

## THE FOUR REGULAR MOSAICS MISSING IN THE ALHAMBRA

R. PÉREZ-GÓMEZ

c/Sevilla, n6, 3A, 18003 Granada, Spain

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**Abstract**—Several investigations assert the existence of regular mosaics as a representation of each of the 17 groups of plane symmetry in The Alhambra (see, for example, Refs [1]–[3]). However, various specialists have only found some of them. Branko Grünbaum *et al.* [4] point out that: (i) it was Coxeter who enhanced the list already acknowledged by Edith Müller (11 groups) by founding two more groups: pm and p31m; (ii) the groups p2, pg, pgg and p3m1 are missing in The Alhambra.

This paper announces the discovery of them in The Alhambra. As a consequence, we agree with Grünbaum's opinion, as stated in Ref. [4], about the infrequent use of these groups.

“On all walls a sea of tiles waves”, Ibn Al-Jatib [5]

### INTRODUCTION

The main decorative elements of any hall in The Alhambra are the floor, the ceiling, the base and the surface of the walls (arabesques, stuccoes etc.). There are also poems which belong to the decoration of the walls, these poems praise God as well as the king who ordered the palace to be built, and describe the room as well as its use. In the poems [5], the floor is once called “cloth” (carpet) due to the fact that it had been floored with glazed tiles, covered with drawings, or with plain ceramic floortiles, sometimes interlaced with wires. The remainders of these are rare and they are in bad condition. The bases of the walls are said to be tiles, and finally, the coffered ceilings are considered to have no details.

Part of these decorative elements are in the Museum of The Alhambra, which has been open to the public since 1943, and the other great part has been restored lately. The reasons why Edith Müller did not find the groups pointed out by Grünbaum [4] could be:

1. The Museum was opened to the public one year before she published her study.
2. The tiling named in Ref. [4], group pm, is in the “Golden Yard”. It is a restoration made during 1965 (there are photographs [6]; this tiling did not exist when Edith Müller visited The Alhambra).

### REMARKS AND A DESCRIPTION OF EACH DISCOVERED GROUP

#### *The p2 group*

The mosaic shown in Fig. 1, is an example to be classified in this group. It belongs to the Museum of The Alhambra, and it is catalogued with the number 1361. Figure 1 is an interlace pattern with two colours, black and white. There are two types of halfturn centres. There are neither 4-fold centres nor reflection axes; indeed, these are destroyed because the layout presents one superposition line in each 4-fold centre possible, as well as in each halfturn centre (see Fig. 2). The base is a cross ruling, and the lace width is constant (see Fig. 3).

Curiously, notice that the  $\sqrt{2}$  was computed from the number 7/5, and we believe it decides the width of the lace, 1/5. The generators for the plane symmetry group p2 are:

- (i) a rotation of centre O and  $\pi$  rad;
- (ii) two translation vectors  $\mathbf{u} = 2 \cdot \mathbf{AB}$  and  $\mathbf{v} = \mathbf{AD}$ ;

A, B, C and D being the vertices of the generating region and O the middle point of AD.

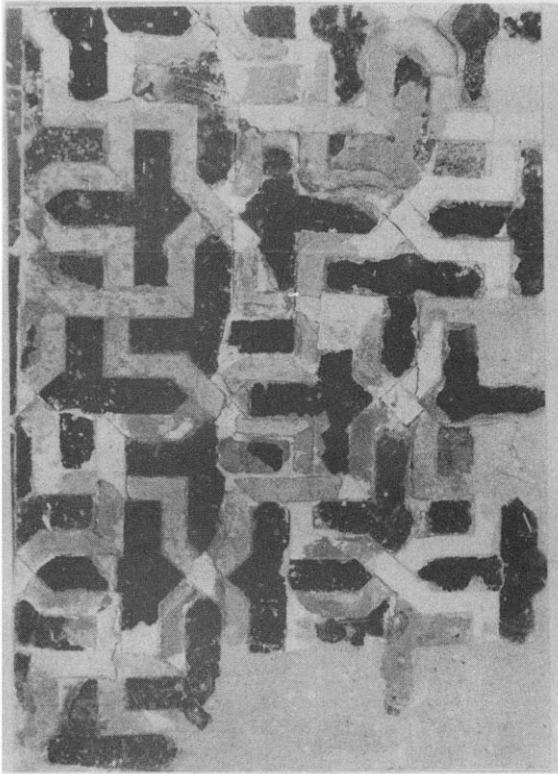


Fig. 1

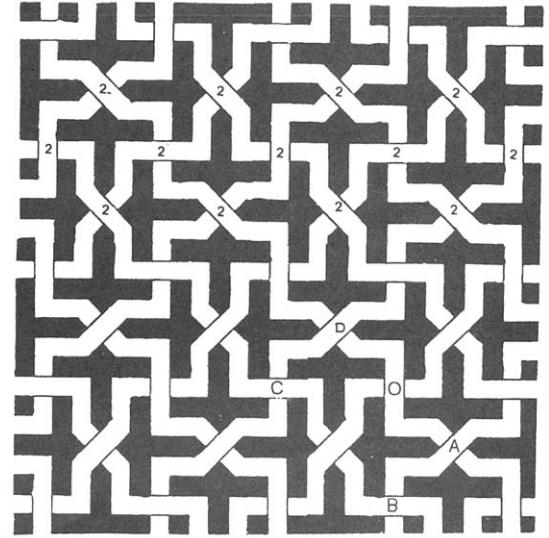


Fig. 2

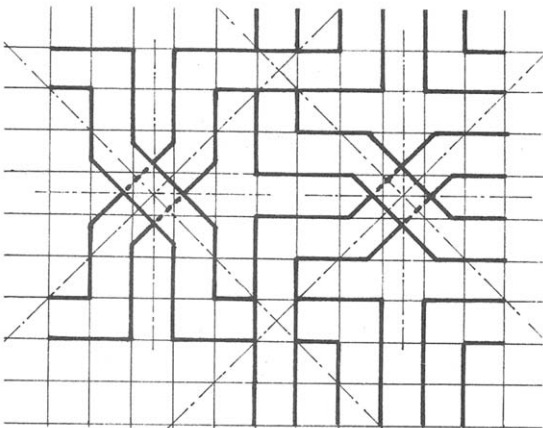


Fig. 3

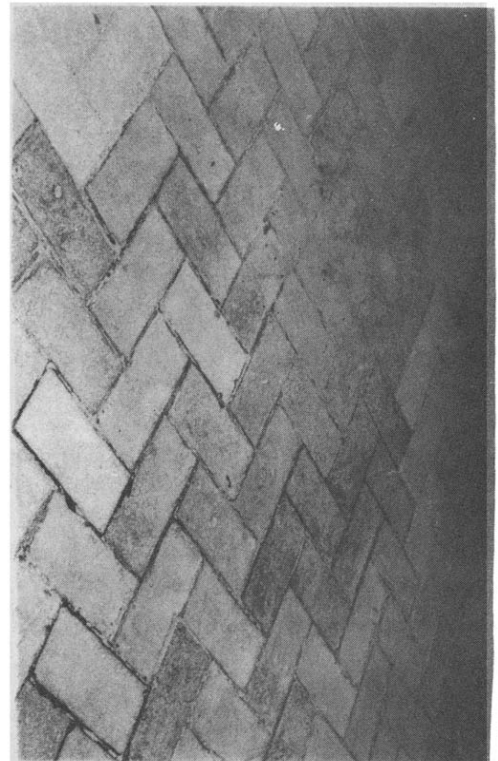


Fig. 4

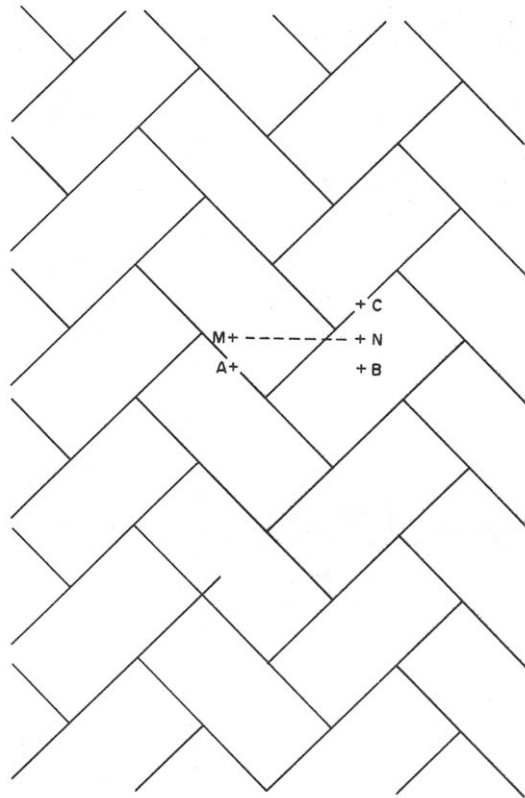
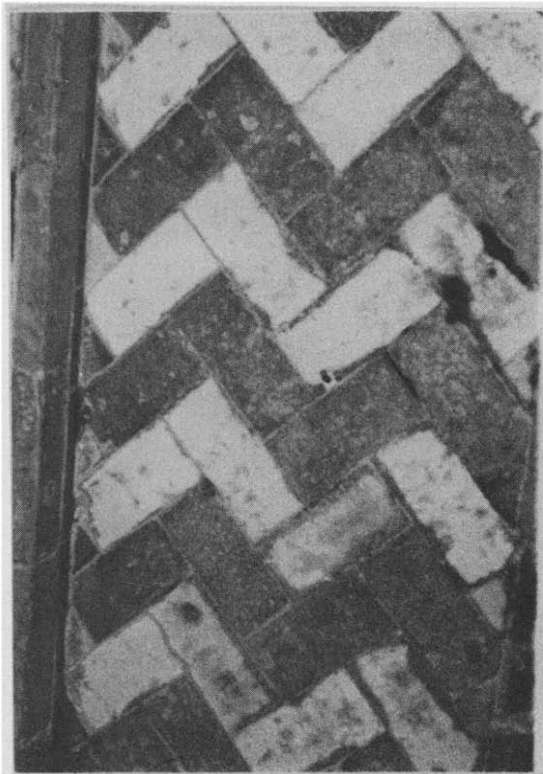
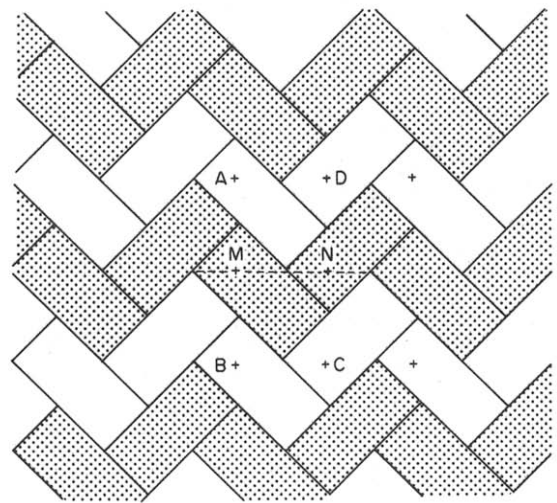


Fig. 5



(a)



(b)

Fig. 6

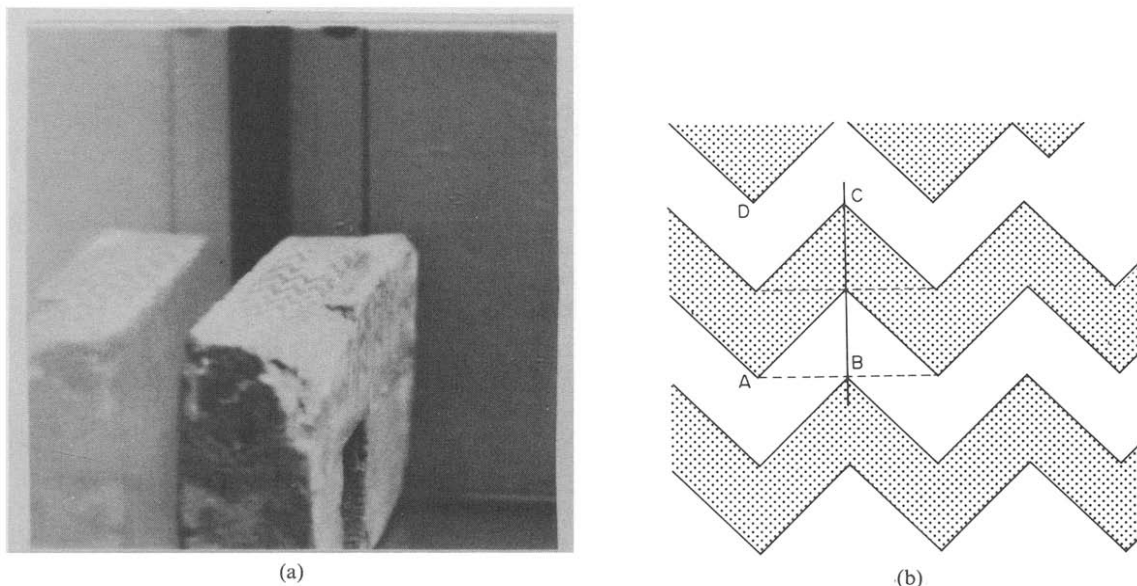


Fig. 7

The next three patterns are based on the same idea. In the first two the laying of the tiles and their colours decide different patterns, *pgg* and *pg*, so far not seen in The Alhambra. The third is a fresco.

#### *The pgg group*

This mosaic is presented as an example for its classification in this group; it can be found in several floors of halls (i.e. Hall of the "Aljimeces") and ceilings (i.e. The Door of the Vine).

The lattice in Fig. 4 is a rectangle, it presents 2-fold rotations, and it does not have reflection axes but it does have glide reflection axes which are drawn with a discontinuous line in Fig. 5.

The generators for the plane symmetry group *pgg* are:

- (i) a non-trivial glide reflection, axis  $MN$  and vector  $\mathbf{AB}$ ;
- (ii) a rotation of centre  $C$  (or  $A$ ) and  $\pi$  rad.

As a consequence, the translation vectors are  $\mathbf{u} = 2 \mathbf{AB}$  and  $\mathbf{v} = 2 \mathbf{AD}$ ;  $A$ ,  $B$ ,  $C$  and  $D$  being the vertices of the generating region;  $M$  (resp.  $N$ ) is the middle point of  $AD$  (resp.  $BC$ ).

#### *The pg group*

We have a representation of this group in the ceramic floor of the lateral chamber of the Hall of the Abencerrajes, belonging to the Palace of the Lions. It is formed by two coloured ceramic tiles, white and green. There are no halfturns centres [those indicated in the previous pattern are destroyed by the colouring (see Fig. 6)].

The generators for the plane symmetry group *pg* are

- (i) a non-trivial glide reflection, axis  $MN$  and vector  $\mathbf{BC}$ ;
- (ii) a translation vector:  $\mathbf{v} = \mathbf{AB}$  (orthogonal to  $\mathbf{BC}$ ).

As a consequence, the translation vectors are  $\mathbf{u} = 2 \mathbf{BC}$  and  $\mathbf{v}$ .  $A$ ,  $B$ ,  $C$  and  $D$  are the vertices of the generating region;  $M$  (resp.  $N$ ) is the middle point of  $AD$  (resp.  $BC$ ).

#### *The pmg group*

We point out a third variant about the previous patterns, *pg* and *pgg*. It is a fresco which is exhibited in the Museum of Alhambra (Hall IX, show window). It's obtained by leaving a part of the borders of the tiles and then by considering only the colouring (see Fig. 7).

There are halfturns centres, parallel reflection axes and two independent translations.

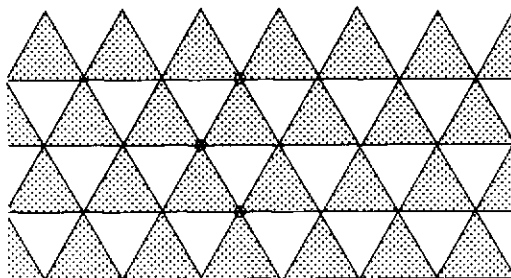


Fig. 8

### The $p3m1$ group

During a recent visit, Professor J. Montesinos found one representation of the group  $p3m1$  in a chair which was part of the furniture of The Alhambra. This chair is in the Museum of The Alhambra. When we looked at it, we remembered that there are some patterns in the "mucarnas" of entry arc (south side) to the Hall of the "Abencerrajes". It has small dimensions (about  $10 \times 5$  cm) and there are several of them.

The lattice is a parallelogram. It presents 3-fold rotations and it has reflection axes passing through any centre. The generators for the plane symmetry group  $p3m1$  are:

- (i) a rotation of centre  $O$  and  $2\pi/3$  rad;
- (ii) two translation vectors  $\mathbf{u} = \mathbf{OO}'$  and  $\mathbf{v} = \mathbf{OO}''$ ;

$O, O', O''$  being the 3-fold rotation centres. This group can be distinguished from the  $p31m$  group since any rotation centre belongs to some reflection axis (see Fig. 8).

This discussion can be summarized as follows: the 17 wallpaper groups occur as symmetry groups of ornaments in The Alhambra.

### REFERENCES

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