

MATH 751

INTRODUCTORY TOPOLOGY

FALL 2010

TR 9:30-10:45

PHYSICS 149

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OFFICE HOURS: TR 2:00 - 2:50, and other times by appointment.

COURSE NOTES: available at <https://pantherfile.uwm.edu/ancel/www/>

OPTIONAL TEXT: *Topology: Second Edition* by James R. Munkres

COURSE OBJECTIVES: Topology is the study of sets equipped with structures called "topologies" which allow one to take limits. A set equipped with a topology is called a "topological space". Topological spaces exist in fantastic variety, illustrating many different phenomena. One type of topological space - manifolds - is a primary focus of modern topology. A manifold is a space which looks locally like a Euclidean space or, more generally, like a topological vector space. Manifolds include Euclidean spaces, Hilbert and Banach spaces, Lie groups, etc. Because of the ubiquity of manifolds, topology finds application in analysis, algebra, geometry and beyond that in physics, chemistry, economics, etc.

Today, topology is broken into three major divisions: general topology, algebraic topology, and geometric topology. General topology investigates the axiomatic foundations of topological spaces and the effects of varying the axioms on these spaces. It delves into properties such as compactness, connectedness and continuity which are fundamental to the rest of topology, as well as to analysis, geometry and parts of algebra. Geometric topology is devoted to the study of manifolds in all their manifestations by intrinsic means. Extrinsic algebraic objects such as groups and rings can be naturally associated to a manifold in a way that illuminates its global structure. The study of these algebraic objects is the preserve of algebraic topology. A newer subarea of topology called geometric group theory in which methods of topology and geometry are used to study algebraic objects called groups has recently become a focus of research for topologists at UWM.

The primary goal of the Math 751 is to acquaint students with the fundamental concepts and results of general topology so that they can apply these tools in future studies of other parts of topology and other areas of mathematics and science. The course surveys basic ideas of general topology: topological spaces, bases, separation properties, metric spaces, continuity, compactness, and connectedness. [This corresponds to parts of chapters 2 – 4 of the optional text.] The sequel course, Math 752, continues with several additional topics from general topology. Then, as time permits, it introduces some basic ideas from algebraic and geometric topology.

An important pedagogical goal of the course is to develop in students an appreciation for and a facility with the method by which pure mathematics acquires knowledge: the

regime of conjecture followed by either a proof (resulting in a theorem) or a counterexample. In particular, the course aims to nurture students' ability to understand and create sophisticated mathematical proofs.

CLASSROOM ACTIVITIES: The goal of teaching topology while developing the student's facility with the conjecture–proof–counterexample regime will be pursued in the classroom. The students will be handed notes containing a list of topology problems. The students are expected to work on these problems, to claim solutions to those problems which they believe they have solved correctly, and to present their solutions orally in class. (They will also hand in written forms of their solutions for critical evaluation.) Each week, a large proportion of classroom time will be devoted to student presentations of their solutions of the problems from the class notes.

GRADES AND EXAMS. Each student's grades will be based with equal weight on

- his/her in-class presentation of solutions to problems in the course notes and
- on his/her performance on two in-class exams and the final exam.

The tentative exam dates are:

EXAM 1: Tuesday, October 5

EXAM 2: Tuesday, November 9

FINAL EXAM: Monday, December 20, 10:00 – 12:00 noon

ADDITIONAL PROBLEMS: Also a list of more challenging "Additional Problems" will be distributed. Students are encouraged to solve these and write up and hand in their solutions for extra credit.

SUGGESTIONS FOR PREPARING PROOFS: When you prepare an oral or written proof, please consider the following issues. **The burden is on you to convince your audience that you know how to do the proof.**

Do not make unsupported assertions. Do not expect your audience to read your mind. If you do not present the correct argument for an assertion, I will assume that you don't know a correct argument. If the argument in question is similar to an argument you have presented earlier, you should either repeat or refer to the earlier argument so that your audience has a clear idea of what is in your mind. (This is particularly important on written tests. Remember that when I grade tests, I read many proofs, and may not recall your earlier argument.)

Do not introduce notation without defining or explaining it.

Proofread your work carefully. I will deduct points for careless errors.

For work prepared outside of class: don't submit your first draft. Prepare a neat final draft.

The burden is on you to make a well organized and neat presentation of the proof. I will deduct points for sloppiness and logical disorganization.