Math 810: Numerical Analysis I
Fall 2023

Department of Mathematical Sciences, UWM

Lecture Time and Place

4:00 PM - 5:15 PM, Monday and Wednesday. EMS Room E424A

Instructor

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Computer Lab Place

EMS Building, Room E416.

Textbook

Numerical Analysis, Richard L. Burden and J. Douglas Faires, 10th edition, Thomson, Brooks/Cole, 2015. The book supplemental materials (e.g., Matlab programs) are available on the book companion website:
https://sites.google.com/site/numericalanalysis1burden/

Goals and Learning Outcomes

Students who successfully complete this course will be able to:

1. Gain deep insights into the foundation of numerical algorithms and simulations.
2. Understand the basic concepts of numerical complexity, convergence, and stability.
3. Develop polynomial interpolant and numerical quadrature for approximating functions and computing derivatives and integrals numerically based on experimental data sets.
4. Use numerical algorithms to solve nonlinear algebraic equations and systems of linear algebraic equations.
5. Implement numerical algorithms in a programming language (MATLAB or Python).

Prerequisites

Graduate standing; Math 413(P); Math 417(P); Math 521(P); or Math 621(P); or consent of instructor.

Course Description

This course introduces polynomial interpolation and approximation; numerical differentiation and integration; direct and iterative methods for linear systems; and iterative methods for nonlinear algebraic equations.

Required Software

All computer work can be done either in MATLAB or Python. MATLAB is freely available on UWM computer labs; Jupyter Notebook can be downloaded for free at *https://jupyter.org/*.

Topics Covered

Lectures are given in Instructor's teaching notes based on the textbook. Homework assignments consist of theoretical problems, numerical experiments, and programing problems in MATLAB or Python script language. The following topics will be covered:

- **Numerical solution of nonlinear equations:** Bisection method, fixed point iteration, and Newton's method.
- **Interpolation and polynomial approximation:** Lagrange polynomial, Newton's divided-difference, Hermite interpolation, and spline interpolation.
- **Numerical differentiation and integration:** Derivative approximation formulas, numerical integration formulas, and Gaussian quadrature.
- **Direct and iterative methods for solving linear systems:** Gaussian elimination, pivoting strategies, Cholesky algorithm, LU factorization, Jacobi iterative method, Gauss-Seidel iterative method, and successive-over-relaxation (SOR) method.

Tentative Schedule

Week 1	Introduction (Chapter 1); analysis of Heron's method for computing the square root of a positive real number; part 1 for HW1. *(First class meets on September 6, 2023.)*
Week 2	Bisection method (Section 2.1); fixed-point iteration (Section 2.2); convergence (Section 2.4); part 2 for HW1.
Week 3	Newton's method and secant method (Section 2.3); convergence; rate of convergence; part 3 of HW1
Week 4	Nonlinear equation systems (Chapter 10); norms of vectors and matrices (Sections 7.1); part 1 of HW2.
Week 5	Fixed-point iteration for nonlinear system (Section 10.1); Newton's method (Section 10.2); convergence; part 2 of HW2.
Week 6	Chord method; quasi-Newton methods (Section 10.3); convergence of Chord method; part 3 of HW2.
Week 7	Finite difference methods for nonlinear problems (Section 11.4); a system of nonlinear algebraic equations arisen from a two-point nonlinear boundary value model problem; part 3 of HW2.
Week 8	Direct methods for linear algebraic systems (Chapter 6); Gaussian elimination method; LU-factorization; part 1 of HW3.
Week 9	Cholesky factorization; application to band matrices and tridiagonal matrices; part 2 of HW3.
Week 10	Iterative methods for linear algebraic systems (Chapter 7); Jacobi's method (Section 7.3); Gauss-Seidel method

	(Section 7.3); the successive-over-relaxation (SOR) method (Section 7.4); part 1 of HW4.
Week 11	Iterative methods for finite difference systems for 2D Poisson boundary value problems; part 2 of HW4.
Week 12	Polynomial interpolation approximation problems; Lagrange polynomial (Section 3.1); Newton's divided differences (Section 3.3); Hermite interpolation; error analysis; part 1 of HW5.
Week 13	Numerical stability of polynomial interpolation; spline interpolation (Section 3.5); part 2 of HW5.
Week 14	Numerical differentiation (Section 4.1); Numerical integration (Section 4.2); composite numerical integration; part 3 of HW5.
Week 15	Adaptive quadrature (Section 4.6); numerical multiple integrals (Section 4.8); numerical improper integral (Section 4.9).

Assignments and Grading

- Five homework evaluations: 50 %. Each for 10 %.
- Attendance: 10 %
- Discussion: 10 %
- Final Exam: 30 %
- Grading scale: A (93-100), A- (88-92), B+ (83-87), B (78-82), C+ (73-77), C (65-72), D (55-64), F (below 55)

Important Dates

- Homework due dates: Announced in the class.
- September 6: First class.
- October 2: Last day to drop without "W".
- November 12: Last day to drop.
- November 22 - 26: Thanksgiving Break.
- December 13: Last Class.
- Final Exam: 4:00 PM -6:00 PM, Dec. 20, 2023.

Credit hour policy and time allocation

The expected workload for this 3-credit course is 148 hours: 35 hours in the classroom (including lecture attendance and in-class midterm), 2 hours for the final exam, and 111 hours of homework and exam preparation (at 3 hours per class hour).

Class attendance

Attendance is required. Attendance will be taken after the first week. Each class missing results in a point reduction. There is no provision for absences, missing examinations, and missing homework due to vacations, family outings, social activities, other special plans and appointments, etc. Absences due to illness require a medical excuse on Physician's letterhead, signed by a physician.

Homework policies

Late homework will not be accepted. Collaboration among students is allowed but each student must submit his/her own work.

Discussion

Homework discussions will be held on selected problems. Each student needs at least two presentations on his/her work to earn the discussion points.

Make-up Policies

No make-up is given for the final exam, but an incomplete grade can be given to eligible students.

University Policies

- Students with disabilities: http://uwm.edu/arc
- A notation of "incomplete" may be given in lieu of a final grade to a student who has carried a subject successfully until the end of a semester but who, because of illness or other unusual and substantiated cause beyond the student's control, has been unable to take or complete the final examination or to complete some limited amount of term work.
- Consultation, Complaints, and Appeals: https://uwm.edu/deanofstudents/assistance/complaints/
- Discrimination conduct and consensual relationships policy: https://apps.uwm.edu/secu-policies/storage/other/SAAP%205-1.%20Discriminatory%20Conduct%20Policy.pdf
- Academic misconduct. https://uwm.edu/deanofstudents/academic-misconduct-2/

Note The instructor reserves the right to make changes to the syllabus as needed as the course progresses.