Some Relevant Additional Problems on Product Spaces

Problem V.1+. Prove that if X is a separable regular space, then X has a basis $\mathscr{B}$ such that $\mathscr{B} \preceq \mathbb{R}$.

In Problems V.2+, V.3+ and V.4+, let X be a set and let $\{ 0, 1 \}^X$ have the product topology.

Problem V.2+. Prove that if $X \preceq \mathbb{R}$, then $\{ 0, 1 \}^X$ is separable.

Problem V.3+. Suppose $X \succ \mathbb{R}$ and $\mathscr{B}$ is any basis for $\{ 0, 1 \}^X$.

a) Prove $\mathscr{B} \succ \mathbb{R}$.

b) Prove $\{ 0, 1 \}^X$ is not separable.

Problem V.4+. Prove that every pairwise disjoint collection of open subsets of $\{ 0, 1 \}^X$ is countable.