(z)$ and then looks for the zeros of q, since every zero of q is a zero of p. Now our problem is to find an intelligent way to find some of the zeros.