TITLE: Are all high-dimensional contractible open manifolds Gabai splittable?

ABSTRACT: A space which is homeomorphic to $\mathbb{R}^n$ is called an *open n-ball.* A contractible open n-manifold is *Gabai splittable* or *Gabeyed* if it is the union of two open n-balls U and V such that U ∩ V is also an open n-ball. (• ( ) •)

Known results:

D. Gabai 2011: The Whitehead contractible open 3-manifold is Gabeyed.

D. Garity, D. Repovs, D. Wright 2013: **a)** There exist uncountably many topologically distinct Gabeyed contractible open 3-manifolds. **b)** There exist uncountably many topologically distinct contractible open 3-manifolds which are not Gabeyed. (These contractible open 3-manifolds are *not* the union of two open 3-balls.)

P. Sparks 2013: There exist uncountably many topologically distinct Gabeyed contractible open 4-manifolds.

Observations (F. Ancel and C. Guilbault)**:** Let $n \geq 5$.

- The interior of every compact contractible n-manifold is Gabeyed.
- There exist uncountably many topologically distinct Gabeyed contractible open n-manifolds.
- Every n-dimensional Davis manifold is Gabeyed.
- Every contractible open n-manifold is the union of two open n-balls U and V such that U ∩ V is contractible.

Questions:

- For $n \geq 5$, is every contractible open n-manifold Gabeyed?
- Is every contractible open 4-manifold the union of two open 4-balls?