Circles and Spheres (C)

A circle is the set of all points (in a plane) at a given distance (radius) for a given point (center). If the radius is r and the center is (a,b) then (x,y) is on the circle if and only if

$$\sqrt{(x-a)^2+(y-b)^2}=r,$$

which is the same as

$$(x-a)^2 + (y-b)^2 = r^2. \tag{3}$$

(3) is called the standard equation of a circle.

A equation of the form

$$Ax^2 + Bx + Ay^2 + Cy + D = 0$$

may be the equation of a circle. To see, try to reduce it to (3) by completing the square.

Example: Is $2x^2 + 4x + 2y^2 + 12y + 4 = 0$ an equation of a circle?

The equation is now in standard form, and we can see that the center is at (-1,-3) and the radius is $2\sqrt{2}$. To graph the circle, plot the center and the endpoints of the horizontal and vertical diameters.

Exercises

1. Give the standard equation for each of the following circles. Graph each circle.
   (a)
   Center at (5,-3) and radius 5;
   (b)
   Center at (0,0) and radius 1;
   (c)
   Center at (-3, -7) and passing through the origin;
   (d)
   Center at the intersection of $3x+2y = 5$ and $2x - 7y = -5$, radius 3.

2.

Determine which of the following are circles by trying to put the equations into standard form. Graph each circle, and if the equation is not a circle explain why.

(a)

$x^2 + 2x + y^2 + 4y = 0$;

(b)

$2x^2 + 4x + 2y^2 - 4y = 0$;

(c)

$4x^2 + 2x + y^2 + 4y = 0$;

(d)

$4x^2 + 2x + 4y^2 + 4y = 40$;

(e)

$x^2 + 2x + y^2 + 4y + 10 = 0$;

3.

Illustrate graphically the solution to the inequality $x^2 + 4x + y^2 + 12x < 0$.

A sphere is the set of all points in space at a given distance (radius) from a given point (center). The standard equation of a sphere with center (a,b,c) and radius r is

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2 \qquad (4)$$

where (x,y,z) is any point on the sphere.

Exercises:

1.

Give the standard form for the equation for each of the following spheres:

(a)

Center at (0,0,0) and radius 1;

(b)

Center at (1,-2,3) and radius 8;

(c)

Center at (2,3,4) and containing (5,-3,5).

2.

By completing the square, find the center and radius of the sphere with the given equation:

(a)

$x^2 + 2x + y^2 - 2y + z^2 - 4z = 2$;

(b)

$x^2 + 4x + y^2 + 6y + z^2 + 8z = 0$

(c)

$x^2 + y^2 + 2z = -z^2$.

3.

Describe geometrically the solution to the inequality $x^2 + y^2-2y + z^2-4z > 0$.