Notes on *Linear Models and Design*

Theorem 5.18 (Bose decomposition), pages 164–5: On page 165 we entertain the idea of defining $U_\emptyset = \langle 1 \rangle$. If we do this, then the statement of the theorem is simplified:

- Part (a) is still true if I or J is empty.
- In (b) the statement (5.19) is trivially true if $I = \emptyset$, and the decomposition (5.20) becomes $\mathbb{R} = \oplus_I U_I$, summing over *all* subsets, not just nonempty ones.
- Part (c) holds for the empty product, since both sides equal 1 (the empty product).

There is a note following the discussion that touches on this. Note that the basic condition of Lemma 5.7 (page 156) is automatic, as every partition is independent of the trivial partition.

This way of defining $U_\emptyset$ would also simplify the proof of Theorem 6.43 (the Fundamental Theorem of Aliasing), since letting $J = \emptyset$ in part (a) would automatically imply the final sentence (one might preface it with the phrase, "In particular"). The decomposition in Corollary 6.47 could omit $\langle 1 \rangle$ and the word "nonempty".

Caution: We are indexing the subspaces U by the index set I in the partition $\mathcal{C}_I = \vee_{i \in I} \mathcal{C}_i$. The blocks are defined on page 162 (see especially equation (5.16)). According to this definition, $\mathcal{C}_\emptyset = \{T\}$, the trivial partition, and so its incidence space (page 154) is $M_\emptyset = \langle 1 \rangle$, which would mean that $U_\emptyset = \{0\}$ if U is the space of contrast vectors in M. Thus defining $U_\emptyset = \langle 1 \rangle$ is technically inconsistent.

Definition 6.25 (page 238), the definition of resolution, should require $R \geq 1$. Resolution $R = 2$ means that main effects are preserved in the fraction, if in the definition we understand an effect with no factors to be defined by the space of constant functions – described on page 165 as "the effect of the grand mean". A fraction thus has (maximum) resolution 1 if some main effects are not preserved in the fraction.

Proposition 6.31, page 243: This is given by Cheng [1, Theorem 9.3], and should have been acknowledged.

Index, page 337: Combine the R(X) entries in the notation list.

References

[1] Ching-Shui Cheng. *Theory of Factorial Design: Single- and Mult-Stratum Experiments.* CRC Press, Boca Raton, FL, 2014.