

MATH 910, Spring, 2024: *Topics in Numerical Analysis* Numerical Methods for Differential Equations

Instructor

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Office Hours: MW 2:30 PM -3:30 PM; or by appointment

Class meeting time and location

MW, 4:00 PM-5:15 PM, EMS E140

Prerequisites

Graduate standing; Math 715 (P); or consent of instructor.

Course Description

This course introduces numerical methods for initial value problems of ordinary differential equations and boundary value and initial value problems of partial differential equations. Finite difference and finite element methods will be introduced to approximate elliptic and parabolic partial differential equations. The SOR and preconditioned conjugate gradient methods and preconditioning techniques will be studied for solving a large scale sparse linear system produced by finite difference and finite element methods.

Mode of delivery

This course will be delivered in face-to-face format. Assignments will be distributed via Canvas.

Required Textbook

- *Numerical solution of partial differential equations by the finite element method*, Claes Johnson, Cambridge, 1987, **ISBN-13: 978-0486469003**.
- *Solving PDEs in Python*, Hans Peter Langtangen and Anders Logg, Springer Open on the website <https://link.springer.com/content/pdf/10.1007%2F978-3-319-52462-7.pdf>.
- *Numerical Analysis*, Richard L. Burden and J. Douglas Faires, Thomson, Brooks/Cole, 10th Edition, 2016.

Required Software

All computer work is done in Python based on the software libraries *Scipy* (<https://scipy.org>) and *FEniCS* (<https://fenicsproject.org>).

Topics Covered

Lectures are given in Instructor's teaching notes. Homework assignments consist of theoretical problems, numerical experiments, and programming problems. The following topics are covered:

- **Initial value problems for ordinary differential equations:** Euler's method, higher-order Taylor methods, Runge-Kutta methods, multistep methods, variable step size strategies, convergence, and stability analysis.
- **Boundary value and initial value problems for elliptic and parabolic partial differential equations:** Finite difference method and finite element methods for Elliptic partial differential equations and parabolic partial differential equations.
- **Iterative and preconditioning techniques for solving large scale sparse linear systems:** Successive-over-relaxation (SOR) method, conjugate gradient method, preconditioned conjugate gradient method, symmetric Gauss-Seidel preconditioner, symmetric SOR preconditioner, and incomplete LU preconditioner.
- **Machine learning techniques for solving differential equations:** Read selected articles and discuss related research topics, which students can study in their projects.

(The instructor reserves the right to make changes to the syllabus as needed.)

Assignments and Grading

- Three homework evaluations: 60 %
- Discussion: 10 %
- Final Project: 30 %
- Grading scale: A (93-100), A- (88-92), B+ (83-87), B (78-82), C+ (73-77), C (65-72), D (55-64), F (below 55)

Credit hour policy and time allocation

The expected workload for this 3-credit course is 148 hours: 35 hours in the classroom (including lecture attendance and in-class midterm), 2 hours for the final exam, and 111 hours of homework and exam preparation (at 3 hours per class hour).

Class attendance

Attendance is required. Attendance will be taken after the first week. Each class missing is resulted in one point reduction. There is no provision for absences and missing homework due to vacations, family outings, social activities, or other special plans and appointments, etc. Absences due to illness require medical excuse on Physician's letterhead, signed by a physician.

Homework policies

Homework is required to hand in on time. Late homework has 5-points reduction out of the full 20 points per homework evaluation.

Discussion

Homework discussions will be held on selected problems. Each student needs at least two presentations on his/her work to earn the discussion points.